

Solución Tarea N°2

Problema Estudiar mediante transformadas el llenado/vaciado de un estanque con válvula electrónica.

Ecuaciones dinámicas.

$$v_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_a$$

$$t_e = J_m \cdot \frac{d\omega_m}{dt} + \frac{1}{n} \cdot t_r$$

Ecuaciones de estado.

$$v_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + k_\phi \cdot I_f \omega_m$$

$$k_\phi \cdot I_f i_a = J_m \cdot \frac{d\omega_m}{dt} + \frac{1}{n} \cdot k_t \cdot \theta_r$$

$$\frac{\omega_m}{n} = \frac{d\theta_r}{dt}$$

$$\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2 \cdot \frac{dh}{dt} = k_e \cdot \theta_r - k_v \cdot \sqrt{g \cdot h}$$

$$x_1 = i_a \quad x_2 = \omega_m$$

$$x_3 = \theta_r \quad x_4 = h$$

$$\dot{\omega}_r = \frac{d\theta_r}{dt}$$

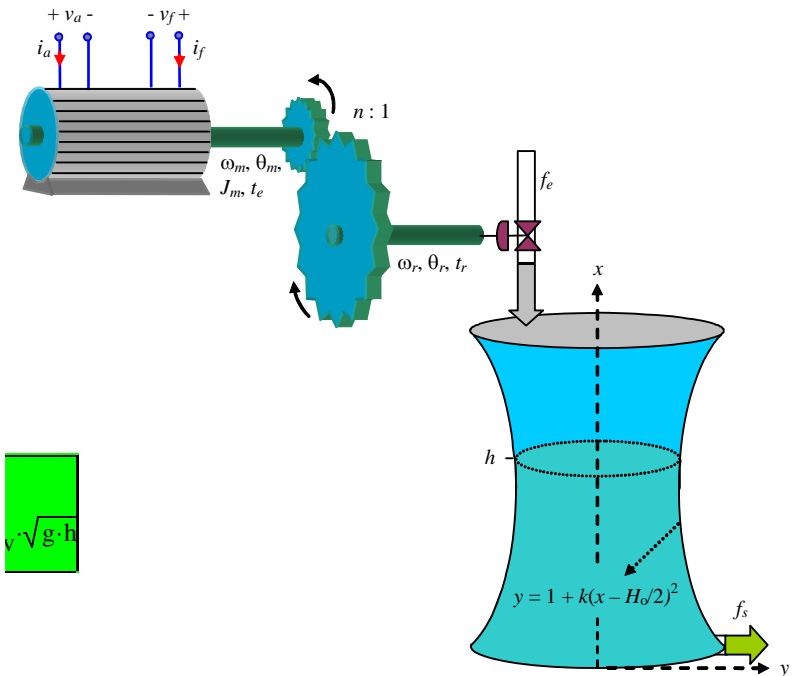
$$\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2 \cdot \frac{dh}{dt} = f_e - k_v \cdot \sqrt{g \cdot h}$$

$$\frac{di_a}{dt} = \frac{-R_a}{L_a} \cdot i_a - \frac{k_\phi \cdot I_f}{L_a} \cdot \omega_m + \frac{1}{L_a} \cdot v_a$$

$$\frac{d\omega_m}{dt} = \frac{k_\phi \cdot I_f i_a}{J_m} - \frac{1}{n \cdot J_m} \cdot k_t \cdot \theta_r$$

$$\frac{d\theta_r}{dt} = \frac{1}{n} \cdot \omega_m$$

$$\frac{dh}{dt} = k_e \cdot \frac{\theta_r}{\left[\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2 \right]} - k_v \cdot \frac{\sqrt{g \cdot h}}{\left[\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2 \right]}$$



donde las cantidades están relacionadas como sigue:

$$e_a = k_\phi \cdot I_f \omega_m \quad t_r = k_t \cdot \theta_r \quad \omega_m = n \cdot \omega_r$$

$$t_e = k_\phi \cdot I_f i_a \quad f_e = k_e \cdot \theta_r$$

$$v_a = R_a \cdot i_a + k_\phi \cdot I_f \omega_m$$

$$k_\phi \cdot I_f \frac{v_a - k_\phi \cdot I_f \omega_m}{R_a} = J_m \frac{d\omega_m}{dt} + \frac{1}{n} \cdot k_t \cdot \theta_r$$

$$\frac{\omega_m}{n} = \frac{d\theta_r}{dt}$$

$$\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2 \cdot \frac{dh}{dt} = k_e \cdot \theta_r - k_v \cdot \sqrt{g \cdot h}$$

Expresiones para h constante igual a H.

