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A Risk-Free Portfolio with Risky Assets

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Abstract

Portfolio theory has traditionally started from the assumption that a portfolio can be separated into risk-free and risky assets. Jarrow (1988) proposes an alternative to the Capital Asset Pricing Model (CAPM) and the Security Market Line (SML) based on the definition given by Harry Markowitz for the mean-variance efficient frontier, and then uses the same mean-variance methodology to elaborate a test to measure the efficient frontier without the existence of risk-free assets. In a recent study, Parada (2008) develops some propositions for building a portfolio made up of risky assets to substitute a risk-free asset, further determining the proportions that should be invested to generate this portfolio and analyzing the construction of a portfolio to substitute the market portfolio. The present article builds on this earlier work to develop the implications of forming risk-free portfolios made up solely of risky assets.

Keywords: Risk-free portfolio, alternative portfolio, CAPM, mean-variance, efficient frontier
JEL Classification Codes: G11

Introduction

The title of this article appears to be a play on words, since it seems strange – a conceptual contradiction – to maintain that the performance of a risk-free asset can be obtained with a portfolio made up of risky assets. Nonetheless Parada (2008) shows that such situations can occur, generating a more global look at portfolio theory and the regulatory definition of risk-free assets. This issue is tackled herein from a more practical perspective – how to obtain a risk-free portfolio – allowing the theory to be broadened regarding the consideration of only "zero Beta" assets as risk-free. The study is extended to include the generation of portfolios that are alternatives to the market portfolio when this has been exhausted, explaining how to obtain this new portfolio.

The Capital Asset Pricing Model (CAPM) is centered on the Security Market Line (SML), on which the financial assets are located. Also important is the Capital Market Line, which is obtained by investing a proportion of the resources in the market portfolio and another proportion in a risk-free asset, such that the returns from this new asset are the weighted returns of the proportion of each individual asset. Black (1972) demonstrated that the CAPM model is met even without the existence of a risk-free asset; this asset is defined as having $\beta = 0$. In the model, the market portfolio is also defined as a portfolio that is a risky asset with $\beta = 1$ (Jarrow, 1988). CAPM requires risk-free assets (that is, with $\beta = 0$) and assets from the market portfolio (that is, with $\beta = 1$).

Jarrow (1988) poses an alternative to the CAPM and the Capital Market Line based on Markowitz's (1958) definition of the mean-variance efficient frontier (M-V) and, using the same M-V methodology, re-elaborates the Capital Market Line using non-linear optimization. Kandel (1984) elaborates a test to measure the efficient frontier without the existence of risk-free assets. Elton and Gruber (1995) develop the case of the efficient frontier based on "short sales" and classic methods of determination. Parada (2008) reconsiders the matter based on these approaches, coming to the conclusion that it is possible to form a portfolio with returns equivalent to those of a risk-free asset by mixing risky assets, and that this new portfolio has zero risk. This portfolio consists of a risky asset and is financed with own resources and loans or disinvesting in a risky asset.

I. Equivalent Portfolio for a Risk-free Asset

An equivalent portfolio is understood to be a portfolio made up of two risky assets that, on average, generate the same return as a risk-free asset. This new portfolio has zero risk and a Beta equal to 0. Generating a portfolio equivalent to a risk-free portfolio requires meeting the following suppositions: CAPM and M-V must be met and the two financial assets chosen must be located exactly on the SML; that is, having perfectly correlated performances. These assets can have different Beta coefficients and these are not zero.

The above implies that two risky assets (1 and 2) can exist with the following returns: $E(R_1) = R_F + \beta_1 [E(R_m) - R_F]$ and $E(R_2) = R_F + \beta_2 [E(R_m) - R_F]$ and only systematic risk, given by the following expressions: $\sigma_1^2 = \beta_1^2 \sigma_m^2$ and $\sigma_2^2 = \beta_2^2 \sigma_m^2$. By applying the methodology of quadratic optimization, Parada (2008) obtains the following results:

$$x^*_1 = \beta_2/(\beta_2 - \beta_1) \tag{1}$$

$$x^*_2 = -\beta_1/(\beta_2 - \beta_1) \tag{2}$$

The above indicates that investing resources in a portfolio made up of risky assets 1 and 2 in the proportions of x^*_1 and x^*_2 gives a portfolio with a performance equivalent to that of a risk-free asset, R_F, with a variance equal to zero, that is, with zero risk. This implies that the new portfolio has a return equivalent to that of a risk-free asset, as well as the same risk.

