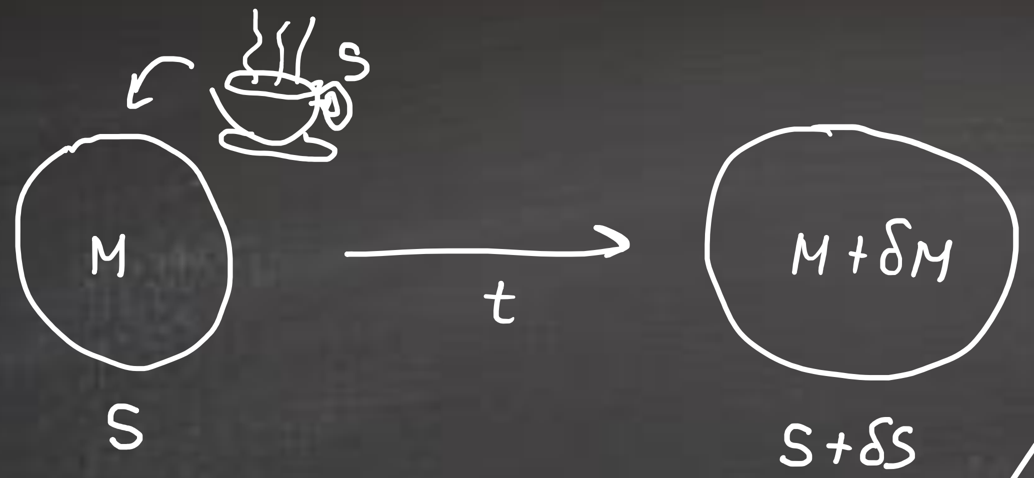
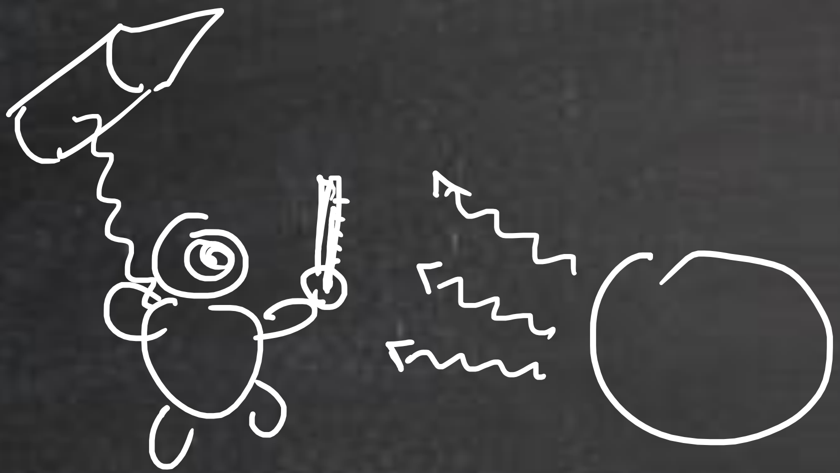
Escuela de Agujeros Negros Clásicos y Cuánticos
(GRAV@UDEC2021)

Universidad de Concepción
Enero de 2021





$$\boxed{S = \left(\frac{1}{4}\right) \frac{k_B\,\text{Area}\,c^3}{\hbar G}}$$

$$dE = T\,dS$$

$$c^2\,dM = T_H\,d\left(\quad\right)$$

$$\boxed{T_H = \frac{\hbar c^3}{k_B\, 8\pi G M}}$$

QFT + GR

$$\int dt \int d^3\tilde{x} \sqrt{\det \eta_{\mu\nu}} = \int dt \int d\phi \int d\theta \sin\theta \, r^2 dr \qquad (c=1)$$

$$\boxed{S = \frac{1}{2\pi} \int dt \int d^3x \left[\frac{1}{2} \underbrace{\partial_\mu \phi \partial^\mu \phi}_{\nabla_\mu \phi \nabla^\mu \phi} - \frac{1}{2} m^2 \phi^2 - \xi R \phi^2 \right] \underbrace{\sqrt{\det(g_{\mu\nu})}}_{\sqrt{g}}} + \frac{1}{16\pi G} \int dt \int d^3x \sqrt{g}\, R$$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$$

