Entanglement Entropy from Holography
Part 3

Ignacio J. Araya
araya.quezada.ignacio@gmail.com
Universidad Andrés Bello

Universidad de Concepción - Concepción - Chile

August 24, 2019

Kounterterms

- *Kounterterms* is an alternative counterterm series whose main characteristic is its explicit dependence on the extrinsic curvature K_{ij}. [Olea, hep-th/0610230 and hep-th/0504233].
- In even dimensions, they originate from the addition of a topological invariant to the Einstein- Hilbert action $\Rightarrow$ **Topological Renormalization**
- An analogous renormalization scheme is defined in odd-dimensions
- The Kounterterm expansion can be written in closed form.

Kounterterms

- *Kounterterms* is an alternative counterterm series whose main characteristic is its explicit dependence on the extrinsic curvature K_{ij}. [Olea, hep-th/0610230 and hep-th/0504233].
- In even dimensions, they originate from the addition of a topological invariant to the Einstein- Hilbert action $\Rightarrow$ **Topological Renormalization**
- An analogous renormalization scheme is defined in odd-dimensions
- The Kounterterm expansion can be written in closed form.

Kounterterms

- *Kounterterms* is an alternative counterterm series whose main characteristic is its explicit dependence on the extrinsic curvature K_{ij}. [Olea, hep-th/0610230 and hep-th/0504233].
- In even dimensions, they originate from the addition of a topological invariant to the Einstein- Hilbert action $\Rightarrow$ **Topological Renormalization**
- An analogous renormalization scheme is defined in odd-dimensions
- The Kounterterm expansion can be written in closed form.

Kounterterms

- *Kounterterms* is an alternative counterterm series whose main characteristic is its explicit dependence on the extrinsic curvature K_{ij}. [Olea, hep-th/0610230 and hep-th/0504233].
- In even dimensions, they originate from the addition of a topological invariant to the Einstein- Hilbert action $\Rightarrow$ **Topological Renormalization**
- An analogous renormalization scheme is defined in odd-dimensions
- The Kounterterm expansion can be written in closed form.

Topological Renormalization in even dimensions

- EH action+Euler term:

$$I_{ren} = \frac{1}{16\pi G}\int_M d^{2n}x\sqrt{-G}\left(R-2\Lambda + \alpha_{2n}\delta^{[\nu_1\ldots\nu_{2n}]}_{[\mu_1\ldots\mu_{2n}]}R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\right)$$

for

$$\alpha_{2n} = (-1)^n\frac{\ell^{2n-2}}{2^n n(2n-2)!}$$

- Euler theorem in $D=2n$ dimensions:

$$\int_M d^{2n}x\mathcal{E}_{2n} = (4\pi)^n n!\chi(M) + \int_{\partial M} d^{2n-1}xB_{2n-1}$$

- Renormalized action:

$$I_{ren} = I_{EH} + \frac{c_{2n-1}}{16\pi G}\int_{\partial M} d^{2n-1}xB_{2n-1}(h,K,\mathcal{R})$$

Topological Renormalization in even dimensions

- EH action+Euler term:

$$I_{ren} = \frac{1}{16\pi G} \int_M d^{2n}x\sqrt{-G}\,\Big(R - 2\Lambda + \alpha_{2n}\delta^{[\nu_1\dots\nu_{2n}]}_{[\mu_1\dots\mu_{2n}]} R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\Big)$$

for

$$\alpha_{2n} = (-1)^n \frac{\ell^{2n-2}}{2^n n\,(2n-2)!}$$

- Euler theorem in $D = 2n$ dimensions:

$$\int_M d^{2n}x\mathcal{E}_{2n} = (4\pi)^n\, n!\,\chi(M) + \int_{\partial M} d^{2n-1}xB_{2n-1}$$

- Renormalized action:

$$I_{ren} = I_{EH} + \frac{c_{2n-1}}{16\pi G}\int_{\partial M} d^{2n-1}xB_{2n-1}(h, K, \mathcal{R})$$

