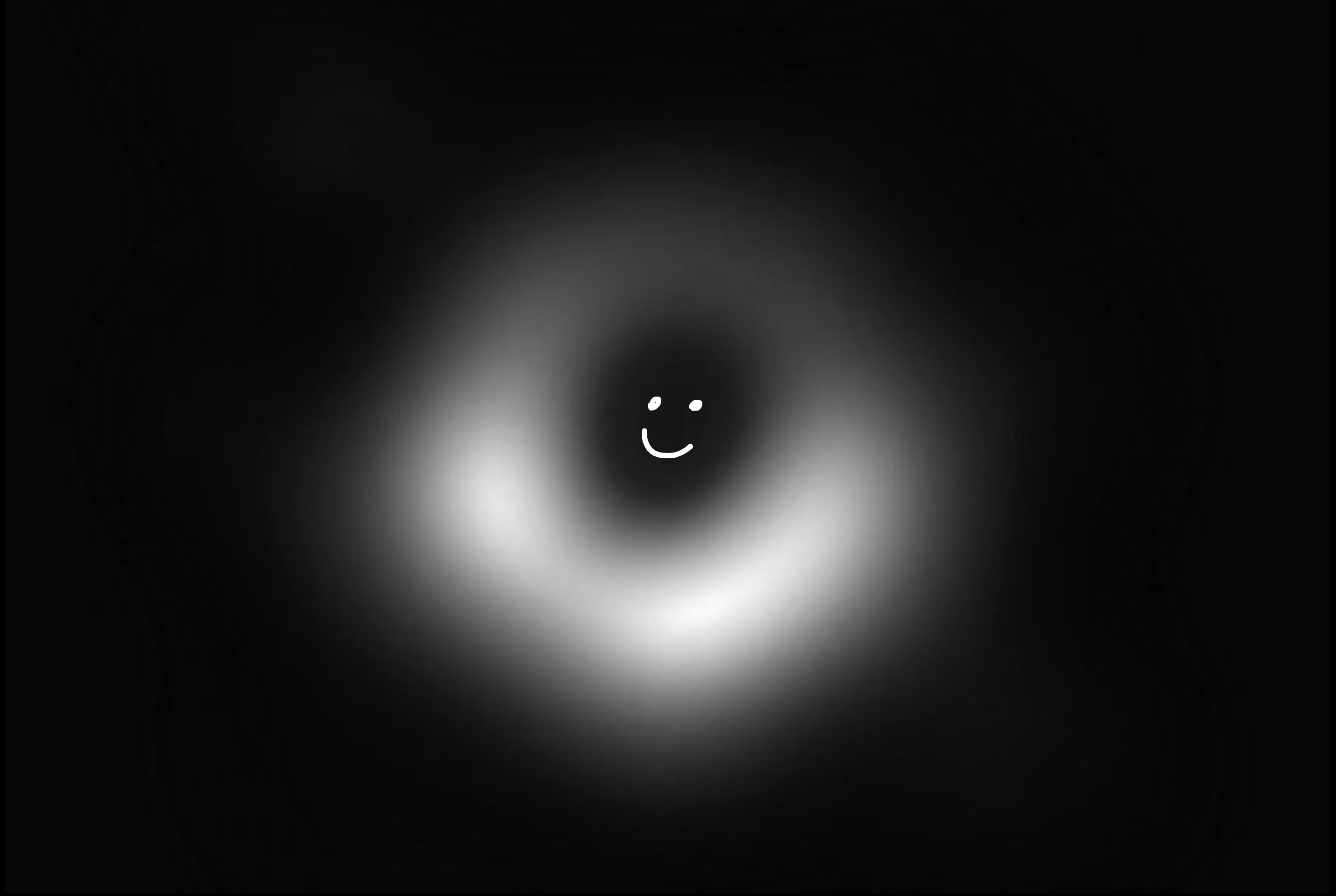
Escuela de Agujeros Negros Clásicos y Cuánticos

(GRAV@UDEC2021)



Universidad de Concepción

Enero de 2021

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{g(r)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \rightarrow$$

