U N I V E R S I D A D D E C O N C E P C I O N
FACULTAD DE CIENCIAS FISICAS Y MATEMATICAS
DEPARTAMENTO DE MATEMATICA

Listado 1
Soluciones
Cálculo III (521227)

1. $\overline{A} = \{(x,y)\in IR^2 : 0 \le (x-3)^2 + (y+4)^2 \le 16\} \cup \{(3,5)\} \cup \{(x,y)\in IR^2 : x = 0 \wedge 7 \le y \le 8\}$

$A' = \{(x,y)\in IR^2 : 0 \le (x-3)^2 + (y+4)^2 \le 16\} \cup \{(x,y)\in IR^2 : x = 0 \wedge 7 \le y \le 8\}$

$\overset{\circ}{A} = \{(x,y)\in IR^2 : 0 < (x-3)^2 + (y+4)^2 < 16\}$

$Fr(A) = \{(x,y)\in IR^2 : (x-3)^2 + (y+4)^2 = 16\} \cup \{(x,y)\in IR^2 : x = 0 \wedge 7 \le y \le 8\} \cup \{(3,5),(3,-4)\}$

$\overline{B} = \{(x,y,z)\in IR^3 : x^2 + y^2 \le z \le 4\} = B'$

$\overset{\circ}{B} = \{(x,y,z)\in IR^3 : x^2 + y^2 < z < 4\}$

$Fr(B) = \{(x,y,z)\in IR^3 : z = x^2 + y^2 \wedge z < 4\} \cup \{(x,y,z)\in IR^3 : x^2 + y^2 \le 4 \wedge z = 4\}$

$\overline{C} = \{(x,y)\in IR^2 : y \ge 3\} \cup \{(0,2)\} \cup \{(x,y)\in IR^2 : 0 \le x \le 1 \wedge y = 0\}$

$\overset{\circ}{C} = \{(x,y)\in IR^2 : y > 3\}$

$\overline{C} = \{(x,y)\in IR^2 : y \ge 3\} \cup \{(x,y)\in IR^2 : 0 \le x \le 1 \wedge y = 0\}$

$Fr(C) = \{(x,y)\in IR^2 : y = 3\} \cup \{(0,2)\} \cup \{(x,y)\in IR^2 : 0 \le x \le 1 \wedge y = 0\}$

$\overset{\circ}{D} = \{(x,y)\in IR^2 : 0 < (x-2)^2 + y^2 < 1\}$

$\overline{D} = \{(x,y)\in IR^2 : 0 \le (x-2)^2 + y^2 \le 1\} \cup \left\{(x,y)\in IR^2 : x = \frac{1}{n} \wedge y = 0, n \in IN\right\} \cup \{(0,0)\}$

$D' = \{(x,y)\in IR^2 : 0 \le (x-2)^2 + y^2 < 1\} \cup \{(0,0)\}$

$Fr(D) = \{(x,y)\in IR^2 : (x-2)^2 + y^2 = 1\} \cup \left\{(x,y)\in IR^2 : x = \frac{1}{n} \wedge y = 0, n \in IN\right\} \cup \{(0,0)\}$

$\overset{\circ}{E} =]1,2[$

$\overline{E} = [1,2] \cup \{0\}$

$E' =]1,2] \cup \left\{x \in IR : x = \frac{1}{n}, n \in IN\right\} \cup \{(0,0)\}$

$Fr(E) = \{0,2\} \cup \left\{x \in IR : x = \frac{1}{n}, n \in IN\right\}$

$$\overset{\circ}{F} = \{(x,y,z,u) \in IR^4 : 1 < x^2 + y^2 + z^2 + u^2 < 4\}$$
$$\overline{F} = \{(x,y,z,u) \in IR^4 : 1 \le x^2 + y^2 + z^2 + u^2 \le 4\} \cup \{(0,0,0,0);(1,1,1,2)\}$$
$$F' = \{(x,y,z,u) \in IR^4 : 1 \le x^2 + y^2 + z^2 + u^2 \le 4\}$$
$$Fr(F) = \{(x,y,z,u) \in IR^4 : 1 = x^2 + y^2 + z^2 + u^2 \vee x^2 + y^2 + z^2 + u^2 = 4\} \cup \{(0,0,0,0);(1,1,1,2)\}$$

$$\overset{\circ}{G} = \{(x,y,z) \in IR^3 : z > 3\}$$
$$\overline{G} = \{(x,y,z) \in IR^3 : z \ge 3\} \cup \{(0,0,2)\} \cup \{(x,y,z) \in IR^2 : 0 \le z \le 1, x = 0 \wedge y = 0\}$$
$$G' = \{(x,y,z) \in IR^3 : z > 3\} \cup \{(x,y,z) \in IR^2 : 0 \le z \le 1, x = 0 \wedge y = 0\}$$
$$Fr(G) = \{(x,y,z) \in IR^3 : z = 3\} \cup \{(0,0,2)\} \cup \{(x,y,z) \in IR^2 : 0 \le z \le 1, x = 0 \wedge y = 0\}$$

$$\overset{\circ}{H} = \{(x,y) \in IR^2 : 0 < |x+y| < 1 \wedge xy > 0\}$$
$$\overline{H} = \{(x,y) \in IR^2 : 0 \le |x+y| \le 1 \wedge xy \ge 0\} = H'$$
$$Fr(H) = \{(x,y) : |x+y| = 1 \wedge xy \ge 0\} \cup \{(x,0) : |x| \le 1\} \cup \{(0,y) : |y| < 1, y \ne 0\}$$

$$\overline{I} = \{(x,y) : x^2 \le y \le |x|\} = I'$$
$$\overset{o}{I} = \{(x,y) : x^2 < y < |x|\}$$
$$Fr(I) = \{(x,y) : y = x^2 \wedge |x| \le 1\} \cup \{(x,y) : y = |x| \wedge 0 < |x| < 1\}$$

4.

- **i)** 0
- **ii)** No existe límite.
- **iii)** 2
- **iv)** 0
- **v)** 0
- **vi)** No existe límite.

5.

- **i)** f continua en IR^2.
- **ii)** f continua en IR^2.
- **iii)** f continua en todo IR^2 excepto en el origen.
- **iv)** f continua en IR^2.
- **v)** f continua en todo IR^3 excepto en el origen.
- **vi)** f continua en IR^3.
- **vii)** f continua en IR^3.