



UNIVERSIDAD DE CONCEPCION
FACULTAD DE CIENCIAS FISICAS Y MATEMATICAS
DEPARTAMENTO DE MATEMATICA

Solucion Listado 1

Calculo III(521227)

1.

$$A^\circ = \{(x, y) \in \mathbb{R}^2 : 0 < (x - 2)^2 + y^2 < 1\}$$

$$A^- = \{(x, y) \in \mathbb{R}^2 : 0 \leq (x - 2)^2 + y^2 \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = \frac{1}{n} \wedge y = 0, n \in \mathbb{N}\} \cup \{(0, 0)\}$$

$$A' = \{(x, y) \in \mathbb{R}^2 : 0 \leq (x - 2)^2 + y^2 \leq 1\} \cup \{(0, 0)\}$$

$$Fr(A) = \{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + y^2 = 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = \frac{1}{n} \wedge y = 0, n \in \mathbb{N}\} \cup \{(0, 0)\}$$

$$B^\circ = \{(x, y, z) \in \mathbb{R}^3 : z > 3\}$$

$$B^- = \{(x, y, z) \in \mathbb{R}^3 : z \geq 3\} \cup \{(0, 0, 2)\} \cup \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 1, x = 0 \wedge y = 0\}$$

$$B' = \{(x, y, z) \in \mathbb{R}^3 : z \geq 3\} \cup \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 1, x = 0 \wedge y = 0\}$$

$$Fr(B) = \{(x, y, z) \in \mathbb{R}^3 : z = 3\} \cup \{(0, 0, 2)\} \cup \{(x, y, z) \in \mathbb{R}^3 : 0 \leq z \leq 1, x = 0 \wedge y = 0\}$$

$$C^\circ = \{x \in \mathbb{R} : 5 < x < 8\}$$

$$C^- = \{x \in \mathbb{R} : 5 \leq x \leq 8\} \cup \{x \in \mathbb{R} : x = \frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$$

$$C' = \{x \in \mathbb{R} : 5 \leq x \leq 8\} \cup \{0\}$$

$$Fr(C) = \{5, 8\} \cup \{x \in \mathbb{R} : x = \frac{1}{n}, n \in \mathbb{N}\} \cup \{0\}$$

$$D^\circ = \{(x, y) \in \mathbb{R}^2 : 0 < (x + y)^2 < 1 \wedge xy > 0\}$$

$$D^- = \{(x, y) \in \mathbb{R}^2 : 0 \leq (x + y)^2 \leq 1 \wedge xy \geq 0\} = D'$$

$$Fr(D) = \{|x + y| = 1 \wedge xy \geq 0\} \cup \{y = 0 \wedge -1 \leq x \leq 1\} \cup \{x = 0 \wedge -1 \leq y \leq 1\}$$

$$E^{\circ} = \{(x, y) \in \mathbb{R}^2 : 0 < x^2 + y^2 < 16 \wedge x \neq y\}$$

$$E^{-} = \{(x, y) \in \mathbb{R}^2 : 0 \leq x^2 + y^2 \leq 16\} = E'$$

$$Fr(E) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 16 \vee y = x, -2\sqrt{2} \leq x \leq 2\sqrt{2}\}$$

$$F^{\circ} = \{(x, y, z, u) \in \mathbb{R}^4 : 1 < x^2 + y^2 + z^2 + u^2 < 4\}$$

$$F^{-} = \{(x, y, z, u) \in \mathbb{R}^4 : 1 \leq x^2 + y^2 + z^2 + u^2 \leq 4\} \cup \{(0, 0, 0, 0); (1, 0, 0, 0); (1, 1, 1, 2)\}$$

$$F' = \{(x, y, z, u) \in \mathbb{R}^4 : 1 \leq x^2 + y^2 + z^2 + u^2 \leq 4\}$$

$$Fr(F) = \{(x, y, z, u) \in \mathbb{R}^4 : 1 = x^2 + y^2 + z^2 + u^2 \vee x^2 + y^2 + z^2 + u^2 = 4\} \cup \{(0, 0, 0, 0); (1, 0, 0, 0); (1, 1, 1, 2)\}$$

2.

$$(i) D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \wedge y \neq -x\}$$

$$D^{-} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\} = D'$$

$$D^{\circ} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1, y \neq -x\}$$

$$Fr(D) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1 \vee y = -x\}$$

$$(ii) D = \{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + (y + 1)^2 \geq 25\} = D^{-} = D'$$

$$D^{\circ} = \{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + (y + 1)^2 > 25\}$$

$$Fr(D) = \{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + (y + 1)^2 = 25\}$$

$$(iii) D = \{(x, y, z) \in \mathbb{R}^3 : 0 < x + y + z < \pi\} = D^{\circ}$$

$$D^{-} = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x + y + z \leq \pi\} = D'$$

$$Fr(D) = \{(x, y, z) \in \mathbb{R}^3 : 0 = x + y + z \vee x + y + z = \pi\}$$

$$(iv) D = \{(0, 0)\} = D^{-} = Fr(D)$$

$$D^{\circ} = \emptyset = D'$$

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4.

(i) $\|X\|\|Y\|$

$\|X\|$

(ii) $\|(x, y)\|^n$

(iii) $\|(x, y)\|^{n+m}$

(iv) $|P + Q| \leq |P| + |Q|$ (v) 1

(vi) 1

x

5.

(i) 0

(ii) *No existe*

(iii) 5

(iv) 2

(v) *No existe*

(vi) 0

(vii) 0

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