. estático

muchas veces

$$1 + \frac{2\Phi(r)}{c^2} = \underbrace{f(r)}_{-g_{tt}} = g(r)$$

$$\mathcal{E}_{elect} = \int \underbrace{dvol}_{r^2} \underbrace{|\vec{E}|^2}_{\frac{1}{r^4}}$$

$$\mathcal{E}_{\Lambda} = \int dvol\, \frac{\Lambda}{8\pi G}$$

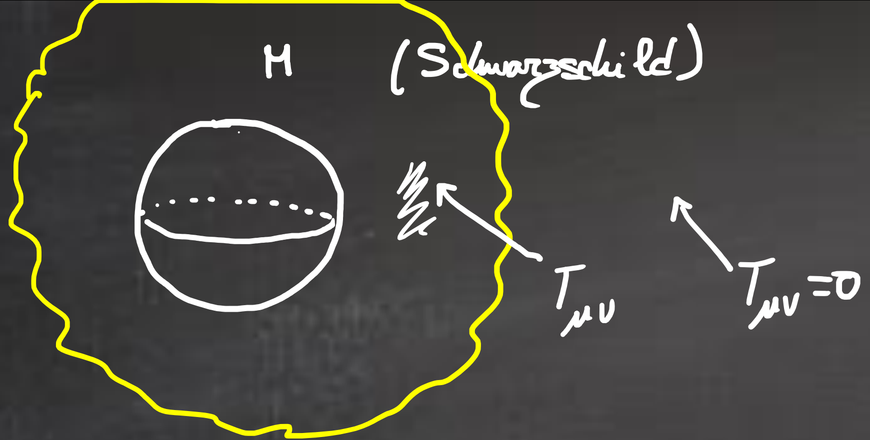
$$f(r) = 1 - \frac{2G\cancel{M}}{c^2}\frac{1}{r}\left(M + \int_0^r \underbrace{dvol}_{4\pi\, dr\, r^2} \cancel{4\pi}\, T^0_0(r)\right) = 1 - \frac{2MG}{c^2 r} + \frac{GQ^2}{r^2} - \frac{\Lambda}{3} r^2 = f(r)$$

$$J = 0 = Q = \Lambda$$

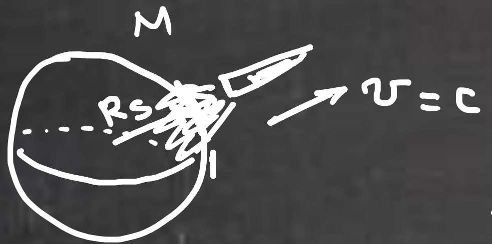
M

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \\ \nabla_\mu F^{\mu\nu} = 0 \end{cases}$$

Maxwell



$$\boxed{\begin{array}{c} f(r_H)=0 \\ \| \\ R_s \end{array}}$$



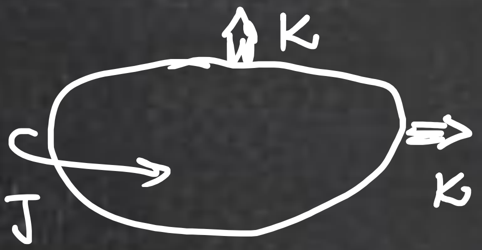
$$\frac{1}{2} m \underset{c^2}{\underset{\|}{v^2}} = \frac{GMm}{R_s} \rightarrow \boxed{R_s = \frac{2MG}{c^2}}$$

R. Sch.

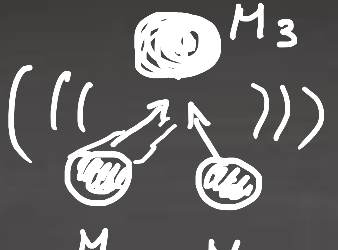
$$F = m a_{sup}$$

$$m\frac{MG}{R_s^2} = m a_{sup} \Rightarrow \boxed{\underbrace{a_{sup}}_{K} = \frac{c^4}{4MG}}$$

Surf. grav.



A) Hawking (circa 1970)



Energía: $M_1 + M_2 \rightarrow M_3 + \text{ondas}$

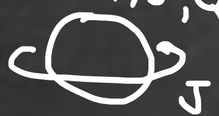
$$M_1 + M_2 \geq M_3$$

$$M_1^2 + M_2^2 + 2M_1M_2 \geq M_3^2$$

Hawking: $M_1^2 + M_2^2 \leq M_3^2$

$$\left.\begin{array}{l} R = \dfrac{2GM}{c^2} \\ \text{Area} = 4\pi R^2 \end{array}\right\} \boxed{\text{Area}_1 + \text{Area}_2 \leq \text{Area}_3}$$

