Journal of Applied Finance & Banking, vol. 3, no. 1, 2013, 31-43
ISSN: 1792-6580 (print version), 1792-6599 (online)
Scienpress Ltd, 2013

Simple, Compound And Continuous Interest Discounts. A Comparative Study.

José Rigoberto Parada-Daza[1]

Abstract

This article makes an analytical comparison between simple, compound and continuous interest discount factors. It studies the equivalency relations between the three discount factors. An analysis is performed for a limited time period and of normal economy interest rates. It makes use of the Taylor and Maclaurin series to analytically compare the three factors, then performs a numerical analysis that includes econometric and statistical verification whose results are consistent with those stipulated within the mathematical argument. The central observation is that, regarding interest rates within a normal economy and within a maximum period of three years, the three factors are statistically similar and therefore very close. Some implications are obtained from the analyzed results.

JEL classification numbers: E430, G120 and G210
Keywords: Simple discount, compound interest, continuous interest, discount, capitalization.

1 Introduction

In both business as well as financial and economic theory the concepts of simple, compound and continuous interest have been used as separate elements. The idea of simple interest has been gradually abandoned, although its use has remained central for very short periods of at most a year and has theoretical support from a well-established mathematical relationship which compares the function of simple and compound capitalization. These equalize only between a period comprised of an initial time equal to zero and a final time equal to one, hence the basis for the use of simple interest in

---

[1]Faculty of Administrative and Economic Sciences, Universidad de Concepción-Chile.
e-mail: rparada@udec.cl

Article Info: *Received :* October 21, 2012. *Revised :* November 19, 2012.
Published online : January 15, 2013

operations of up to one year (Insolera, 1937, pp. 155).
In theoretical models of economic and financial theory, continuous and compound interest concepts are more widely used while financial literature essentially focuses on the concepts and implications of compound interest. Simple discount is usually omitted even for short-term operations. In this paper it is shown that for fixed periods of between one and three years, the use of any of the three discount factors are interchangeable thus simple interest continues to have mathematical support for periods of over one year. Analysis will focus on discount factors. Interest theory or its philosophical evolution will not be discussed here.
Discount factors are essentially used to try to determine the present value of future cash flow and to see its influence on the assessment of economic and financial investments and have no relevance when determining ex post values. This is important with respect to the basis of this article because whatever method is used to calculate the present value of a set of economic flows whether it be simple, compound or continuous discount, all are approximations to a probable expected event. There is no determinism and it is necessary to put emphasize on an analysis that implicates the assumption that approximations exist and can be conclusive to see in what form discount factors should be used whether it be for theoretical models to explain price formation or in order to quantify the likely economic benefits of investments in business practice.
The concepts to be firstly addressed are those of the discount factors of each of the three types of capitalizations: simple, compound and continuous. It will be mathematically shown that the discount factors of the last two are equivalent and approximately equal to the first. In a second stage an analysis will show that the three discount factors are statistically similar for short periods with a maximum of three years and interest rates of a normal economy ranging from 2% to 12% per year.
The objective of this work is to provide a theoretical and analytical background which proves that a simple discount factor can be used in evaluations involving discounts for periods longer than one year and that should be analyzed in the formalization of normative models of financial asset prices when these involve periods longer than one year and less than three years as well as to generate discussion about the practical use of the simple discount factor in financial transactions for periods greater than one year.

2 Methodology

Firstly, the analytical method based on the Taylor series will be used and in particular the Maclaurin series to reduce the discount factor in both continuous and compound interest as a mathematical expression equivalent to that of the simple interest discount factor. The approximation of the last two discount factors to that of the simple discount factor will be shown through these series. The equivalence between the three factors will be analyzed according to the algebraic definition of equivalence relation.
In a second stage a numerical method will be used to prove with a high degree of statistical confidence that the average discount factors for simple, continuous and compound interest are statistically equal. This will be carried out for monthly, bimonthly, quarterly and semi-annual capitalization and within a total analysis period of three years. This numerical analysis is based on normal interest rates in the economy of different countries ranging from 2% to 12% annually according to empirical evidence.
In a third stage of the article the degree of dependence will be examined between the three

discount factors, this will take data with daily and monthly capitalizations and will use the Ordinary Least Squares (OLS) method to prove linear relations between the three discount factors for a total analysis period of three years.