Topological Renormalization in even dimensions

- EH action+Euler term:

$$I_{ren} = \frac{1}{16\pi G}\int_M d^{2n}x\sqrt{-G}\,\Big(R - 2\Lambda + \alpha_{2n}\delta^{[\nu_1\ldots\nu_{2n}]}_{[\mu_1\ldots\mu_{2n}]} R^{\mu_1\mu_2}_{\nu_1\nu_2}\cdots R^{\mu_{2n-1}\mu_{2n}}_{\nu_{2n-1}\nu_{2n}}\Big)$$

for

$$\alpha_{2n} = (-1)^n \frac{\ell^{2n-2}}{2^n n\,(2n-2)!}$$

- Euler theorem in $D = 2n$ dimensions:

$$\int_M d^{2n}x\mathcal{E}_{2n} = (4\pi)^n\, n!\chi\,(M) + \int_{\partial M} d^{2n-1}xB_{2n-1}$$

- Renormalized action:

$$I_{ren} = I_{EH} + \frac{c_{2n-1}}{16\pi G}\int_{\partial M} d^{2n-1}xB_{2n-1}\,(h, K, \mathcal{R})$$

Surface term: n-th Chern form

- The n-th Chern form is given by

$$B_{2n-1} = 2n\sqrt{h}\int_0^1 dt\,\delta^{[i_1\ldots i_{2n-1}]}_{[j_1\ldots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3}\right)\times$$
$$\ldots \times \left(\frac{1}{2}\mathcal{R}^{j_{2n-2} j_{2n-1}}_{i_{2n-2} i_{2n-1}}(h) - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}}\right)$$

  and the coefficient

$$c_{2n-1} = \frac{(-1)^n \ell^{2n-2}}{n(2n-2)!}$$

- The surface term B_{2n-1} is the boundary correction of the Euler theorem.

Surface term: n-th Chern form

- The n-th Chern form is given by

$$\begin{aligned} B_{2n-1} =& 2n\sqrt{h}\int_0^1 dt\delta_{[j_1\dots j_{2n-1}]}^{[i_1\dots i_{2n-1}]} K_{i_1}^{j_1}\left(\frac{1}{2}\mathcal{R}_{i_2 i_3}^{j_2 j_3}(h) - t^2 K_{i_2}^{j_2} K_{i_3}^{j_3}\right)\times \\ & \dots \times \left(\frac{1}{2}\mathcal{R}_{i_{2n-2} i_{2n-1}}^{j_{2n-2} j_{2n-1}}(h) - t^2 K_{i_{2n-2}}^{j_{2n-2}} K_{i_{2n-1}}^{j_{2n-1}}\right) \end{aligned}$$

  and the coefficient

$$c_{2n-1} = \frac{(-1)^n \ell^{2n-2}}{n(2n-2)!}$$

- The surface term B_{2n-1} is the boundary correction of the Euler theorem.

Kounterterms in odd dimensions

- Kounterterm-renormalized Einstein-AdS action in odd-D given by

$$I_{EH}^{ren} = I_{EH} + \frac{c_{2n}}{16\pi G} \int_{\partial M} B_{2n}$$

$$c_{2n} = \frac{(-1)^n \, \ell^{2(n-1)}}{2^{2(n-1)} n \left[(n-1)!\right]^2}$$

- The extrinsic counterterm B_{2n} is

$$B_{2n} = -2nd^{2n}x\sqrt{-h}\delta^{[j_1 \cdots j_{2n}]}_{[i_1 \cdots i_{2n}]} \int_0^1 dt \int_0^t ds \delta^{i_1}_{j_1} K^{i_2}_{j_2} \left(\frac{1}{2} \mathcal{R}^{i_3 i_4}_{j_3 j_4} - t^2 K^{i_3}_{j_3} K^{i_4}_{j_4} + \frac{s^2}{\ell^2} \delta^{i_3}_{j_3} \delta^{i_4}_{j_4} \right) \cdots \left(\frac{1}{2} \mathcal{R}^{i_{2n-1} i_{2n}}_{j_{2n-1} j_{2n}} - t^2 K^{i_{2n-1}}_{j_{2n-1}} K^{i_{2n}}_{j_{2n}} + \frac{s^2}{\ell^2} \delta^{i_{2n-1}}_{j_{2n-1}} \delta^{i_{2n}}_{j_{2n}} \right)$$