B) Israel - Carter



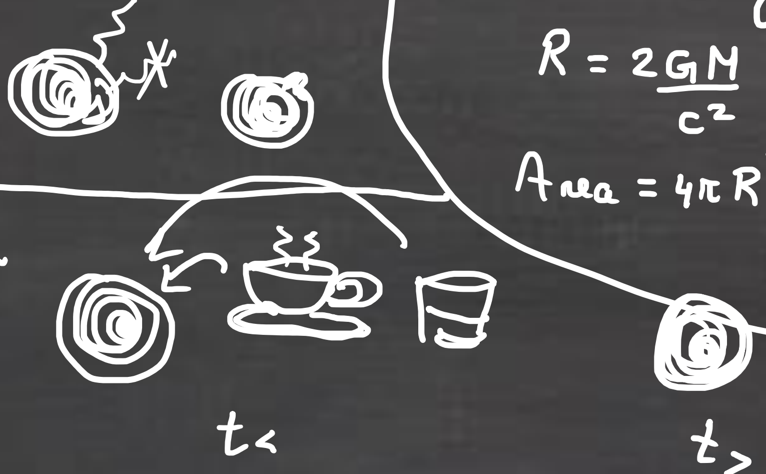
(K-N) ⊃ K ⊃ S

(M, J, Q) (M, J) (M)

Teorema de NO-hair

C) Wheeler - Bekenstein

¿S?



D) Bardeen - Hawking - Carter { analogías entre Termo. y BH's

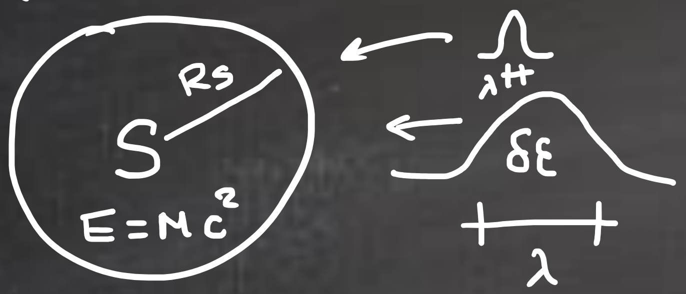
Termo	BH's
1) 0 p.	1) κ es la misma
2) 1er p. $dE = T\,dS - P\,dV$ (dq)	$\rightarrow d\,Mc^2 = \frac{\kappa}{2\pi}\,dA_{\text{...}} + \ldots$
3) 2do p. $dS > 0$	$dA > 0$
4) 3 do p. →	...

E) P. Penrose

$S = -I\,k_B$

$$R_s = \frac{2MG}{c^2} \quad , \quad E = Mc^2$$

$G, c, \hbar, k_B$



$\boxed{\lambda \simeq 2R_s}$

$$\delta E = h\nu = \hbar\omega = \frac{2\pi\hbar c}{\lambda}$$

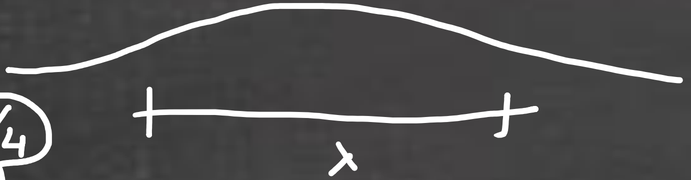
$$\left\{ \frac{\omega}{k} = c \right.$$

$$k = \frac{2\pi}{\lambda}$$

$\delta E = c^2 \delta M$

$$\frac{\delta E}{\delta \text{bit}} = \frac{\delta E}{\delta I} = \frac{\cancel{2}\pi\hbar c}{(\cancel{2}R_s)} = \frac{\pi\hbar c^3}{2GM}$$

λ de un bit



$1/4$

$$\boxed{S \simeq \mathcal{O}(1)\,\frac{c^3\,\text{Area}\,k_B}{\hbar G}}$$

Bekenstein - Hawking

$$\frac{\delta M}{\delta I} = \frac{\pi\hbar c}{2GM} \longrightarrow \underbrace{\int \delta I}_{-I = \frac{S}{k_B}} = \left[\frac{\pi\hbar c}{2G}\right]^{-1} \underbrace{\int M\,\delta M}_{\frac{M^2}{2}}$$

$\#$

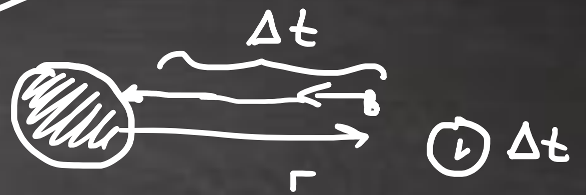
$$S = \frac{k_B G}{\pi\hbar c} M^2 = \frac{g_s}{16\pi^2}\,\frac{k_B c^3\,(4\pi R_s^2)}{\hbar G}$$