$$k_\phi \cdot I_f \frac{V_a - k_\phi \cdot I_f \Omega_m}{R_a} = \frac{1}{n} \cdot k_t \cdot \Theta_r \quad \frac{\Omega_m}{n} = 0$$

$$\Theta_r = n \cdot \frac{k_\phi}{k_t} \cdot I_f \frac{V_a}{R_a}$$

$$\Omega_m = 0$$

$$x_1 = \omega_m \quad x_2 = \theta_r \quad x_3 = h$$

$$i_a = \frac{v_a - k_\phi \cdot I_f \omega_m}{R_a}$$

$$\frac{d\omega_m}{dt} = \frac{k_\phi \cdot I_f}{J_m \cdot R_a} \cdot v_a - \frac{k_\phi^2 \cdot I_f^2}{J_m \cdot R_a} \cdot \omega_m - \frac{1}{n \cdot J_m} \cdot k_t \cdot \theta_r$$

$$\frac{d\theta_r}{dt} = \frac{1}{n} \cdot \omega_m$$

$$\frac{dh}{dt} = k_e \cdot \frac{\theta_r}{\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2} - k_v \cdot \frac{\sqrt{g \cdot h}}{\pi \cdot \left[1 + k \cdot \left(h - \frac{H_0}{2} \right)^2 \right]^2}$$

$$0 = k_e \cdot \Theta_r - k_v \cdot \sqrt{g \cdot H}$$

$$H = \frac{1}{g} \cdot \left(\frac{k_e}{k_v} \cdot n \cdot \frac{k_\phi}{k_t} \cdot I_f \frac{V_a}{R_a} \right)^2$$

$$V_a = (H \cdot g)^{\frac{1}{2}} \cdot R_a \cdot k_t \cdot \frac{k_v}{I_f k_\phi \cdot n \cdot k_e}$$

Linealización.

$$A(H) := \begin{bmatrix} \frac{k_\phi^2 \cdot I_f^2}{J_m \cdot R_a} & -\frac{1}{n \cdot J_m} \cdot k_t & 0 \\ \frac{1}{n} & 0 & 0 \\ 0 & \frac{k_e}{\text{Area}(H)} & -\frac{1}{2} \cdot \frac{k_v}{(\text{g} \cdot H)^{\frac{1}{2}}} \cdot \frac{g}{\text{Area}(H)} \end{bmatrix}$$

$$b := \begin{pmatrix} \frac{k_\phi \cdot I_f}{J_m \cdot R_a} \\ 0 \\ 0 \end{pmatrix}$$

$$c := (0 \ 0 \ 1)$$

$$\text{Area}(H) := \pi \cdot \left[1 + k \cdot \left(H - \frac{H_0}{2} \right)^2 \right]^2$$

Parte B Respuesta Matemática a Impulso en la Entrada $\Delta u(t) = d(t)$.

$$\Delta h_{hva}(s) := c \cdot (s \cdot \text{identity}(3) - A(H_1))^{-1} \cdot b \rightarrow 2 \cdot n \cdot \frac{k_e}{\left[2 \cdot s \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot \text{Area}(H_1) + k_v \cdot g \right] \cdot \left(s^2 \cdot n^2 \cdot J_m \cdot R_a + s \cdot n^2 \cdot k_\phi^2 \cdot I_f^2 + k_t \cdot R_a \right)} \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot k_\phi \cdot I_f$$

$$\Delta h_{hva}(s) = \frac{2 \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot k_\phi \cdot I_f}{n \cdot J_m \cdot R_a} \cdot \frac{1}{k_v \cdot g} \cdot \frac{k_e}{\left[2 \cdot s \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot \frac{\text{Area}(H_1)}{k_v \cdot g} + 1 \right] \cdot \left(s^2 + s \cdot k_\phi^2 \cdot \frac{I_f^2}{J_m \cdot R_a} + k_t \cdot \frac{1}{n^2 \cdot J_m} \right)}$$

$$\Delta h_{hva}(s) = k_p \cdot \frac{\omega_n^2}{(s \cdot \tau + 1) \cdot (s^2 + s \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2)}$$

$$k_p = \frac{2 \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot k_\phi \cdot I_f}{R_a} \cdot \frac{k_e \cdot n}{k_v \cdot k_t \cdot g}$$

$$\omega_n = \sqrt{k_t \cdot \frac{1}{n^2 \cdot J_m}}$$

$$\tau = 2 \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot \frac{\text{Area}(H_1)}{k_v \cdot g}$$

$$\xi = \frac{k_\phi^2}{2 \cdot \sqrt{k_t \cdot \frac{1}{n^2 \cdot J_m}}} \cdot \frac{I_f^2}{J_m \cdot R_a}$$

Tomando Laplace Inversa.

$$\Delta h_{hva}(t) := \frac{k_p}{-2 \cdot \xi \cdot \omega_n \cdot \tau + \omega_n^2 \cdot \tau^2 + 1} \cdot \omega_n^2 \cdot \left[\left(-\tau \cdot \cos(\omega_n \cdot \sqrt{1 - \xi^2} \cdot t) + \frac{1 - \tau \cdot \xi \cdot \omega_n}{\omega_n \cdot \sqrt{1 - \xi^2}} \cdot \sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot t) \right) \cdot \exp(-\xi \cdot \omega_n \cdot t) + \tau \cdot \exp\left(\frac{-t}{\tau}\right) \right]$$