Kounterterms in odd dimensions

- Kounterterm-renormalized Einstein-AdS action in odd-D given by

$$I_{EH}^{ren} = I_{EH} + \frac{c_{2n}}{16\pi G}\int\limits_{\partial M} B_{2n}$$

$$c_{2n} = \frac{(-1)^n \, \ell^{2(n-1)}}{2^{2(n-1)}n\left[(n-1)!\right]^2}$$

- The extrinsic counterterm B_{2n} is

$$B_{2n} = -2nd^{2n}x\sqrt{-h}\delta_{[i_1\cdots i_{2n}]}^{[j_1\cdots j_{2n}]}\int\limits_0^1 dt\int\limits_0^t ds\delta_{j_1}^{i_1}K_{j_2}^{i_2}\left(\frac{1}{2}\mathcal{R}_{j_3j_4}^{i_3i_4} - t^2K_{j_3}^{i_3}K_{j_4}^{i_4} + \frac{s^2}{\ell^2}\delta_{j_3}^{i_3}\delta_{j_4}^{i_4}\right)\cdots\left(\frac{1}{2}\mathcal{R}_{j_{2n-1}j_{2n}}^{i_{2n-1}i_{2n}} - t^2K_{j_{2n-1}}^{i_{2n-1}}K_{j_{2n}}^{i_{2n}} + \frac{s^2}{\ell^2}\delta_{j_{2n-1}}^{i_{2n-1}}\delta_{j_{2n}}^{i_{2n}}\right)$$

Example, thermodynamics of 5D Schwarzschild-AdS

- We consider

$$ds^2_{Sch} = \frac{dr^2}{f^2(r)} - f^2(r)\,dt^2 + r^2\sigma_{mn}(y)\,dy^m dy^n$$

$$f^2 = k + \frac{r^2}{\ell^2} - \frac{\mu}{r^2}\ ;\ \left(f^2\right)' = \frac{2r}{\ell^2} + \frac{2\mu}{r^3}\ ;\ \left(f^2\right)'' = \frac{2}{\ell^2} - \frac{6\mu}{r^4}$$

- Riemann and extrinsic curvature (radial foliation) are

$$K^i_j = -\frac{1}{2N} h^{ik}\partial_r h_{kj} = \begin{bmatrix} -f' & 0 \\ 0 & -\frac{f}{r}\delta^m_n \end{bmatrix}$$

$$R^{tr}_{tr} = -\frac{1}{2}\left(f^2\right)''\ ;\ R^{tn}_{tm} = R^{rn}_{rm} = -\frac{1}{2r}\left(f^2\right)'\delta^n_m$$

$$R^{mn}_{kl} = \frac{1}{r^2}\left(k - f^2\right)\delta^{[mn]}_{[kl]}\ ;\ \mathcal{R}^{n_1 n_2}_{m_1 m_2} = \frac{k}{r^2}\delta^{[n_1 n_2]}_{[m_1 m_2]}$$

Example, thermodynamics of 5D Schwarzschild-AdS

- We consider

$$ds^2_{Sch} = \frac{dr^2}{f^2(r)} - f^2(r)\,dt^2 + r^2\sigma_{mn}(y)\,dy^m dy^n$$

$$f^2 = k + \frac{r^2}{\ell^2} - \frac{\mu}{r^2}\ ;\ \left(f^2\right)' = \frac{2r}{\ell^2} + \frac{2\mu}{r^3}\ ;\ \left(f^2\right)'' = \frac{2}{\ell^2} - \frac{6\mu}{r^4}$$