$4\pi R_s^2 \rightarrow$ Area

$$\boxed{S = \frac{1}{4}\,\frac{c^3\,\text{Area}\,k_B}{\hbar G}}$$

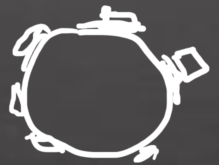
i) $S = S(\underbrace{c, \hbar, G}, k_B)$

ii) $S \propto \text{Area}$

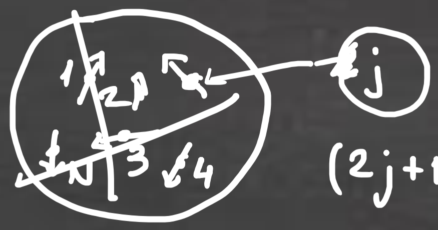


$$ds^2 = 0 = -f(r)\,dt^2 + \frac{dr^2}{f(r)}$$

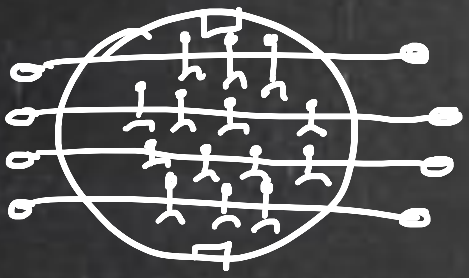
$$\Delta t = \int_0^{\Delta t} dt = \int_{r_i}^{R_S} \frac{dr}{f(r)} = \infty$$



$\cancel{S_{ideal} \propto \text{Vol.}}$



$(2j+1)$



$$\Omega = (2j+1)^N$$

$$S = k_B \log \Omega \propto k_B N \propto \text{Vol}$$

iii) $S \sim 1/\hbar$

iv) $S \sim 1/G$

$Mc^2=E$ $\quad S \quad$ ¿T? $\qquad \underline{dE} = \underline{T}\, d\underline{S}$

$\left\{ R_S = \frac{2GM}{c^2}, \quad \text{Area} = 4\pi R_S^2 \right.$

$$d(Mc^2) = T_H \, d\left(\frac{c^3\, \text{Area}\, k_B}{4\hbar G}\right)$$

i) $T_H \sim 1/M \sim 1/E \qquad \frac{dE}{dT} = C_v < 0$

$$\boxed{T_H = \frac{\hbar c^3}{8\pi G M k_B}}$$

Hawking

$$\boxed{S_{BH} = \frac{k_B c^3\, \text{Area}}{4G\hbar}}$$

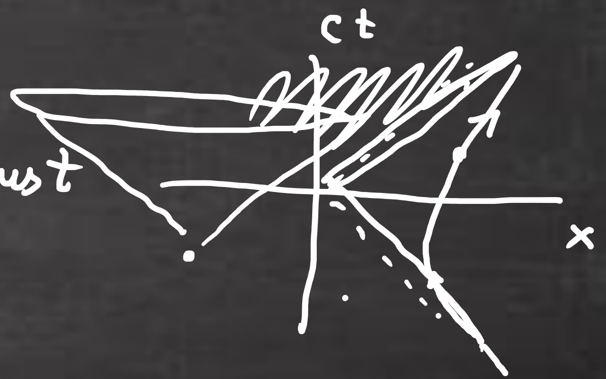
Bekenstein - Hawking

ii) $T_H \sim \hbar$

iii) $T_H = \frac{\kappa}{2\pi}\left(\frac{\hbar}{k_B c}\right)$

Unruh $\quad$ obs $\quad \vec{a} = \frac{d^2\vec{x}}{d\tau^2} = \text{const}$

$$T_H \sim \frac{a\hbar}{2\pi k_B c}$$



- $\boxed{T\big|_{M_{sol}} \simeq 2 \cdot 10^{-7}\,K}$ Cfr. $T_{CMB} \simeq 2{,}7\ K$