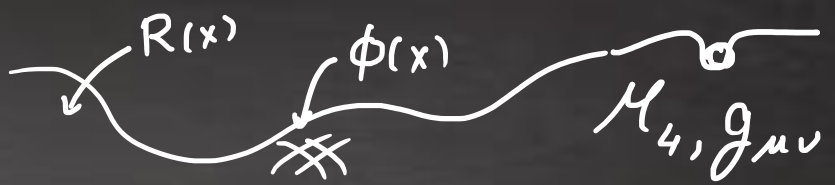
- $\boxed{\Box \phi + m^2 \phi + \xi R \phi = 0}$ $\quad R_{\mu\nu} g^{\mu\nu}$ $\quad \cancel{\left(R_{\mu\nu} R^{\mu\nu}\right)^{DNI} \phi^{\widetilde{DNI}}}$
-

$$\Box \phi = \nabla_\mu \nabla^\mu \phi = \nabla_\mu \partial^\mu \phi = \nabla_\mu g^{\mu\nu} \partial_\nu \phi = g^{\mu\nu} \nabla_\mu \partial_\nu \phi =$$
$$= \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g}\, g^{\mu\nu} \partial_\nu \right) \phi$$

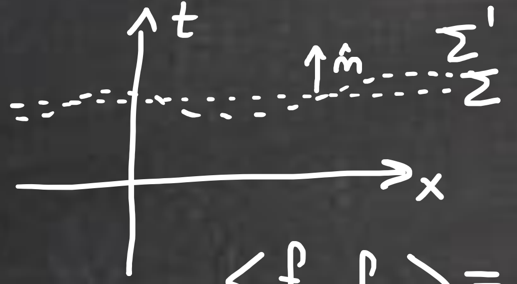
$$\nabla_\mu \partial^\mu f + m^2 f + R f = 0$$

sol. KG

$$\boxed{\Box\phi + m^2\phi + \xi\phi R = 0}$$ K-G



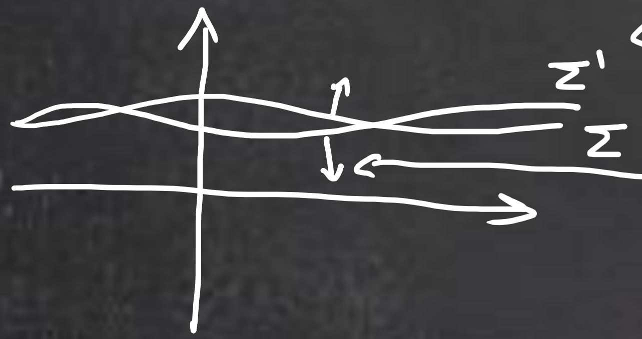
$$\langle f_1, f_2\rangle_\Sigma \equiv i\int \underbrace{d\Sigma^\mu}_{d\Sigma\, n^\mu} \left(\underbrace{f_2^*\partial_\mu f_1 - f_1^*\partial_\mu f_2}_{f_2^* \overleftrightarrow{\partial_\mu} f_1}\right)$$



$$\|$$

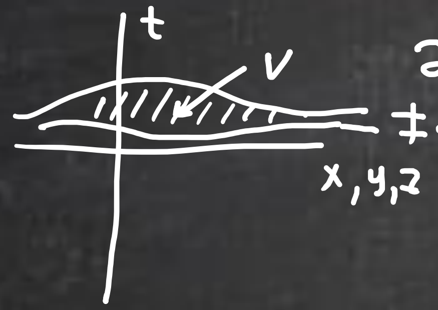
$$\langle f_1, f_2\rangle_{\Sigma'} \equiv i\int d\Sigma'^\mu \left(f_2^*\partial_\mu f_1 - f_1^*\partial_\mu f_2\right)$$

$$\langle f_1, f_2\rangle_\Sigma - \langle f_1, f_2\rangle_{\Sigma'} \overset{?}{=} 0$$



dim 3 dim = 4

$$\langle f_1,f_2\rangle_\Sigma - \langle f_1,f_2\rangle_{\Sigma'} = i\oiint_{\partial V}\left(f_2^*\overleftrightarrow{\partial_\mu} f_1\right) d\Sigma^\mu \overset{\text{Stokes}}{=} i\iiiint_V \nabla_\mu\left(f_2^*\overleftrightarrow{\partial_\mu} f_1\right) dV =$$



$\partial V = \Sigma \cup \Sigma' \cup \dots$

$f_{1,2}$ vayan a cero a $|x| \to \infty$

$$= i\iiiint_V dv\left[\nabla_\mu\left(f_2^*\partial_\mu f_1 - f_1\partial_\mu f_2^*\right)\right]$$

$$= f_2^*\underbrace{\nabla_\mu\partial^\mu}_{\Box} f_1 - f_1\underbrace{\nabla_\mu\partial^\mu}_{\Box} f_2^*$$

$$+ \cancel{\nabla f_2\,\partial f_1} - \cancel{\nabla f_2\,\partial f_1} =$$

f_1, f_2 satisfacen K-G

$$= +\iiiint_V dv\left(f_2^*\left(\cancel{-m^2} - \xi R\right) f_1 + f_1\left(\cancel{m^2} + \xi R\right) f_2^*\right)$$

$$= 0.$$

$\phi = \int dk \left(a_k e^{i\cdots} + a_k^* e^{-i\cdots} \right)$

f

$\langle e^{ik x}, e^{ik' x} \rangle = \delta_{kk'}$

f_1

$S = \int d^4x \sqrt{g} \underbrace{\left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + \xi \phi^2 \right)}_{\mathcal{L}}$