3 Discount Factors According To The Taylor And Maclaurin Series

3.1 Simple Discount

For this the following situation is defined. Let C be the value of equal periodical installments to be paid by an initial debt D, for n periods at a rate of i per period. If this debt is paid at simple interest, then the relationship between debt and payment is reduced to the following:

$$D = C(1-i) + C(1-2i) + C(1-3i) + \ldots\ldots\ldots\ldots + C(1-ni) = \sum_{t=1}^{n} C(1-ti) \qquad (1)$$

Equation N° 1 means that each payment is included in the interest charged by a lender to a lender and the period interest is therefore reduced. The relevance of this operation to simple interest is that interest is not paid on top of interest but only on the initial value of the debt, De La Mazaand Levenfel (1997, pp. 47). From (1) simple discount is defined as a factor in the following expression:

$$F_{i,t}^{Sm} = 1 - ti, \qquad \forall t < 1/i \qquad (2)$$

$F_{i,t}^{Sm}$ = Simple discount factor at periodic rate i in time t.

3.2 Compound Interest Discounting.

To define compound interest capitalization, suppose that an initial value of M that capitalized in period t to a compound interest rate of i, the amount of money at the end of period t, assuming that compound capitalization changes in the following relationship: $M_t = M(1 + i)^t$, in which case the discount factor is defined as:

$$F_{i,t}^{Cm} = (1 + i)^{-t}, \qquad \forall\ t \geq 0 \qquad (3)$$

$F_{i,t}^{Cm}$ = Discount factor of compound interest to periodic rate i in time t.

Clearly (2) and (3) correspond to different mathematical expressions, except for that which is considered in the initial period t=0 or to a hypothetical interest rate of i = 0. While the former has a linear behavior, the second has an exponential behavior. However, it can be expressed, through the use of series, that both expressions are mathematically equivalent.

The following proposition is presented: The discount factor for a financial or economic transaction using compound interest is equivalent and approximately equal to that of the simple discount factor.

This proposition is proved by $f(i)= (1 + i)^{-t}$. For verification it is of interest to search the form of this binomial through a series and so for this the Maclaurin series is used which is a particular case of the Taylor series when this is centered on a = 0 and is defined as:

f(x) = $\sum_{n=0}^{\infty}\frac{f^{(n)}}{n!}(x-a)^n$. Thus, for the Maclaurin series it is known that:

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+...+\frac{f^{(n)}(0)}{n!}x^n+...$$

(4)

Where f(x=i) = (1 + i)$^{-t}$, and calculating the respective derivatives, the following is given:

$$f'(i)=-t(1+i)^{-t-1} \quad\Rightarrow\quad f'(0)=-t$$
$$f''(i)=t(t+1)(1+i)^{-t-2} \quad\Rightarrow\quad f''(0)=t(t+1)$$
$$f'''(i)=-t(t+1)(t+2)(1+i)^{-t-3} \quad\Rightarrow\quad f'''(i)=-t(t+1)(t+2)$$
$$\vdots \qquad\qquad \vdots$$
$$f^{(k)}(i)=(-1)^k t(t-1)(t-2)...(t-k+1)(1+i)^{-t-k} \quad\Rightarrow\quad f^{(k)}(0)=(-1)^k t(t-1)(t-2)...(t-k+1)$$

Replacing the previous relations in (4) the following is given:

$$(1+i)^{-t}=1-ti+\frac{t(t+1)}{2!}i^2-\frac{t(t+1)(t+2)}{3!}i^3+...+\frac{(-1)^k t(t+1)(t+2)...(t+k-1)}{k!}i^k+... \qquad (5)$$

In series N°5 there is a usual tendency to reject the remainder starting from the third term due to the tendency for a slight limit. Thus, from (5) the following is obtained

$$(1+i)^{-t}\approx 1-ti \qquad (6)$$

Expression N°6 is equal to N°2, which shows that the compound interest discount factor approximately coincides with that of the simple discount factor.
Furthermore, the Maclaurin series is a convergent series. In effect, applying the criteria of the quotient (Quotient Test) to analyze the convergence of series the following is given:

$$\rho=\lim_{n\to\infty}\left|\frac{(-1)^n t(t+1)(t+2)...(t+n-1)i^n}{n!}\frac{(n-1)!}{(-1)^{n-1}t(t+1)(t+2)...(t+n-2)i^{n-1}}\right|$$
$$=\lim_{n\to\infty}\left|\frac{(-1)(t+n-1)i}{n}\right|$$
$$=|i|\lim_{n\to\infty}\left|\frac{(-1)(t+n-1)}{n}\right|$$
$$=|i|$$

The previous series is absolutely convergent for | i | < 1 and diverges for | i | > 1. In this case to define an absolute convergent series it is known that: $\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n-1}}\right|<1$.