Parámetros

$R_a := 250$ $L_a := 1$ $k_\phi := 200$ $I_f := 10 \cdot 10^{-3}$ $n := 50$ $k_t := 25$ $k_v := 1$ $g := 10$ $k_e := 35$ $J_m := 5 \cdot 10^{-3}$ $H_o := 4$ $k := 0.25$

$H_1 := 3$ $\Omega_{m1} := 0$ $\Omega_{m1} = 0$ $V_{a1} := (g \cdot H_1)^{\frac{1}{2}} \cdot R_a \cdot k_t \cdot \frac{k_v}{I_f \cdot k_\phi \cdot n \cdot k_e}$ $V_{a1} = 9.781$ $\Theta_{r1} := n \cdot \frac{k_\phi}{k_t} \cdot I_f \cdot \frac{V_{a1}}{R_a}$ $\Theta_{r1} \cdot \frac{180}{\pi} = 8.966$

$$\text{Area}(H) := \pi \cdot \left[1 + k \cdot \left(H - \frac{H_0}{2} \right)^2 \right]^2$$

$$k_p := \frac{2 \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot k_\phi \cdot I_f \cdot k_e \cdot n}{R_a \cdot k_v \cdot k_t \cdot g}$$

$$\omega_n := \sqrt{k_t \cdot \frac{1}{n^2 \cdot J_m}}$$

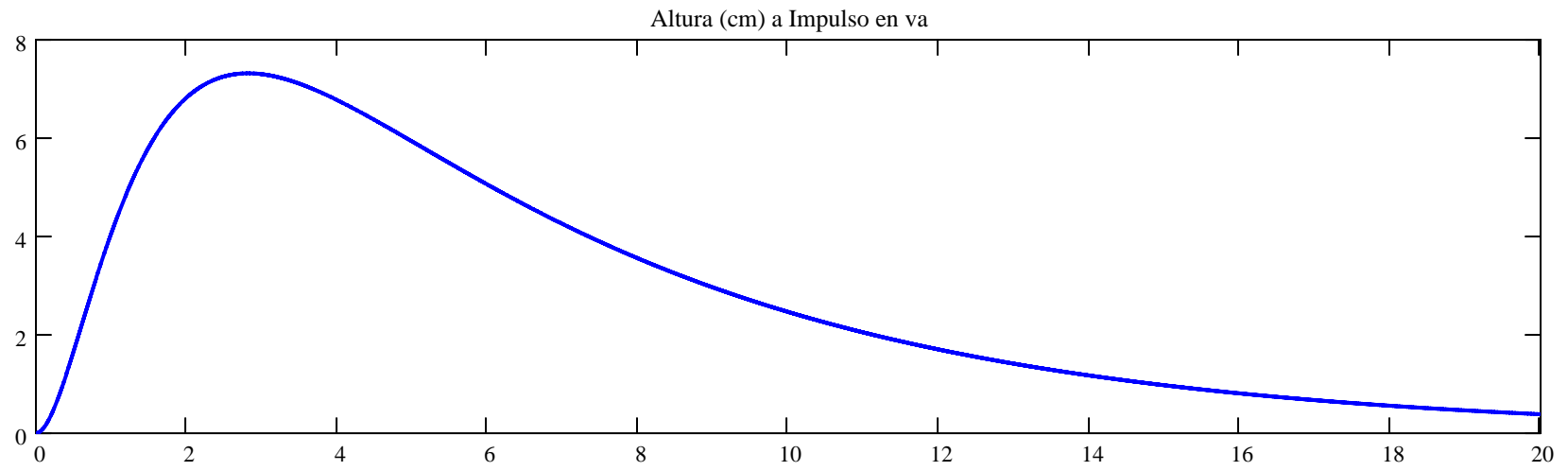
$$\tau := 2 \cdot (g \cdot H_1)^{\frac{1}{2}} \cdot \frac{\text{Area}(H_1)}{k_v \cdot g}$$

$$\xi := \frac{k_\phi^2}{2 \cdot \sqrt{k_t \cdot \frac{1}{n^2 \cdot J_m}}} \cdot \frac{I_f^2}{J_m \cdot R_a}$$

$\text{Area}(H_1) = 4.909$
 $k_p = 0.613$
 $\omega_n = 1.414$
 $\tau = 5.377$
 $\xi = 1.131$

$$\Delta h_{hva}(t) := \frac{k_p}{-2 \cdot \xi \cdot \omega_n \cdot \tau + \omega_n^2 \cdot \tau^2 + 1} \cdot \omega_n^2 \cdot \left[\left(-\tau \cdot \cos(\omega_n \cdot \sqrt{1 - \xi^2} \cdot t) + \frac{1 - \tau \cdot \xi \cdot \omega_n}{\omega_n \cdot \sqrt{1 - \xi^2}} \cdot \sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot t) \right) \cdot \exp(-\xi \cdot \omega_n \cdot t) + \tau \cdot \exp\left(\frac{-t}{\tau}\right) \right]$$