Let us suppose that the market portfolio return is $(R_M) = 9\%$ and the return of a risk-free asset $(R_F) = 4\%$. We have two risky financial assets with different Beta coefficients and whose returns are obtained according to the CAPM model. Table 1 shows the proportions (x_1 and x_2) that should be invested in two assets to form any eight portfolios whose Betas are known; the proportions to invest in each one of these is calculated according to Formulas 1 and 2 and the risk of each new portfolio is calculated considering only the systematic risk, that is, $\beta_i\sigma_i^2$.[1]

[1] The risk of the portfolio is calculated according to the formula: $(x^*_1)^2(\sigma_1)^2 + (x^*_2)^2(\sigma_2)^2 + 2(x^*_1)(x^*_2)\,\sigma_{1,2}$. For this case, as the assets are on the SML, then they comply with $(\sigma_i)^2 = (\beta_i\sigma_M)^2$ and $\sigma_{1,2} = \beta_1\beta_2\sigma^2_M$

Table 1: Risk-free portfolios made up of risky assets

Portfolios		1	2	3	4	5	6	7	8
β_1		0.0	0.4	0.6	0.8	1.0	1.1	1.5	1.8
β_2		0.1	0.5	0.8	0.9	0.8	0.8	1.2	1.5
x^*_1	(1)	1.0	5	4	9	-4	-2.66	-4	-5
x^*_2	(1)	0	-4	-3	-8	5	3.66	5	6
R_1	(2)	4.0%	6.0%	7.0%	8.0%	9.0%	9.5%	11.5%	13.0%
R_2	(2)	4.5%	6.5%	8.0%	8.5%	8.0%	8.0%	10.0%	11.5%
R_p=R_F	(2)	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%
σ_p^2	(3)	0%	0%	0%	0%	0%	0%	0%	0%
β_p	(4)	0	0	0	0	0	0	0	0

(1) $x^*_1 = \beta_2/(\beta_2 - \beta_1)$ and $x^*_2 = -\beta_1/(\beta_2 - \beta_1)$

(2) R_i =Return from Asset i = $R_F + \beta_i(R_M - R_F)$ and R_p = Return from Portfolio = $x^*_1R_1 + x^*_2R_2$

(3) σ_p^2 = Risk of Portfolio = $(x^*_1)^2(\beta_1\sigma_M)^2 + (x^*_2)^2(\beta_2\sigma_M)^2 + 2(x^*_1)(x^*_2)(\beta_1\beta_2\sigma^2_M)$

(4) $\beta_p = (x^*_1)\,\beta_1 + (x^*_2)\,\beta_2$

Table 1 shows that, for portfolios made up of different risky assets – and thus with different Betas – the return is equal to that of a risk-free asset and the risk is equal to zero. For example, portfolio 2 is made up of two assets, one with β = 0.4 and the other with β = 0.5; the proportion to invest in the first is 5 and in the second –4, resulting in 4% return and 0% risk. The same return (4%), that of a risk-free asset, and a risk of 0% are obtained for portfolio 8, with very different Betas (1.8 and 1.5). This occurs in any of the portfolios constituted.

By solving algebraically for (1) and then graphing it, we get:

$$x_1 = \frac{1}{1-\beta_1/\beta_2} \tag{3}$$

Graph 1

Graph 1 reveals the two following situations:

a) If: $0 < \beta_1/\beta_2$ risk < risk 1, then we should invest in asset 1 and disinvest in asset 2 to make up a portfolio that has a return equivalent to that of a risk-free asset and that has zero risk: in other words, an equivalent portfolio. We should invest $1/(1 - \beta_1/\beta_2)$ in asset 1 and disinvest in asset 2 in a proportion equal to $-(\beta_1/\beta_2)/(1 - \beta_1/\beta_2)$.

b) If $\beta_1/\beta_2 > 1$, we should invest in asset 2 in a proportion of: $-(\beta_1/\beta_2)/(1 - \beta_1/\beta_2)$ and disinvest in asset 1 in a proportion equal to $1/(1 - \beta_1/\beta_2)$.