The previous analysis can also be directly made through the Binomial Theorem which was formulated in 1946 by Richardson and Miller (1946, pp. 52.) as a satisfactory approximation used to calculate time equated when debts may be settled by making a single payment. In Schneider (1944) a similar idea to approximate the Internal Rate of Return is observed. It is obvious that both cases refer to compound interest capitalization.

Using series to estimate performance, as Marie (1890) put it, to calculate present value is even older. Currently these calculations are less relevant given the technological ease of accurately calculating rates of return with compound interest; however the underlying conceptual and analytical arguments have important implications for the development of normative price formation models of short-term financial assets.
Both in economics and finance, normal interest rates are well below 100% so you can be assured that this series is suitable when used to represent the discount factor in a real economy to be absolutely convergent. Due to the existence of the Taylor and Maclaurin series remainder, there may be differences in the equivalence between the two discount factors but these are closer to a hundred of the factor's values, these are measured in decimal fraction. This can be clarified by the following example where the discount factor of compound interest is calculated at an annual rate of 4% over three years using equation N° 5, that is:

$$\begin{aligned} 1.04^{-3} &= 1-3(0.04)+\frac{3(4)}{2!}(0.04)^2-\frac{3(4)(5)}{3!}(0.04)^3+\frac{3(4)(5)(6)}{4!}(0.04)^4 \\ &= 1-0.12+0.0096-0.00064+0.000038 \\ &= 0.888998 \end{aligned}$$

By directly calculating the discount factor is 0.888996. By taking the sum of the first two terms, this is 0.88.

3.3 Continuous Interest Discounting.

Capitalization of continuous interest means that both the principal and interest accumulate within one year and in extremely short periods such as this the number of capitalizations tend to be extremely large. If in one year there are j capitalizations to an annual interest rate of i, then a continuous capitalization is when $j \to \infty$. Starting with compound interest capitalization it can be deduced, assuming a total period of n, a capitalization of continuous interest as follows:

Let $f(i)= (1 + i/j)^{jn}$, then continuous interest capitalization is $\lim_{j\to\infty}\left(1+i/j\right)^{jn}$. Calculating this limit by the L'Hopital Rule, $e^{in} = (1 + i/j)^{jn}$ is obtained. Its discount factor equivalent is: $e^{-in}=(1 + i/j)^{-jn}$, Henderson and Quandt (1980, pp. 342). This last expression is of interest for this article. The continuous interest discount factor is expressed as follows:

$$F_{i,t}^{Cn} = e^{-it} \tag{7}$$

$F_{i,t}^{Cn}$ = Discount factor for continuous capitalization at periodic rate i in time t.

The following proposition is put forth: The continuous interest discount factor is the same as if made with discount factors of both simple and compound interest for a certain period therefore these discount factors are approximately equal.
To show this statement let $f(i)= e^{-it}$ and recall the Maclaurin Series:

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+...+\frac{f^{(n)}(0)}{n!}x^n+... \tag{8}$$

Calculating the derivative of the function, the following is obtained:

$$\begin{array}{lll} f(i)=e^{-it} & \Rightarrow & f(0)=1 \\ f'(i)=-te^{-it} & \Rightarrow & f'(0)=-t \\ f''(i)=t^2e^{-it} & \Rightarrow & f''(0)=t^2 \\ \vdots & & \vdots \\ f^{(k)}(i)=(-t)^k e^{-it} & \Rightarrow & f^{(k)}(0)=(-t)^k \end{array}$$

Replacing the previous values in (8) one has:

$$e^{-it}=f(i)=1+-ti+\frac{t^2}{2!}i^2-\frac{t^3}{3!}i^3+\ldots+\frac{(-t)^n}{n!}i^n+\ldots$$

By the Quotient Convergence Criteria, it is shown that this series is convergent for any value of i. Indeed, it is given that:

$$\rho=\lim_{n\to\infty}\left|\frac{a_n}{a_{n-1}}\right|=\lim_{n\to\infty}\left|\frac{(-t)^n i^n}{n!}\frac{(n-1)!}{(-t)^{n-1}i^{n-1}}\right|=\lim_{n\to\infty}\left|\frac{-it}{n}\right|=|it|\lim_{n\to\infty}\left|\frac{1}{n}\right|=0<1$$

From which the series is convergent for all values of i. This implies that this series is convergent with any interest rate.

Thus, assuming that the remainder of the Maclaurin series tends to be small, the following expression for the continuous discount factor is given:

$$e^{-it}\approx 1-it \tag{9}$$

Expression N° 9 is equal to N° 2 and N° 6, which shows that the continuous interest discount factor is equivalent to that of a discount to simple and compound interest. This equivalence is clearer for certain numbers of periods and certain interest rates, as in the Taylor and Maclaurin series there is however a remainder that can vary the accuracy of equivalence within the hundredths, the discount factors are expressed as decimal fractions.