$t_f := 20$
 $m_f := 5000$
 $m := 0 .. m_f$
 $t := 0, \frac{t_f}{m_f} .. t_f$



$$A(H) := \begin{bmatrix} \frac{k_\phi^2 \cdot I_f^2}{J_m \cdot R_a} & -\frac{1}{n \cdot J_m} \cdot k_t & 0 \\ \frac{1}{n} & 0 & 0 \\ 0 & \frac{k_e}{\text{Area}(H)} & \frac{-1}{2} \cdot \frac{k_v}{\frac{1}{\text{Area}(H)}} \cdot \frac{g}{(g \cdot H)^2} \end{bmatrix}$$

$$b := \begin{pmatrix} \frac{k_\phi \cdot I_f}{J_m \cdot R_a} \\ 0 \\ 0 \end{pmatrix}$$

$$c := (0 \ 0 \ 1)$$

Simulación.

$$\text{del}_T(t, T) := \frac{1}{T} \cdot (\Phi(t) - \Phi(t - T))$$

$$u(t, T) := \text{del}_T(t, T)$$

$$D_1(t, x) := A(H_1) \cdot (x_1 \ x_2 \ x_3)^T + b \cdot u(t, 2.00)$$

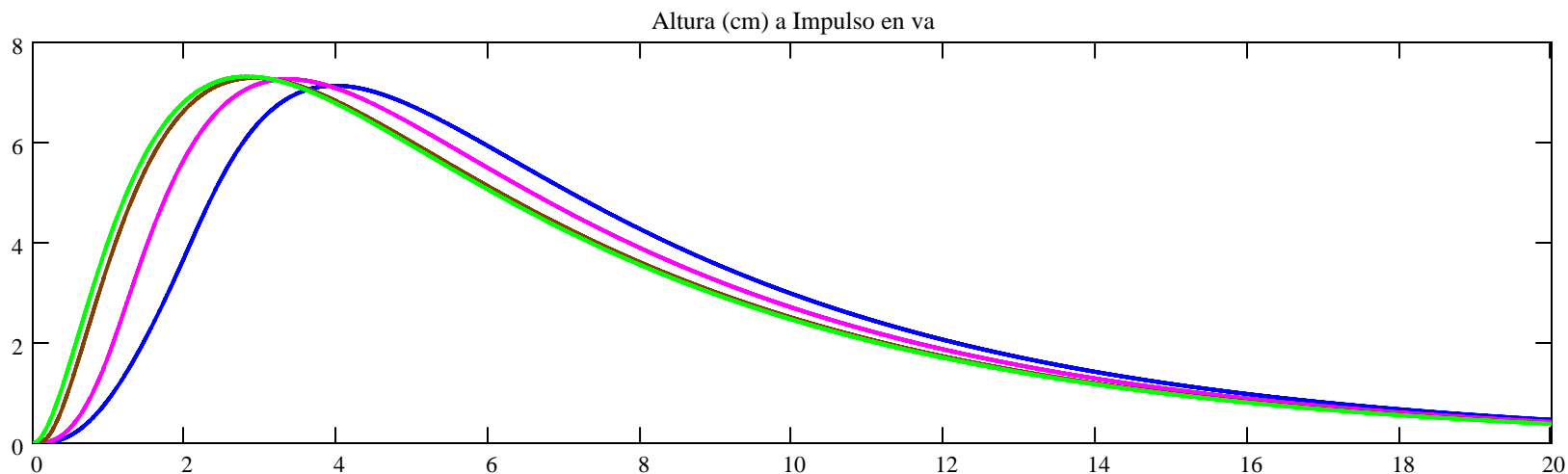
$$CI := (0 \ 0 \ 0)^T \quad Z_{u1} := \text{rkfixed}(CI, 0, t_f, m_f, D_1)$$