In order to clarify the above with data in money, let us suppose that we have two risky assets with $\beta_1 = 0.45$ and $\beta_2 = 0.65$ and we have \$500 in own resources to invest in a portfolio and that the returns of this are obtained through the CAPM model. What is the equivalent portfolio to a risk-free asset? Let us suppose that: $R_F = 0.10$ and $E(R_m) = 0.15$; by replacing these values in equations (1) and (2), we get:

$x^*_1 = 0.65/(0.65-0.45) = 3.25$ $x^*_2 = -0.45/(0.65-0.45) = -2.25$

This indicates that for each \$1 of own resources, we should borrow (or sell, if this asset is in the current portfolio) 2.25 times that in order to finance the investment of 3.25 in a risky asset and obtain a net result of a return rate equal to the risk-free rate. The financing (loan or sale of the asset) is assumed to be obtained at a cost set according to the CAPM model. In the case of disinvestment in this asset, the cost would be the earnings lost due to the sale whereas, in the case of a loan, it would be the lender's charges according to the Beta. The following table presents this situation.

	Asset	Proportion	Money
Investment:	1	3.25	\$1,625 (3.25x\$500)
Financing:			
Loan or disinvestment	2	2.25	\$1,125 (2.25x\$500)
Own resources		1.00	\$ 500 (1.00x\$500)

Total financing	**3.25**	**$1,625**

Net Result of a portfolio equivalent to a risk-free asset:		
Cash Flow from return of investment:	1.625[0.10+0.45(0.15-0.10)]	= $199.0625
-Cash Flow from cost of financing:	**1.125[0.10+0.65(0.15-0.10)]**	**= $149.0625**
Net Cash Flow:	**199.0625-$149.0625**	**= $ 50.0000**

Returns from own resources = Net Cash Flow/Own Resources = $50/500=0.10=$R_F$. That is, the alternative portfolio provides the same return as if the investor had invested the $500 in a risk-free asset. In the example, the investment of x_1 generates a return of 12.25%, or $199.0625 in money, and external financing (x_2) has a cost of 13.25%, or $149.0625 in money. This has two interpretations: one for the case of disinvestment in this asset (equivalent to earnings lost due to selling) and another for the case of a loan (the cost charged by the lender that has a given Beta) and requires at least the return that will be obtained according to the determination of the CAPM model.

Given the simplicity of formulas (1) and (2), we can tabulate the data for different combinations of two assets with different Betas. In fact, Table 2 shows the proportion (as a percent) that should be invested or disinvested (or loaned) in asset 1 for different combinations of two financial assets with their respective Betas. Obviously, the difference is the proportion to invest or disinvest in asset 2, applying the following equation: $x^*_1 + x^*_2 = 1$. For example, for a portfolio made up of a first asset with $\beta = 0.6$ and the second asset with $\beta = 0.8$, we should invest 4 in asset 1 and, hence, –3 in asset 2. Note that, when $\beta = 0$, which is the classic definition of a risk-free asset, then we should invest 100% in this asset. The table indicates that that this definition is used only in the particular case of a risk-free asset, since this can be obtained with a combination of risky assets, as shown in the first row or first column of Table 2.

Table 2: Investment in a risky asset (x_1) for a known B_1 and B_2 To obtain a risk-free portfolio

B2 \ B1	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0	-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.2	1.000	-	-1.000	-0.500	-0.333	-0.250	-0.200	-0.167	-0.143	-0.125	-0.111
0.4	1.000	2.000	-	-2.000	-1.000	-0.667	-0.500	-0.400	-0.333	-0.286	-0.250
0.6	1.000	1.500	3.000	-	-3.000	-1.500	-1.000	-0.750	-0.600	-0.500	-0.429
0.8	1.000	1.333	2.000	4.000	-	-4.000	-2.000	-1.333	-1.000	-0.800	-0.667
1.0	1.000	1.250	1.667	2.500	5.000	-	-5.000	-2.500	-1.667	-1.250	-1.000
1.2	1.000	1.200	1.500	2.000	3.000	6.000	-	-6.000	-3.000	-2.000	-1.500
1.4	1.000	1.167	1.400	1.750	2.333	3.500	7.000	-	-7.000	-3.500	-2.333
1.6	1.000	1.143	1.333	1.600	2.000	2.667	4.000	8.000	-	-8.000	-4.000
1.8	1.000	1.125	1.286	1.500	1.800	2.250	3.000	4.500	9.000	-	-9.000
2.0	1.000	1.111	1.250	1.429	1.667	2.000	2.500	3.333	5.000	10.000	-