The next point will approach the numeric relation where this equivalency has a statistical validity for a determined number of periods and determined range of interest rates. However, it can be verified, using the definitions of algebraic equivalence relations, that these discount factors are equivalence relations between themselves and following the definitions of Jaisingh and Ayres (2004, pp. 18) set S is used, which is expressed as: $S=\left\{F_{i,t}^{Sm},\ F_{i,t}^{Cm},\ F_{i,t}^{Cn}\right\}$.

The relationships between the elements of the S set is an equivalence relation if the relation $\Re$ between its components is: i) Reflexive, ii) Symmetric and iii) Transitive. It can be deduced from relations N° 2, 6 and 9 that the three conditions are met so as to affirm that there is indeed an equivalence relation between the three discount factors.

i) Reflexive.

This is true as: $F_{i,t}^{Sm}\Re F_{i,t}^{Sm}$, $F_{i,t}^{Cm}\Re F_{i,t}^{Cm}$ and $F_{i,t}^{Cn}\Re F_{i,t}^{Cn}$.

ii) Symmetric.

This is true as: $F_{i,t}^{Sm}\Re F_{i,t}^{Cm} \rightarrow F_{i,t}^{Cm}\Re F_{i,t}^{Sm}$, $F_{i,t}^{Cm}\Re F_{i,t}^{Cn} \rightarrow F_{i,t}^{Cn}\Re F_{i,t}^{Cm}$ and $F_{i,t}^{Cn}\Re F_{i,t}^{Sm} \rightarrow F_{i,t}^{Sm}\Re F_{i,t}^{Cn}$

iii) Transitive.

This is true as: $F_{i,t}^{Sm}\Re F_{i,t}^{Cm} \wedge F_{i,t}^{Cm}\Re F_{i,t}^{Cn} \rightarrow F_{i,t}^{Sm}\Re F_{i,t}^{Cn}$.

Thus expressions N° 2, 6 and 9 show that the three discount factors maintain an equivalence relation.

4 Numerical Analisys of the Equivilancy Between Simple, Compound And Continuous Discount Factors.

Due to the fact that both the Taylor and Maclaurin Series have a remainder that can alter the previous numerical approximations it is necessary to numerically analyze how previous equivalence propositions can be fulfilled in the case of discount rates for simple, compound and continuous interest, between the three discount factors and see how approximate they are. Two forms of analysis will be presented, the first is a statistical approach to a Means Test analysis and the second part is an analysis by way of the Ordinary Least Squares (OLS) method to see how different they are on average and how accurate the approximations are between the three discount factors.

4.1 Discount Factor Averages and Means Test Analysis.

This first stage analyzes how statistically different the discount factor averages are according to simple, compound and continued interest over three years. The analysis is tested at rates between 2% and 12% per year and for a maximum period of three years. These interest rate ranges are based on empirical studies applied to compare interest rates on bank deposits and consumer loans in different countries.

A report by the European Commission (Eurostat) shows that for a set of countries, short-term interest rates range from 1% to 12% per year and also shows that for 28 European countries some Bond Yield rates fluctuated from 2% to 12% annually for the 2000-2009period. A report by the Central Bank of Iceland shows in "Domestic and foreign real interest rate 1990-2001" that rates fluctuate between 2% and 12%. Sepúlveda (2010) analyzes the interest rates for 25 countries from the years 2006-2009, which shows the same range of interest rate variations of between 2% and 12% per year.

The discount factors have been calculated for a total period of three years with monthly, bimonthly, quarterly, semiannual and annual capitalization for interest rates of 2%, 4%, 6%, 8%, 10% and 12%. So, to calculate the averages of each factor data of 36, 18, 12, 6 and 3 is used for monthly, bimonthly, quarterly, semiannual and annual discount factors respectively. For each period the rate of their respective period was used. For example, for the monthly series, the annual interest rate changed to a monthly simple, continuous or compound interest rate depending on the case and according to the appropriate type of capitalization being used.

The discount factors used are those indicated in formulas N° 2, 3 and 6 corresponding to simple, continuous or compound discount factors, respectively. Average and standard deviations have been calculated for each of these data sets and their results are shown in Appendix N°1. A Means Test has been applied that compares with T Student tables and also according to a Normal Distribution test, considering the following assumptions:

H0= There is no difference between the discount factor averages for each rate level according to simple, compound or continuous capitalization.
Ha= The discount factor averages capitalized for each level of interest rate are different depending on the capitalization method used.