$\uparrow t \quad \tilde{t}(t)$

$\Sigma \; (\tilde{t} = \text{const})$

$\frac{d}{d\hat{t}} = \cdot$

$\phi(x) \;,\; \Pi(x) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \xrightarrow{\text{QFT}} \hat{\phi}(x) \;,\; \hat{\Pi}(x)$

$[\hat{\phi}(x), \hat{\Pi}(y)] = i\delta^{(3)}(x-y)$

$\int d^3k$

análogo $e^{ik_\mu x^\mu}$

$\phi = \sum_k \left(a_k f_k + a_k^* f_k^* \right)$

en general

$(\square + m^2 + \xi R) f_k = 0$

$\int d\Sigma \, \delta(x-y) = 1$

$\cdot \hat{\phi} = \sum_k \Big(\underbrace{\hat{a}_k}_{\text{aniquil.}} f_k + \underbrace{\hat{a}^\dagger_k}_{\text{creación}} f_k^* \Big)$

$\exists |0\rangle \,/\, a_k |0\rangle = 0 \;\forall k$

$|n_k\rangle \sim a^\dagger_k |n_k - 1\rangle$

$\cdot \hat{\phi} = \sum_k \left[\hat{b}_k F_k + \hat{b}^\dagger_k F_k^* \right]$

$$(f_j, f_{j'}) = (F_j, F_{j'}) = \delta_{jj'}$$

$$(f_j^*, f_{j'}^*) = (F_j^*, F_{j'}^*) = -\delta_{ij}$$

$$(f_j, f_j^*) = 0$$

$$f_j = \sum_k \left(\alpha_{jk} F_k + \beta_{jk} F_k^*\right)$$

$$\updownarrow$$

$$F_k = \sum_j \left(\alpha^*_{jk} f_j - \beta_{jk} f_j^*\right)$$

$$a_k|0\rangle_{(f)} = 0 \quad \forall k$$

$$b_k|0\rangle_{(F)} = 0 \quad \forall k$$

$$a_j = \sum_k \left(\alpha^*_{jk} b_k - \beta^*_{jk} b_k^\dagger\right)$$

$$b_k = \sum_j \left(\alpha_{jk} a_j + \beta^*_{jk} a_j^\dagger\right)$$

$$\phi = \sum_k \left(a_k f_k + a_k^* f_k^*\right) = \sum_j \left(b_j F_j + b_j^* F_j^*\right)$$