II. Equivalent Portfolio to the Market Portfolio

Parada (2008) also shows that the same methodological exercise can be used to create alternative market portfolios that include risky assets; these portfolios have $\beta \neq 1$ and assume the same risk and return as a market portfolio with a $\beta = 1$. Using the same optimization method and the suppositions of CAPM and M-V, the following proportions should be used for investing in any portfolio made up of two risky assets in order to obtain a performance equivalent to that obtained by the market portfolio:

$$x^*_1 = (\beta_2-1)/(\beta_2-\beta_1) \qquad (4)$$

$$x^*_2 = (1-\beta_1)/(\beta_2-\beta_1) \qquad (5)$$

If a portfolio is made with x^*_1 and x^*_2, the return is $E(R_p)$ and the risk is σ^2_p:

By solving, we get: $E(R_p) = E(R_m)$ and

$$E(R_p) = [R_F + \beta_1(E(R_m) - R_F)](\beta_2 - 1)/(\beta_2 - \beta_1) + [R_F + \beta_2(E(R_m) - R_F)](1 - \beta_1)/(\beta_2 - \beta_1)$$

$$\sigma_p^2 = \left(\frac{\beta_2 - 1}{\beta_2 - \beta_1}\right)^2 (\beta_1\sigma_m)^2 + \left(\frac{1 - \beta_1}{\beta_2 - \beta_1}\right)^2 (\beta_2\sigma_m)^2 + 2\left(\frac{\beta_2 - 1}{\beta_2 - \beta_1}\right)\left(\frac{1 - \beta_1}{\beta_2 - \beta_1}\right)(\beta_1\beta_2\sigma_m^2)$$

The risk is expressed by the following equation:

$$\sigma_p^2 = \left(\frac{\beta_2 - 1}{\beta_2 - \beta_1}\right)^2 (\beta_1\sigma_m)^2 + \left(\frac{1 - \beta_1}{\beta_2 - \beta_1}\right)^2 (\beta_2\sigma_m)^2 + 2\left(\frac{\beta_2 - 1}{\beta_2 - \beta_1}\right)\left(\frac{1 - \beta_1}{\beta_2 - \beta_1}\right)(\beta_1\beta_2\sigma_m^2)$$

By solving and reducing the above expression, this becomes: $\sigma^2_p = \sigma^2_m$

That is, the new portfolio made up of x^*_1 and x^*_2 has an expected return equal to the return of a market portfolio, $E(R_m)$, with the same risk as this portfolio, that is, σ^2_m. In (4) and (5), we can see that a $\beta_i = 1$ would be a particular case, with the SML coinciding exactly with the efficient frontier.

Table 3 shows eight different portfolios made up of two risky assets. Each has a weighted return equal to that of a market portfolio, with the same risk and a weighted $\beta = 1$, the Beta of a market portfolio.

Table 3: Alternative portfolios to the market portfolio

Portfolios	1	2	3	4	5	6	7	8
β_1	0.0	0.4	0.6	0.8	1.0	1.1	1.5	1.8
β_2	0.1	0.5	0.8	0.9	0.8	0.8	1.2	1.5
x^*_1 (1)	-9	-5	-1	-1	1	0.67	-0.67	-1.67
x^*_2 (1)	10	6	2	2	0	0.33	1.67	2.67
R_1 (2)	4.0%	6.0%	7.0%	8.0%	9.0%	9.5%	11.5%	13.0%
R_2 (2)	4.5%	6.5%	8.0%	8.5%	8.0%	8.0%	10.0%	11.5%
R_p=R_F (2)	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%	9.0%
σ_p^2 (3)	σ^2_m	σ^2_m	σ^2_m	σ^2_m	σ^2_m	σ^2_m	σ^2_m	σ^2_m
β_p (4)	1	1	1	1	1	1	1	1