The average of each discount factor was compared first between simple and compound interest and then between simple and continuous discount factors. In both, the calculated tests are identical and show that the Null Hypothesis is accepted, i.e. there is no statistical difference between the average discount factors with a confidence level of 99% according to the Student tables and also according to a Normal Distribution. This development confirms the first part of the paper on the mathematical foundation of almost exact equivalence between the three discount factors for normal economy interest rates, which is between 2% and 12% per year and for any number of capitalization period within one year. As the "z" Test between Compound Interest Discount Factor and Continuous interest are the same as those shown in Table N° 1, then the conclusions are also identical, i.e. on average there is no difference between Compound Interest Discount Factor and the Continuous Interest Factor.
In conclusion, the averages of the three discount factors are statistically similar with 99% statistical confidence interval for a maximum period of three years, regardless of the number of capitalization within a year. The results of index "z" are presented in Table 1.

Table 1: Index "z" between Simple Discount Factor and Compound Interest Factor

	Capitalization Periods Within One Year				
Rate (%)	*Monthly*	*Bimonthly*	*Trimonthly*	*Biannual*	*Annual*
12	1.28	0.93	0.83	0.59	0.47
10	1.08	0.78	0.70	0.49	0.39
8	0.87	0.63	0.56	0.40	0.32
6	0.66	0.48	0.43	0.30	0.24
4	0.44	0.32	0.29	0.20	0.16
2	0.22	0.16	0.14	0.10	0.08
Liberty Grades	70	34	22	10	4
Student (0.99)	2.38	2.438	2.508	2.764	3.74
t Norm. D.(0.99)	-2,58 y 2,58	-2,58 y 2,58	2,58 y 2,58	-2,58 y 2,58	2,58 y 2,58
Number of Data	36	18	12	6	3

The “z” factor is calculated as follows:

$$z = \left(\overline{F_{i,t}^{Cm}} - \overline{F_{i,t}^{Sm}} \right) \Big/ \sqrt{\frac{\sigma_1^{\ 2}}{n_1} + \frac{\sigma_2^{\ 2}}{n_2}}$$

Where: $\overline{F_{i,t}^{Cm}}$ and $\overline{F_{i,t}^{Sm}}$ represent the discount factor average at compound interest and simple respectively. σ_1 and σ_2 represent the standard deviation of the discount factor of compound and simple interest respectively. n_1 and n_2 is the number of data for each of these factors.

4.2 Analysis between Simple Discount Factors, Compound Interest and Continuous Interest using the OLS Method

The analysis will prove that for rates between 2% and 12% per year and a maximum period of three years the level of equivalence between the three types of capitalization is almost exactly equivalent in accordance with that which is stated in the theoretical part of this article. These interest rate ranges are based on an empirical study applied in order to compare interest rates on bank deposits and consumer loans previously identified in 3.1. Proving the following models:

$$\text{Icom} = \alpha_1 + \beta_1 \text{ Isim} \tag{10}$$

$$\text{Icon} = \alpha_2 + \beta_2 \text{ Isim} \tag{11}$$

$$\text{Icon} = \alpha_3 + \beta_3 \text{ Icom} \tag{12}$$

Where: $\text{Isim} = 1 - ti_s$, $\text{Icom} = (1 + i_c)^{-t}$, $\text{Icon} = e^{-ti_{cn}}$, i_s = Simple Interest Rate, i_c = Compound Interest Rate and i_{cn} = Continuous Interest Rate.

A maximum period of three years was analyzed for two cases. The first is for a data set with daily capitalization for the three types of capitalization (simple, compound and continuous) with 1095 data each and in the other case a series of interest rate data is used with monthly capitalizations and 36 data. Analysis has only been performed for these two situations due to data availability needed to provide reliable statistical models. The Ordinary Least Squares (OLS) model has been used as well as the econometric software EViews. The results obtained are shown in Table 2, 3 and 4.

The results of both, the daily and monthly capitalization models analyzed (Table N° 2 and N° 4), indicate a high statistical correlation between the three factors. Thus, all R2 cases are extremely close to one and the F test shows a high statistical significance. A similar situation is observed in the Alpha and Beta coefficients, which also have high statistical significance for different interest rate levels.

These results are consistent with the deduction in points II and III, it is inferred that the three discount factors are equivalent for those levels of interest rates and degrees of approximation between the two are numerically very high.