$$D_2(t, x) := A(H_1) \cdot (x_1 \ x_2 \ x_3)^T + b \cdot u(t, 1.00)$$

$$CI := (0 \ 0 \ 0)^T \quad Z_{u2} := \text{rkfixed}(CI, 0, t_f, m_f, D_2)$$

$$D_3(t, x) := A(H_1) \cdot (x_1 \ x_2 \ x_3)^T + b \cdot u(t, 0.20)$$

$$CI := (0 \ 0 \ 0)^T \quad Z_{u3} := \text{rkfixed}(CI, 0, t_f, m_f, D_3)$$

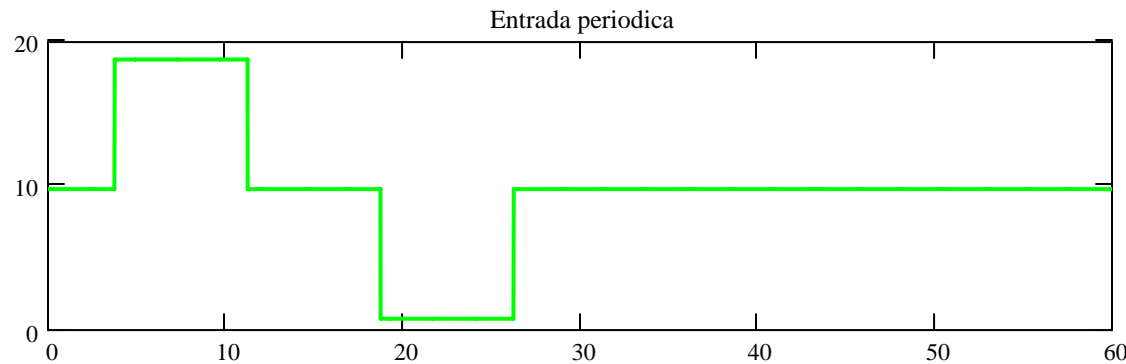


Entrada $u(t)$, un periodo.

$$T_o := 30 \quad t_0 := T_o \cdot \frac{45}{360} \quad t_1 := \frac{T_o}{2} - T_o \cdot \frac{45}{360} \quad t_2 := \frac{T_o}{2} + T_o \cdot \frac{45}{360} \quad t_3 := T_o - T_o \cdot \frac{45}{360} \quad Am := 9$$

$$\Delta u(t) := Am \cdot (\Phi(t - t_0) - \Phi(t - t_1) - \Phi(t - t_2) + \Phi(t - t_3)) \quad \text{La entrada es la suma de escalones.}$$

$$t_f := 2 \cdot T_o \quad t_i := 0 \quad t := t_i, t_i + \frac{t_f - t_i}{m_f} .. t_f \quad F_o := \frac{1}{T_o}$$



Cálculo de la T. de L. de la entrada $\Delta u(t)$,

$$\int_{-\infty}^{\infty} Am \cdot (\Phi(t - t_0) - \Phi(t - t_1) - \Phi(t - t_2) + \Phi(t - t_3)) \cdot \exp(-s \cdot t) dt \quad \left| \begin{array}{l} \text{simplify} \\ \text{float, 6} \end{array} \right. \rightarrow -9 \cdot \exp(-3.75000 \cdot s) \cdot \frac{-1 + \exp(-7.50000 \cdot s) + \exp(-15 \cdot s) - 1 \cdot \exp(-22.5000 \cdot s)}{s}$$

Laplace de la entrada $\Delta u(t)$ que se denotará por $\Delta u_s(s)$ es,

$$\Delta u_s(s) := Am \cdot \frac{\exp(-t_0 \cdot s) - 1 \cdot \exp(-t_1 \cdot s) - 1 \cdot \exp(-t_2 \cdot s) + \exp(-t_3 \cdot s)}{s}$$

Laplace de la respuesta a impulso $\Delta h(t)$ que se denotará por $\Delta h_s(s)$ es,

$$\Delta h_s(s) := k_p \cdot \frac{\omega_n^2}{(s \cdot \tau + 1) \cdot (s^2 + s \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2)}$$

T. de F. de $u(t)$ que se denotará por $u_\omega(\omega)$ es,

$$\Delta u_\omega(\omega) := Am \cdot \frac{\exp(-t_0 \cdot j \cdot \omega) - 1 \cdot \exp(-t_1 \cdot j \cdot \omega) - 1 \cdot \exp(-t_2 \cdot j \cdot \omega) + \exp(-t_3 \cdot j \cdot \omega)}{j \cdot \omega}$$

T. de F. de $h(t)$ que se denotará por $h_\omega(\omega)$ es,

$$\Delta h_\omega(\omega) := k_p \cdot \frac{\omega_n^2}{(j \cdot \omega \cdot \tau + 1) \cdot [(j \cdot \omega)^2 + j \cdot \omega \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2]}$$