(1) $x^*1 = (\beta2 - 1)/(\beta2 - \beta1)$ and $x^*2 = (1 - \beta1)/(\beta2 - \beta1)$

(2) R_i = Return from Asset i = $R_F + \beta_i(R_M - R_F)$ and R_p = Return from Portfolio = $x^*_1R_1 + x^*_2R_2$

(3) σ_p^2 = Risk of Portfolio = $(x^*_1)^2(\beta_1\sigma_M)^2 + (x^*_2)^2(\beta_2\sigma_M)^2 + 2(x^*_1)(x^*_2)(\beta_1\beta_2\sigma^2_M)$

(4) $\beta_p = (x^*_1)\,\beta_1 + (x^*_2)\,\beta_2$

As seen in Table 3, whatever the Beta of the two assets, the final result is that the alternative portfolio has a weighted return of 9%, equal to the return of the market portfolio. At the same time, it has the same risk expressed through the variance of the portfolio and all the portfolios have a weighted $\beta = 1$, the Beta of a market portfolio.

The following example is presented to clarify the above case using money. We have two assets that are on the SML with $\beta_1 = 0.4$ and $\beta_2 = 0.6$ and $1,500 in own resources to invest in a portfolio. In order to generate an alternative portfolio to the market portfolio, the proportions to invest in assets 1 and 2 are calculated as follows: replacing the values of β_1 and β_2 in (4) and (5), we get the following proportions:

$$x_1^* = \frac{\beta_2 - 1}{\beta_2 - \beta_1} = \frac{0.6 - 1}{0.6 - 0.4} = -2 \quad x_2^* = \frac{1 - \beta_1}{\beta_2 - \beta_1} = \frac{1 - 0.4}{0.6 - 0.4} = 3$$

This can be expressed in money as:

	Asset	Proportion	Money
Investment in Asset 2 Financing:	2	3	$4,500 (3x$1,500)
Loan or Disinvestment	1	2	$3,000 (2x$1,500)
Own Resources		1	$1,500 (1x$1,500)
Total Financing		3	$4,500

Net Result of the Alternative Portfolio:

Cash Flow per Return on Investment:	\$4,500[0.10 + 0.6(0.15 - 0.10)] = \$585
Minus: Cash Flow per cost of Financing	\$3,000[0.10 + 0.4(0.15 - 0.10)] =-\$360
Net Cash Flow	\$225

Return of Own Resources= \$225/\$1,500=0.15=$E(R_m)$

The risk of this portfolio is: $(-2)^2(0.4\sigma_m)^2 + 3^2(0.6\sigma_m)^2 + 2(-2)(3)(0.4)(0.6)\sigma_m^2 = \sigma_m^2$

If we had invested \$1,500 of own resources directly in a market portfolio, that is, with a $\beta = 1$, its earnings in money would have been \$1,500 [0.10 + (0.15-0.10)] = \$225, which is equivalent to the return of the alternative to the market portfolio, calculated using expressions (4) and (5).

The two previous cases are particular situations of a more general position that consists of determining alternative portfolios with any Betas and to obtain any return, be it market or risk-free. Brennan (1971) offers an approach for the formation of portfolios with loans at different rates of interest based on the efficient frontier. In another work, Jarrow (1988) presents an efficient frontier approach and its relationship with CAPM through theorems. Both lenses unite the two models. Thus, Parada (2008) poses the following question: Is there, for any efficient portfolio, a point of tangency with the Capital Market Line? The answer is positive and portfolios can be made with two assets with different Betas ($\beta \neq 0$ and $\beta \neq 1$) subject to desired return R_d by the investor. Applying non-linear optimization, we get the following proportions to invest in x_1 and x_2:

$$x_1^{'} = \frac{\beta_2}{\beta_2 - \beta_1} - \frac{(R_d - R_F)}{(R_m - R_F)}\frac{1}{(\beta_2 - \beta_1)} \qquad \textit{with } \beta_1 \neq \beta_2 \textit{ and } R_m \neq R_F \tag{6}$$

$$x_2^{'} = \frac{-\beta_1}{\beta_2 - \beta_1} + \frac{(R_d - R_F)}{(R_m - R_F)}\frac{1}{(\beta_2 - \beta_1)} \tag{7}$$