$M_{sol} \sim 2\ 10^{30}\ Kg$

- ¿M_{BH} / $T_H \simeq T_{CMB} \sim K$? $M_{BH} \simeq 7\ 10^{22}\ Kg = M_{LUNA}$

$$\rho = \frac{M}{vol} \sim \frac{M}{R_s^3} \sim \frac{1}{M^2}$$

$M_{Tierra} \longleftrightarrow R_s \sim 1\ cm$

$M_{sol} \longleftrightarrow R_s \sim 3\ Km$

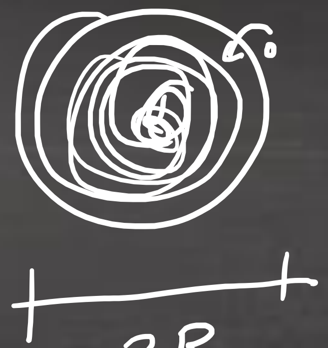
$M_{M87*} \longleftrightarrow R_s \sim R_S$

$$S_{sol} \simeq \cancel{10^{60}\ \frac{erg}{K} \simeq 10^{53}\ \frac{J}{K}}\ 10^{35}\ \frac{J}{K}$$

$$S_{BH\ con\ M_{sol}} \simeq 10^{60}\ \frac{erg}{K} = 10^{53}\ \frac{J}{K}$$

$$\boxed{S_{BH\ M_{sol}} \sim 10^{18}\ S_{sol}}$$

Bound de Bekenstein



2R

$$R = \frac{2MG}{c^2}, \qquad E = Mc^2$$

$$S \le S_{max} = \frac{\overbrace{4\pi R^2}^{\text{Area}}\, k_B c^3}{\hbar\, 4G} = \frac{\pi R\, k_B R c^3}{\hbar G} = \frac{2\pi\, k_B R E \cancel{G}}{\hbar c \cancel{G}}$$

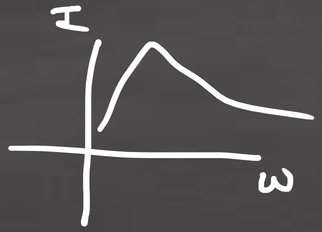
S_{max}: la que un BH tendría en ese volumen

$$\boxed{\frac{S}{E} \le \frac{S_{max}}{E} = \frac{2\pi k_B R}{\hbar c}}$$

No hay G

$$\cancel{\frac{S}{R} \le \frac{S_{max}}{R} = \pi R k_B}$$

$$I_{\text{ntensidad}}^{(\omega)} = \frac{\hbar\omega^3}{2\pi^2c^2}\left(\frac{1}{e^{\frac{\hbar\omega}{k_B T_H}}-1}\right)$$



Planck

$$\boxed{T_H = \frac{\hbar c^3}{8\pi G M k_B}}$$

Agujero Negro es <u>Un cuerpo Negro</u>

$$P = \frac{dE}{dt} = \sigma \, Sup. \, T^4$$

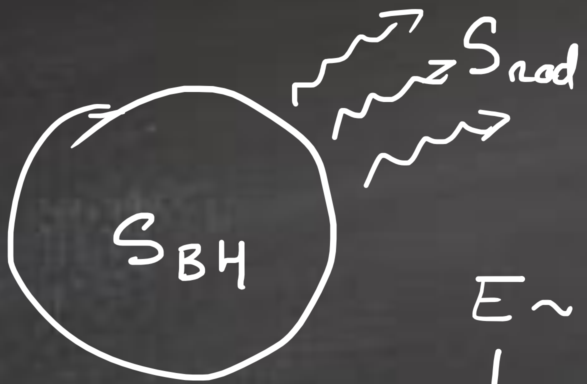
S & B

$E \sim M$
$Sup \sim R_s^2 \sim M^2$
$T^4 \sim 1/M^4$

- $P(M \sim sol) \simeq 10^{-28}\,W$
- tevap

$$-\frac{dE}{dt}\,\frac{1}{Sup} \sim T^4 \longrightarrow \Delta t = \int_0^{\Delta t} dt \propto -\int_{M_i}^{M_f=0} M^2\, dM < \infty$$

$\Delta t = 10^{67}$ años



$$S = \underline{S_{BH}} + \underline{S_{rad}} \quad \text{aumenta}$$

$$E \sim M \sim R \sim \sqrt{A} \sim \sqrt{S_{BH}}$$

$$\Big\downarrow \qquad\qquad\qquad\qquad \Big\downarrow$$

$$\varepsilon \qquad\qquad\qquad\qquad \sqrt{s_{BH}}$$

También pueden ver:

https://www.youtube.com/watch?v=lIpLD_GPme8&t=1720s&ab_channel=GastonGiribet