Laplace de la salida $\Delta y(t)$ que se denotará por $\Delta y_s(s)$ es,

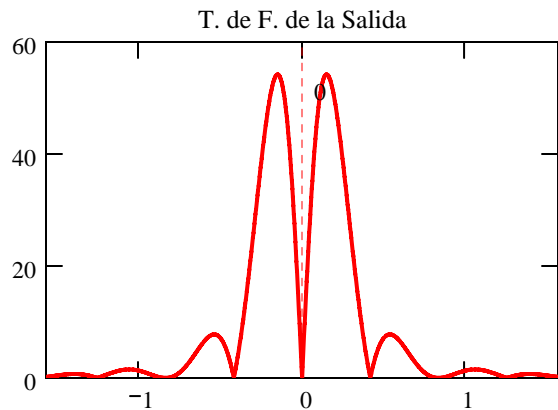
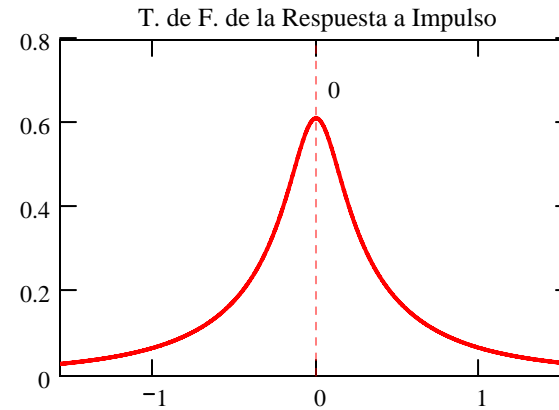
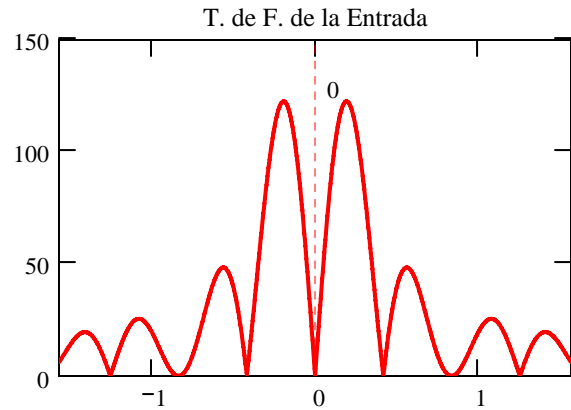
$$\Delta y_s(s) := k_p \cdot \frac{\omega_n^2}{(s \cdot \tau + 1) \cdot (s^2 + s \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2)} \cdot \text{Am} \cdot \frac{\exp(-t_0 \cdot s) - 1 \cdot \exp(-t_1 \cdot s) - 1 \cdot \exp(-t_2 \cdot s) + \exp(-t_3 \cdot s)}{s}$$

T. de F. de $y(t)$ que se denotará por $y_\omega(\omega)$ es,

$$\Delta y_\omega(\omega) := k_p \cdot \frac{\omega_n^2}{(j \cdot \omega \cdot \tau + 1) \cdot [(j \cdot \omega)^2 + j \cdot \omega \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2]} \cdot \text{Am} \cdot \frac{\exp(-t_0 \cdot j \cdot \omega) - 1 \cdot \exp(-t_1 \cdot j \cdot \omega) - 1 \cdot \exp(-t_2 \cdot j \cdot \omega) + \exp(-t_3 \cdot j \cdot \omega)}{j \cdot \omega}$$

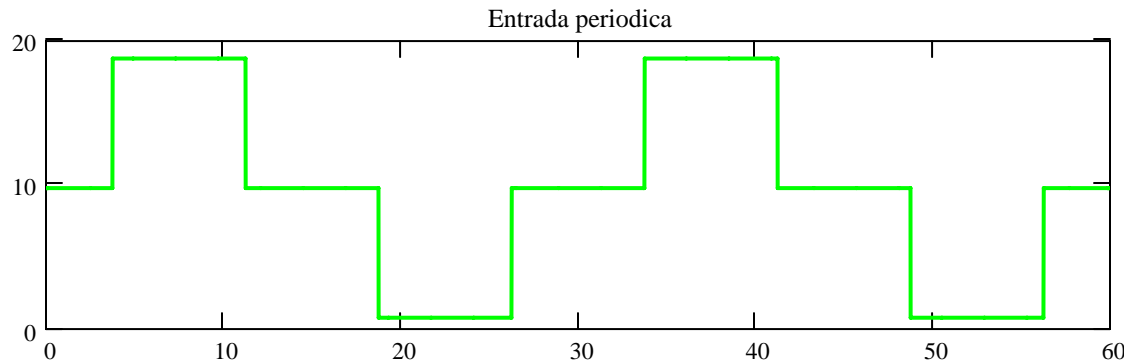
$$\omega_i := -0.5 \cdot \pi \quad \omega_f := 0.5 \cdot \pi \quad n_\omega := 1000$$

$$\omega := \omega_i, \omega_i + \frac{\omega_f - \omega_i}{n_\omega} .. \omega_f$$



$$T_o := T_o \quad \Delta u(t) := Am \cdot (\Phi(t - t_0) - \Phi(t - t_1) - \Phi(t - t_2) + \Phi(t - t_3)) \Delta u_p(t) := \sum_{m=0}^2 \Delta u(t - m \cdot T_o)$$