Table 2: Results of models (10), (11) and (12) for three years with a total of 36 data monthly

	Icom= $\alpha_1 + \beta_1$Isim				Icon = $\alpha_2 + \beta_2$Isim				Icon = $\alpha_3 + \beta_3$Icom			
Rate	α_1	β_1	R_1^2	*F*	α_2	β_2	R_2^2	*F*	α_3	β_3	R_3^2	*F*
12%	0.1956 (39.83)	0.7953 (133.11)	0.9981	1,77E+4	0.1956 (39.84)	0.7953 (133.1)	0.998	1.77E+4	0 (132.5)	1 3.1E+8	1	9.4E+16
10%	0,1689 (38.11)	0,8245 (158.25)	0.9986	2.5E+4	0.1689 (38.12)	0.8245 (158.3)	0.999	2.5E+4	0 (-157)	1 (2.8E+18)	0.99	8.0E+15
8%	0.140 (36.49)	0.8555 (196.9)	0.9991	3.8E+4	0.1400 (36.49)	0.8580 (195.3)	0.999	3.8E+4	0 (-196)	1 (1.5E+10)	1	2.1E+20
6%	0.1094 (34.94)	0.8884 (258.1)	0.9995	6.7$+4	0.1094 (34.94)	0.8884 (258.8)	0.999	6.7E+4	0 (258)	1 2.9E+18	1	8.49E+18
4%	0.0755 (33.48)	0.9233 (384.5)	0.9998	1,5E+6	0.0755 (33.48)	0.9233 (384.5)	0.999	1.47E+5	0 (384)	1 1,65E+6	1	2.71E+14
2%	0.0392 (32.10)	0.9604 (761.5)	0.9999	5.8E+5	0.0392 (32.1)	0.9604 (761.5)	0.970	5.8E+5	0 (384)	1 32.3E+7	1	1.04E+15

*The numbers in parenthesis indicate t value statistics.

Table 3: Average monthly Simple ($\overline{ISim}$), Compound ($\overline{ICom}$) and Continuous Interest ($\overline{ICon}$) and their standard deviations for 36 data. The figures are in decimal fractions.

Rate	$\overline{ISim}$	σ_{Isim}	$\overline{ICom}$	σ_{Icom}	$\overline{ICon}$	σ_{Icon}
12%	0.815000	0.105357	0.843743	0.083871	0.843743	0.083871
10%	0.845833	0.087797	0.866291	0.072441	0.866291	0.072441
8%	0.876667	0.070238	0.890092	0.060116	0.890093	0.060116
6%	0.907500	0.052678	0.915249	0.046810	0.915249	0.046811
4%	0.93833	0.035119	0.941869	0.032429	0.941870	0.032428
2%	0.969167	0.017559	0.970075	0.016865	0.970075	0.016865

Table 4: Results of models (9), (10) and (11) for three years with a total of 1095 data daily

	Icom= $\alpha_1 + \beta_1$Isim				Icon = $\alpha_2 + \beta_2$Isim				Icon = $\alpha_3 + \beta_3$Icom			
Rate	α_1	β_1	R_1^2	*F*	α_2	β_2	R_2^2	*F*	α_3	β_3	R_3^2	*F*
12%	0.1926 (219.8)	0.7989 (753.5)	0.9981	5.7E+5	0.11926 (219.8)	0.7989 (753.5)	0.9981	5.72E+5	0.0 (-750)	1 3.4E+8	1	1.1E+17
10%	0.1662 (210.5)	0.8277 (896)	0.9987	8.02E+5	0.1662 (210.5)	0.8277 (896)	0.9986	8.02E+5	0.0 (-893)	1 3.9E+8	1	1.6E+17
8%	0.1373 (200.9)	0.8586 (1109)	0.9991	1.22E+6	0.1378 (201.8)	0.8582 (1109)	0.9991	1.23E+6	0.0	1	1	3.64E+12
6%	0.1071 (193,4)	0.8905 (1465)	0.9994	2.15E+6	0.1071 (193.4)	0.8904 (1465)	0.9994	2.14E+6	0.0 2E-10	1 4E+9	1	1.6E+19
4%	0.0741 (185.5)	0.9248 (2177)	0.9998	4.7E+6	0.0741 (185.5)	0.9248 (193)	0.9998	4.74E+6	0.0 (2175)	1 1.1E18	1	1.16E+18
2%	0.0386 (178.05)	0.9611 (4311)	0.9999	1.86E+7	0.0337 (154.6)	0.9660 (4289)	0.9999	1.84E+7	-0.005 -(4288)	1.005 (83051)	1	6.9E+11

*The numbers in parenthesis indicate t value statistics.

Table 5: Average daily Discount Factor for Simple ($\overline{ISim}$), Compound ($\overline{ICom}$) and Continuous Interest ($\overline{ICon}$) and their standard deviations for 1095 data, in decimal fractions.