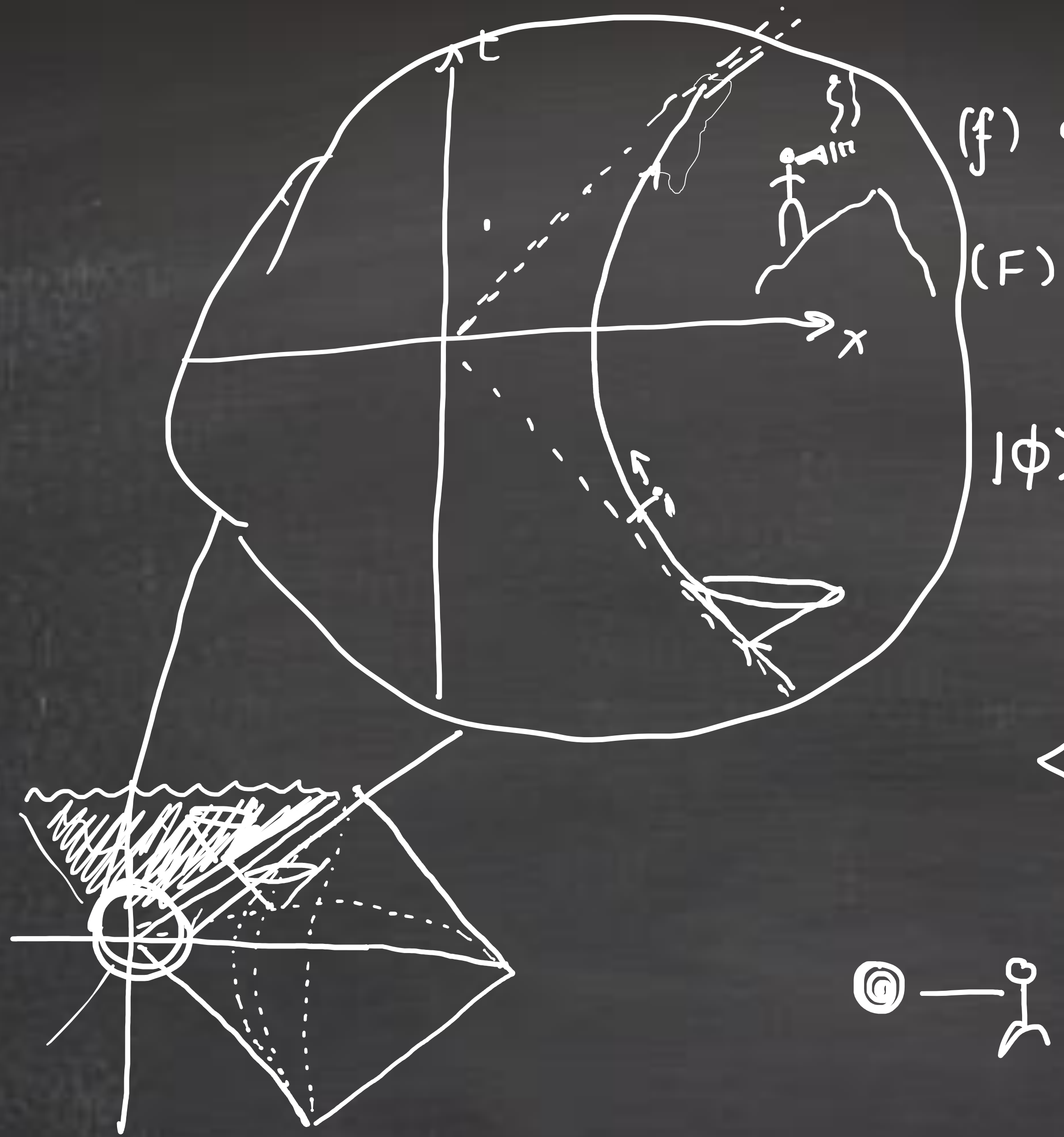
$$\langle \hat{N} \rangle$$

$$a_k|0\rangle_{(f)} = 0$$

$$N_f = {}_{(f)}\langle 0|\sum_k a_k^\dagger \overbrace{a_k|0\rangle_{(f)}}^{0} = 0 \;;\quad {}_{(f)}\langle 0| \overbrace{\sum_k b_k^\dagger \, b_k}^{N_F} \underbrace{|0\rangle_{(f)}}_{?} \overset{?}{=} \sum_k \sum_j |\beta_{jk}|^2$$

$${}_{(F)}\langle 0|\sum_k b_k^\dagger \overbrace{b_k|0\rangle_{(F)}}^{0} = 0$$

$$\boxed{{}_{(f)}\langle 0|N_F|0\rangle_{(f)}} \neq \boxed{{}_{(f)}\langle 0| \underbrace{b_k^\dagger \, b_k}_{{}_{(f)}\langle N_F\rangle_{(f)}} |0\rangle_{(f)} = \sum_j |\beta_{jk}|^2 \neq 0}$$



(f) $ds^2 = -\underbrace{dt^2} + dx^2 \longrightarrow f$

$$\boxed{(F)\quad ds^2 = -\underline{a^2}\,\tilde{x}^2\,\underbrace{d\tau^2} + \underline{d\tilde{x}^2} \longrightarrow F}$$

$$|\phi\rangle = |0\rangle_{(f)} \overset{?}{\neq} |0\rangle_{(F)}$$

$$\langle N \rangle \sim a$$

$$ds^2_{BH} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r_H) = 0 \rightarrow f(r) \simeq \cancel{f(r_H)} + \underset{\neq 0}{f'(r_H)}\,(r - r_H) + \frac{1}{2} f''(r_H)\,(r - r_H)^2 + \dots$$

$$\underset{\uparrow \; r \simeq r_H}{ds^2_{BH} \simeq} -f'(r_H)\,(r - r_H)\,dt^2 + \underbrace{\frac{dr^2}{f'(r_H)\,(r - r_H)}}_{d\rho^2} + r_H^2\, d\Omega^2 + \cancel{\dots}$$

$$= -\underbrace{\left(\frac{f'(r_H)}{2}\right)^2}\rho^2 dt^2 + d\rho^2$$

$$\boxed{\rho = 2\sqrt{\frac{r - r_H}{f'(r_H)}}} \rightarrow (r - r_H) = \frac{f'(r_H)}{4}\rho^2$$

$$a \sim \frac{f'(r_H)}{2} \sim \kappa$$

$$f = 1 - \frac{2MG}{r}$$