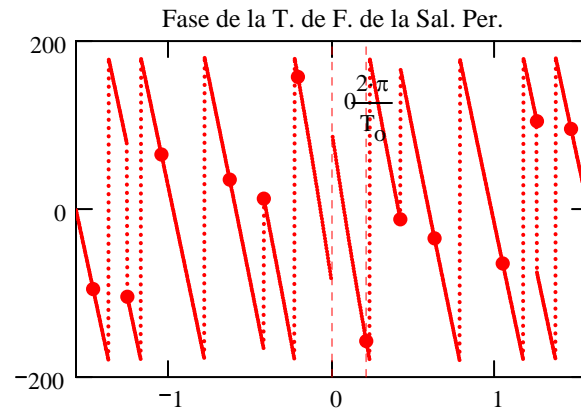
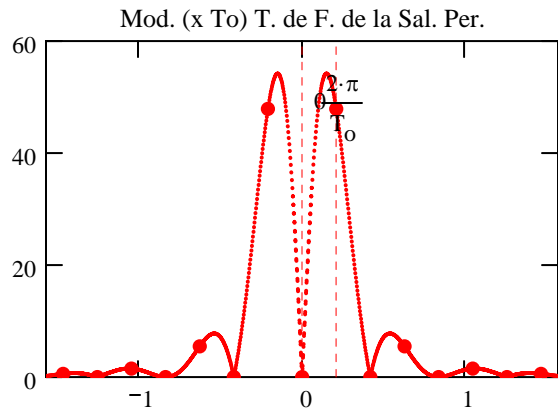
$$t_f := 2 \cdot T_o \quad t_i := 0 \quad t := t_i, t_i + \frac{t_f - t_i}{m_f} \dots t_f \quad F_o := \frac{1}{T_o}$$



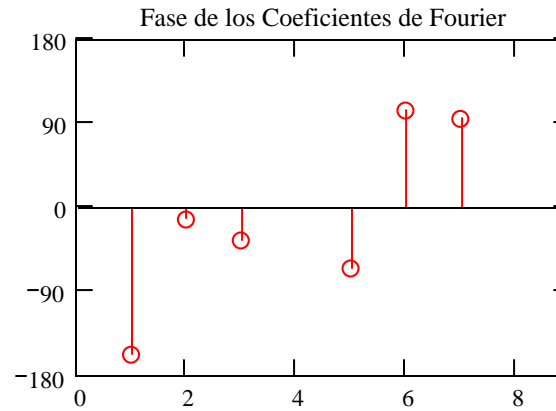
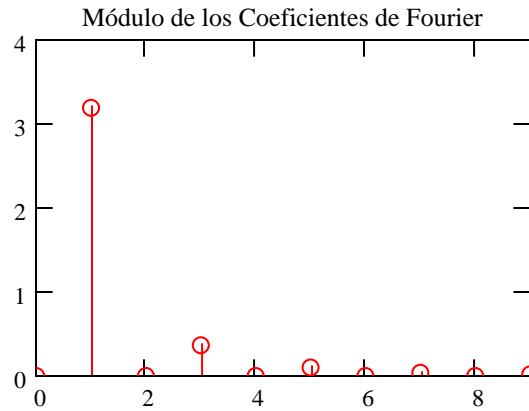
T. de F. de la salida periodica $\Delta y_p(t)$ que se denotará por $\Delta y_{pn}(nn)$ es,

$$\Delta y_{pn}(nn) := \frac{1}{T_o} \cdot k_p \cdot \frac{\omega_n^2}{[j \cdot (2 \cdot nn \cdot \pi \cdot F_o) \cdot \tau + 1] \cdot [j \cdot (2 \cdot nn \cdot \pi \cdot F_o)]^2 + j \cdot (2 \cdot nn \cdot \pi \cdot F_o) \cdot 2 \cdot \xi \cdot \omega_n + \omega_n^2]} \cdot Am \cdot \frac{\exp[-t_0 \cdot j \cdot (2 \cdot nn \cdot \pi \cdot F_o)] \dots + -1 \cdot \exp[-t_1 \cdot j \cdot (2 \cdot nn \cdot \pi \cdot F_o)] \dots + -1 \cdot \exp[-t_2 \cdot j \cdot (2 \cdot nn \cdot \pi \cdot F_o)] + \exp[-t_3 \cdot j \cdot (2 \cdot nn \cdot \pi \cdot F_o)]}{j \cdot (2 \cdot nn \cdot \pi \cdot F_o)}$$

$$n_i := -9 \quad n_f := 9 \quad nn := n_i, n_i + \frac{n_f - n_i}{n_f - n_i} \dots n_f$$



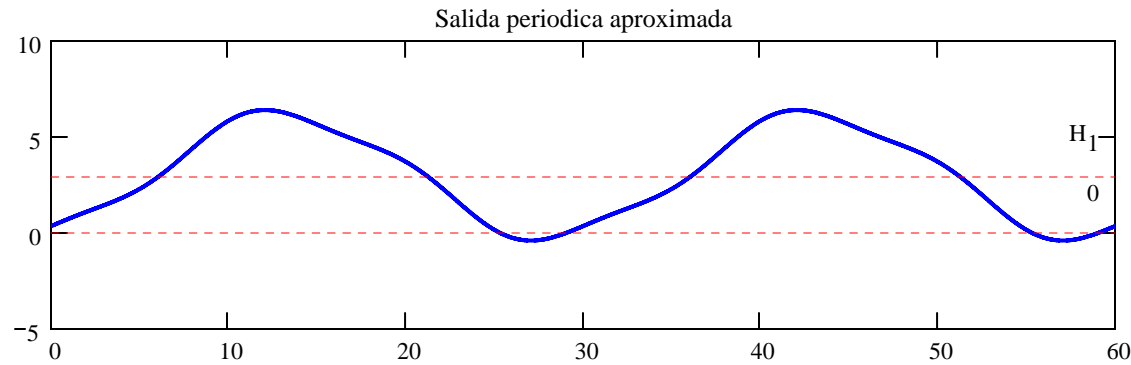
Parte F Coeficientes de Fourier de la salida $\Delta y(t)$ para una entrada periódica $\Delta u(t)$ dada.