The above result shows that, by following an investment strategy, whether for asset 1 (β_1) or asset 2 (β_2), that generates a return of $R_F + \beta_k(E(R_m) - R_F)$ financed with own resources and loans with a cost given by the CAPM model of $R_F + \beta_i[E(R_m) - R_F]$, then these portfolios are located on the efficient frontier on the Capital Market Line, and this new portfolio has a desired return of R_d and a minimum risk. To show whether this portfolio is on the SML, we should test whether this portfolio provides the same desired return (R_d) and the same risk as a portfolio on the efficient frontier, which is easily shown.

Note also that, in x'_1 and x'_2, if $R_d = R_F$, we reach the same conclusion as when searching for an alternative portfolio to a risk-free asset. Now, if $R_d = R_m$, we come to the second part of this article, which is the search for a portfolio equivalent to the market portfolio. On the other hand, for the general expressions (6) and (7), we observed that the definition of a risk-free portfolio with $\beta = 0$ is a highly particular case, as is a market portfolio with $\beta = 1$. Both portfolios can be made up with $\beta \neq 0$ and $\beta \neq 1$ and we obtain the same results, as deduced herein. In order to have this general situation, the suppositions are always that the return of the investment in asset i is: $R_F + \beta_i[E(R_m) - R_F]$ with i = 1 or 2 and the cost of the financing for asset k is: $R_F + \beta_k[E(R_m) - R_F]$ with k = 1 or 2. The return on the own resources is: R_d.

The above situations can be clarified with the following example. There are two risky assets in which we can invest or sell. The data are the following: $\beta_1 = 0.4$; $\beta_2 = 0.6$; $E(R_m) = 0.10$; $R_F = 0.07$; $\sigma^2_m = 0.10$. If the investor wishes a return of 11%, how can we constitute this portfolio with these two assets and how do we finance them, supposing that we have \$1,000 available?

We can obtain the values of x'_1 and x'_2 by replacing in (6) and (7).

$$x_1^{'} = \frac{0.6}{0.2} - \frac{(0.11-0.07)}{(0.10-0.07)}.\frac{1}{0.2} = -3.66666$$

We invest in $x_2 = 1 - x_1$=1 - (-3.6666) and this is financed with x_1 plus own resources, resulting in the following cash flow:

Investment:	\$1000 x 4.6666 = <u>\$4.66666</u>
Financing:	
Loan (or sale)	\$ 1000 x 3.6666 = \$3.66666
Own resources	<u>\$1.00000</u>
Total	<u>\$4.66666</u>

The return on investment of asset 2 is:	4.66666(0.07 + 0.6[0.1-0.07]) = \$410,666
Minus: Cost of Financing:	3.66666(0.07 + 0.4[0.1-0.07]) =-<u>\$300,666</u>
Net result:	\$110,000

The own resources (\$1,000) obtain a return of 11%, which is the return desired by the investor. The risk of this investment is $\sigma^2 = 1.7777\sigma^2_m = 0.1777$, which is obtained by replacing the values of x'_1 and x'_2 in the risk function. In this example, we can simulate different desired returns in order to calculate the adequate proportions, as occurs when $R_d = R_F = 0.07$ or: $R_m = R_F = 0.10$.

Table 4 shows a generalization for determining the proportion to invest in a risky asset (x_1) given two assets with known Betas so that the resulting portfolio will be equivalent to a market portfolio. To use the table, we must look for the intersection between the two known Betas. For example, we have one asset with $\beta = 0.4$ and another with $\beta_2 = 0.2$. The intersection of these two Betas on the table indicates that the proportion to invest in asset 1 is 4, and in asset 2 it is $1-x_1$; that is, 1-4. This shows that asset 2 is financing for -3. For smaller Beta intervals, a linear interpolation must be done.