Rate	$\overline{ISim}$	σ_{Isim}	$\overline{ICom}$	σ_{Icom}	$\overline{ICon}$	σ_{Icon}
12%	0.878890	0.069314	0.89280	0.059541	0.892859	0.059510
10%	0.856884	0.082590	0869625	0.071764	0.869627	0.071764
8%	0.878890	0.069314	0.89280	0.059541	0.892859	0.059510
6%	0.909918	0.051985	0.917401	0.046304	0.917401	0.046304
4%	0.939945	0.034657	0.943359	0.032053	0.943359	0.032053
2%	0.969975	0.017327	0.970852	0.016655	0.970704	0.016739

5 Implications

From the above discussion the following implications emerge:

a) For theoretical purposes in approach to behavior models of short-term prices of stock assets, working with simple interest discount factors can be useful in special case analysis models such as: Option Pricing Theory, Financial Futures Contracts, Term structure of interest rates, Forward Rates, Future Rates, Spot Rate, Bond Prices, Duration Measures and others.
In theoretical work on asset pricing models it is more mathematically elegant to use a continuous interest discount factor, which sometimes aids in understanding the problem, but given the results shown here, also testing what would happen in each model when the simple discount factor is used should not be ruled out as well as analysis of the theoretical and practical implications of both situations.

b) To calculate Internal Rate of Return (IRR) the use of simple discount factors solves the problem of multiple rates of return if the stream of estimated cash flows changes sign more than once, put forth by Lorie and Savage (1955, p. 237) in the "Oil pump problem". Teichroew, et al. (1965) proposed a solution for these cases, however, that situation was a two year investment and used previously exposed mathematical and numerical deductions and the solution to that problem is simpler.
For the same IRR indicator, the use of a simple discount factor as an approximation mitigates the effect of the supposed mathematical simplification that IRR has on reinvestment of intermediate flows at a constant rate, which is a mathematical supposition to solve an unknown in a polynomial of the "nth" degree but not necessarily compatible with the data of a real economy.

c) From a practical point of view it becomes more difficult to replace the current use of the simple interest discount factor for compound interest as it is an integral part of economic and financial practice. However, simple discount factor can facilitate rapid calculation for investors who lack expertise in these matters.

6 Conclusion

The relevant conclusion of this work is that the use of a simple discount factor does not show different values to compound and continuous interest discount factors for a period of three years, regardless of the number of capitalizations per year. The main implication of this conclusion is that the use of simple capitalization is very relevant in either theoretical or empirical analysis of economic and financial transactions that have a maximum three years year duration. Today, compound interest capitalization is predominant including in very short term transaction such as with financial options.

ACKNOWLEDGEMENTS: I acknowledge the comments of two anonymous reviewers of JAFB. I also acknowledge the valuable collaboration of my son Miguel Parada-Contzen.

References

[1] Central Bank of Iceland, Domestic and foreign real interest rate 1990-2001, *Monetary Bulletin 2001/4*, (2001), Iceland.

[2] European Commission (Eurostat), *Exchange rates and Interest rate*, (2011) (http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Exchange_rates_and_interest_rates)

[3] J. M. Henderson and R. E. Quandt, *Microeconomic Theory: A mathematical approach*, Mac Graw-Hill Book Company, New York, 1980.

[4] Filadelfo Insolera, *Corso Di Matematica Finaziaria*, Societá Reale Mutua di Assicurazioni di Torino, Turín, 1937. In Spanish, *Curso de Matemática Financiera y Actuarial*, Editorial Aguilar, Madrid, Spain, 1950.

[5] L. R. Jaisingh, and F. Ayres, *Abstract Algebra*, Second Edition, MacGraw-Hill, N. York, 2004.

[6] Gustavo Levenfeld and Sofía De la Maza, *Matemática de las Operaciones Financieras y de la Inversión*. MacGraw-Hill, Madrid, Spain, 1997

[7] J. H. Lorie, L.J. Savage, Three Problems in Capital Rationing, *Journal of Business*, **Vol. 28**, (1955), 229-239.

[8] L. Marie, *Traité Mathématique et practique des operations financierès*, France, 1890, Pp. 342.

[9] C.H. Richardson, and I.L. Miller, *Financial Mathematics*, D. Van Nostrand Company, Inc. New York, 1946

[10] F. Sepúlveda, *Análisis comparativo internacional de tasas de interés de Crédito de Consumo* (2009), Banco Central de Chile, Minuta N° DPF 2009-2, Gerencia de División de Política Financiera, Santiago, Chile. (www.bcentral.cl/publicaciones/políticas/pdf/MinutasIEF_062010.pdf).