Parte G Reconstrucción de la Salida en base a cuatro de los Coeficientes de Fourier.

$$\text{Sal}_{157}(t) := 2 \cdot |\Delta y_{pn}(1)| \cdot \cos(1 \cdot 2 \cdot \pi \cdot F_o \cdot t + \arg(\Delta y_{pn}(1))) + 2 \cdot |\Delta y_{pn}(3)| \cdot \cos(3 \cdot 2 \cdot \pi \cdot F_o \cdot t + \arg(\Delta y_{pn}(3)))$$

$$t_f := 2 \cdot T_o \quad t_i := 0 \quad m_f := 2000 \quad t := t_i, t_i + \frac{t_f - t_i}{m_f} .. t_f$$



Parte H Simulación del sistema con la Entrada Periodica.

$$\Delta u(t) := \Delta u_p(t) \quad t_f := T_o \cdot 2 \quad m_f := 2000 \quad m := 0 .. m_f$$

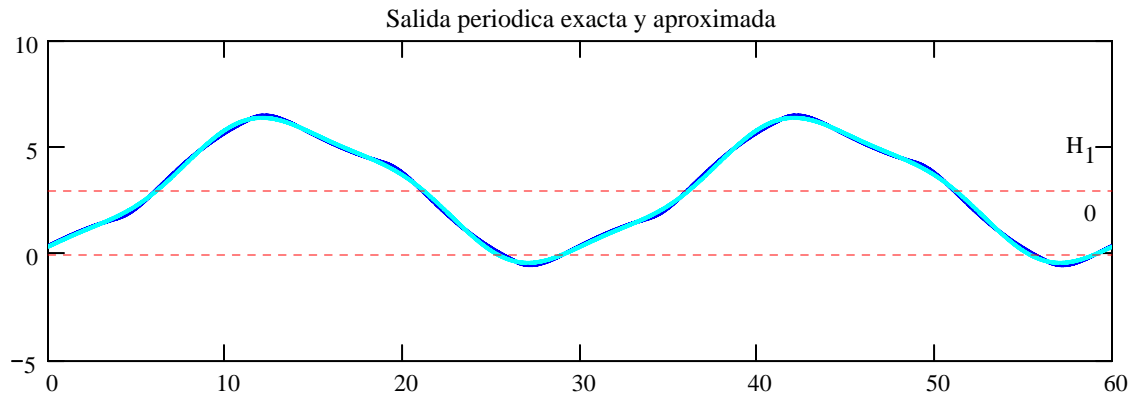
Simulación para encontrar la C.I.

$$D_1(t, x) := A(H_1) \cdot \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T + b \cdot \Delta u(t) \quad CI := (0 \ 0 \ 0)^T$$

$$Z_{p1} := \text{rkfixed}(CI, 0, t_f, m_f, D_1)$$

Simulación

$$D_1(t, x) := A(H_1) \cdot \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T + b \cdot \Delta u(t) \quad CI := \begin{pmatrix} Z_{p1m_f,2} & Z_{p1m_f,3} & Z_{p1m_f,4} \end{pmatrix}^T \quad Z_{p1} := \text{rkfixed}(CI, 0, t_f, m_f, D_1)$$



Parte I Simulación Sistema Original.

$$v_a(t) := V_{a1} + \Delta u(t) \quad u(t) := v_a(t)$$

$$D(t, x) := \begin{bmatrix} \frac{k_\phi \cdot I_f u(t)}{J_m \cdot R_a} - \frac{k_\phi^2 \cdot I_f^2 \cdot x_1}{J_m \cdot R_a} - \frac{1}{n \cdot J_m} \cdot k_t \cdot x_2 \\ \frac{1}{n} \cdot x_1 \\ x_2 \\ k_e \cdot \frac{x_2}{\left[\pi \cdot \left[1 + k \cdot \left(x_3 - \frac{H_0}{2} \right)^2 \right]^2 \right]} - k_v \cdot \frac{\sqrt{g \cdot x_3}}{\left[\pi \cdot \left[1 + k \cdot \left(x_3 - \frac{H_0}{2} \right)^2 \right]^2 \right]} \end{bmatrix}$$

$$Z_{a2} := \text{rkfixed}\left(\begin{pmatrix} \Omega_{m1} & \Theta_{r1} & H_1 \end{pmatrix}^T, 0, t_f, m_f, D\right)$$

$$CI := \begin{pmatrix} Z_{a2m_f,2} & Z_{a2m_f,3} & Z_{a2m_f,4} \end{pmatrix}^T$$

$$Z_{a2} := \text{rkfixed}(CI, 0, t_f, m_f, D)$$