Table 4: Proportion to invest in risky asset (x_1) for a B_1 and B_2 to obtain a portfolio equivalent to a market portfolio

B2 \ B1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
0	-	10.00	5.00	3.33	2.50	2.00	1.67	1.43	1.25	1.11	1.00	0.91	0.83	0.77	0.71	0.67	0.63	0.59
0.1	-9.00	-	9.00	4.50	3.00	2.25	1.80	1.50	1.29	1.13	1.00	0.90	0.82	0.75	0.69	0.64	0.60	0.56
0.2	-4.00	-8.00	-	8.00	4.00	2.67	2.00	1.60	1.33	1.14	1.00	0.89	0.80	0.73	0.67	0.62	0.57	0.53
0.3	-2.33	-3.50	-7.00	-	7.00	3.50	2.33	1.75	1.40	1.17	1.00	0.88	0.78	0.70	0.64	0.58	0.54	0.50
0.4	-1.50	-2.00	-3.00	-6.00	-	6.00	3.00	2.00	1.50	1.20	1.00	0.86	0.75	0.67	0.60	0.55	0.50	0.46
0.5	-1.00	-1.25	-1.67	-2.50	-5.00	-	5.00	2.50	1.67	1.25	1.00	0.83	0.71	0.63	0.56	0.50	0.45	0.42
0.6	-0.67	-0.80	-1.00	-1.33	-2.00	-4.00	-	4.00	2.00	1.33	1.00	0.80	0.67	0.57	0.50	0.44	0.40	0.36
0.7	-0.43	-0.50	-0.60	-0.75	-1.00	-1.50	-3.00	-	3.00	1.50	1.00	0.75	0.60	0.50	0.43	0.38	0.33	0.30
0.8	-0.25	-0.29	-0.33	-0.40	-0.50	-0.67	-1.00	-2.00	-	2.00	1.00	0.67	0.50	0.40	0.33	0.29	0.25	0.22
0.9	-0.11	-0.13	-0.14	-0.17	-0.20	-0.25	-0.33	-0.50	-1.00	-	1.00	0.50	0.33	0.25	0.20	0.17	0.14	0.13
1.0	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.1	0.09	0.10	0.11	0.13	0.14	0.17	0.20	0.25	0.33	0.50	1.00	-	-1.00	-0.50	-0.33	-0.25	-0.20	-0.17
1.2	0.17	0.18	0.20	0.22	0.25	0.29	0.33	0.40	0.50	0.67	1.00	2.00	-	-2.00	-1.00	-0.67	-0.50	-0.40
1.3	0.23	0.25	0.27	0.30	0.33	0.38	0.43	0.50	0.60	0.75	1.00	1.50	3.00	-	-3.00	-1.50	-1.00	-0.75
1.4	0.29	0.31	0.33	0.36	0.40	0.44	0.50	0.57	0.67	0.80	1.00	1.33	2.00	4.00	-	-4.00	-2.00	-1.33
1.5	0.33	0.36	0.38	0.42	0.45	0.50	0.56	0.63	0.71	0.83	1.00	1.25	1.67	2.50	5.00	-	-5.00	-2.50
1.6	0.38	0.40	0.43	0.46	0.50	0.55	0.60	0.67	0.75	0.86	1.00	1.20	1.50	2.00	3.00	6.00	-	-6.00
1.7	0.41	0.44	0.47	0.50	0.54	0.58	0.64	0.70	0.78	0.88	1.00	1.17	1.40	1.75	2.33	3.50	7.00	-

Conclusions

From the presentations and examples used in this paper, we can conclude that the initial situation of the validity of the CAPM depends on the existence of a risk-free asset, or of an asset with $\beta = 0$. This latter is a particular case, since it is possible to form an alternative portfolio with two risky assets, each having $\beta \neq 0$. This alternative portfolio has zero risk and an average return equal to that of a risk-free asset. We can also conclude that portfolios equivalent to a market portfolio can be made by mixing two risky assets, each having $\beta \neq 0$; this alternative portfolio has the same return and risk as the original market portfolio. We also develop a general way to generate portfolios according to the returns that the investors would like to obtain and with the minimum risk. The practical implication of the observations in this article is that such portfolios can be made when it is difficult to find risk-free assets and when clarity is lacking as to what the true market portfolio is.

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