[11] Erich Schneider, *Wirtschaftlichkeitsrechnung: Theorie der Investition*, J.C.B. Mohr (Paul Siebeck), MW BookTübingen, Germany, 1951. The Original Version is in Danish, *Investiringogrente*, Arnold Busck, 1944.

[12] D. Teichroew, A.A. Robicheck and M. Montalbano, Mathematical Analysis of Rates of Return Under Certainty, *Management Science*, (January 1965), 395-403.

[13] D. Teichroew, A.A. Robicheck and M. Montalbano, An Analysis of Criteria for Investment and Financing Decisions Under Certainty. *Management Science*, (November 1965). 151-179.

Appendix 1: Discount factor averages for Simple ($F_{i,t}^{Sm}$), Compound ($F_{i,t}^{Cm}$) and Continuous interest ($F_{i,t}^{Cn}$) and their respective variances ($\sigma_{\text{ISim}}, \sigma_{\text{ICom}}$ y σ_{ICon}) for a three year period.

Annual Rate (Interest)	*Average and Stand. Dev.*	*Monthly Capitalization*	*Bimonthly Capitalization*	*Quarterly Capitalization*	*Semiannual Capitalization*	*Annual Capitalization*
2%	$F_{i,t}^{Sm}$	0.969167	0.968333	0.967500	0.965000	0.960000
	σ_{ISim}	0.017559	0.017795	0.018028	0.018708	0.020000
	$F_{i,t}^{Cm}$	0.970075	0.969274	0.968475	0.966070	0.961294
	σ_{ICom}	0.016865	0.017078	0.017286	0.017895	0.019035
	$F_{i,t}^{Cn}$	0.970075	0.969274	0.968475	0.960770	0.961294
	σ_{ICon}	0.016865	0.017078	0.017286	0.017895	0.019035
4%	$F_{i,t}^{Sm}$	0.938333	0.936667	0.935000	0.930000	0.920000
	σ_{ISim}	0.035119	0.035590	0.036056	0.037417	0.040000
	$F_{i,t}^{Cm}$	0.941869	0.940330	0.938792	0.93419	0.925030
	σ_{ICom}	0.032429	0.032811	0.033185	0.034269	0.036273
	$F_{i,t}^{Cn}$	0.941569	0.94033	0.938792	0.93419	0.925030
	σ_{ICon}	0.032429	0.032811	0.033185	0.034269	0.036273
6%	$F_{i,t}^{Sm}$	0.907500	0.905000	0.902500	0.895000	0.880000
	σ_{ISim}	0.052678	0.053385	0.054083	0.056125	0.060000
	$F_{i,t}^{Cm}$	0.915249	0.913027	0.910808	0.904174	0.891004
	σ_{ICom}	0.046810	0.047324	0.047860	0.049268	0.051896
	$F_{i,t}^{Cn}$	0.915249	0.913027	0.910808	0904174	0.891004
	σ_{ICon}	0.046811	0.047324	0.047860	0.049268	0.051896
8%	$F_{i,t}^{Sm}$	0.876667	0.873333	0.870000	0.860000	0.840000
	σ_{ISim}	0.070238	0.071181	0.072111	0.074833	0.080000
	$F_{i,t}^{Cm}$	0.890092	0.887238	0.884390	0.875882	0.859032
	σ_{ICom}	0.060116	0.060728	0.061323	0.063023	0.066063
	$F_{i,t}^{Cn}$	0.890093	0.887238	0.884390	0.875882	0.859032
	σ_{ICon}	0.060116	0.060728	0.061323	0.063023	0.066063
10%	$F_{i,t}^{Sm}$	0.845833	0.837500	0.825000	0.800000	0.760000
	σ_{ISim}	0.087797	0.090139	0.093540	0.100000	0.120000
	$F_{i,t}^{Cm}$	0.866291	0.859419	0.849181	0.828951	0.800610
	σ_{ICom}	0.072441	0.073782	0.075648	0.078980	0.090587
	$F_{i,t}^{Cn}$	0.866290	0.859419	0.849181	0.828951	0.800610
	σ_{ICon}	0.072441	0.073782	0.075648	0.078918	0.090587
12%	$F_{i,t}^{Sm}$	0.815000	0.810000	0.805000	0.790000	0.760000
	σ_{ISim}	0.105357	0.106771	0.108167	0.112250	0.120000
	$F_{i,t}^{Cm}$	0.843473	0.839759	0.835787	0.823948	0.800610
	σ_{ICom}	0.083871	0.084594	0.085293	0.087248	0.090587
	$F_{i,t}^{Cn}$	0.843743	0.839759	0.835787	0.823948	0.800610
	σ_{ICon}	0.083871	0.084594	0.085293	0.087248	0.090587
N° Data		36	18	12	6	3