- Riemann and extrinsic curvature (radial foliation) are

$$K^i_j = -\frac{1}{2N} h^{ik}\partial_r h_{kj} = \begin{bmatrix} -f' & 0 \\ 0 & -\frac{f}{r}\delta^m_n \end{bmatrix}$$

$$R^{tr}_{tr} = -\frac{1}{2}\left(f^2\right)''\ ;\ R^{tn}_{tm} = R^{rn}_{rm} = -\frac{1}{2r}\left(f^2\right)'\delta^n_m$$

$$R^{mn}_{kl} = \frac{1}{r^2}\left(k - f^2\right)\delta^{[mn]}_{[kl]}\ ;\ \mathcal{R}^{n_1 n_2}_{m_1 m_2} = \frac{k}{r^2}\delta^{[n_1 n_2]}_{[m_1 m_2]}$$

Example, thermodynamics of 5D Schwarzschild-AdS

Euclidean E-H action gives

$$I_{EH}=\frac{1}{16\pi G}\int_{M} d^5x\sqrt{-G}\left(R+\frac{12}{\ell^2}\right)$$

$$I_{EH}^{E}=\frac{1}{16\pi G}\int_0^{\beta} d\tau\int_{\Sigma_{k,3}}\sqrt{\sigma}d^3y\int_{r_h}^{\infty} drr^3\left[\left(f^2\right)''+\frac{3\left(f^2\right)'}{r}\right]$$

And considering the form of $f^2(r)$,

$$I_{EH}^{E}=\frac{\beta Vol\left(\Sigma_{k,3}\right)}{16\pi G}\left[\left(f^2\right)' r^3\right]\Big|_{r_h}^{\infty}=-S+\frac{2}{3}\beta M+\frac{\beta Vol\left(\Sigma_{k,3}\right)}{16\pi G}\lim_{r\to\infty}\left[\frac{2r^4}{\ell^2}\right]$$

$$S=\frac{Vol\left(\Sigma_{k,3}\right)r_h^3}{4G}=\frac{Area\left[\mathcal{H}\right]}{4G}\ ;\ M=\frac{3Vol\left(\Sigma_{k,3}\right)\mu}{16\pi G}$$

Example, thermodynamics of 5D Schwarzschild-AdS

Euclidean Einstein-Hilbert part missing $\frac{1}{3}$ of the mass, it also has a volume divergence. B_3 fixes both issues. Explicitly:

$$I_{ren} = I_{EH} + I_{B_4} \; ; \; I_{B_4} = \frac{c_4}{16\pi G} \int\limits_{\partial M} B_4 \; ; \; c_4 = \frac{\ell^2}{8}$$

$$B_4 = -4 d^4 x \sqrt{-h} \int\limits_0^1 dt \int\limits_0^t ds \delta^{[j_1 \cdots j_4]}_{[i_1 \cdots i_4]} \delta^{i_1}_{j_1} K^{i_2}_{j_2} \left(\frac{1}{2} \mathcal{R}^{i_3 i_4}_{j_3 j_4} - t^2 K^{i_3}_{j_3} K^{i_4}_{j_4} + \frac{s^2}{\ell^2} \delta^{i_3}_{j_3} \delta^{i_4}_{j_4} \right)$$

$$I^E_{B_4} = \frac{1}{16\pi G} \frac{3\ell^2}{2} \beta Vol\left(\Sigma_{k,3}\right) \lim_{r\to\infty} \left[\begin{array}{c} r^3 \left(f^2\right)' \left(\frac{k-f^2}{r^2} + \frac{1}{3\ell^2}\right) + \\ \left(2f^2 - r\left(f^2\right)'\right) \left(\frac{k}{2} - \frac{f^2}{4} + \frac{r^2}{4\ell^2}\right) \end{array} \right]$$

$$I^E_{B_4} = \frac{\beta Vol\left(\Sigma_{k,3}\right)}{16\pi G} \lim_{r\to\infty} \left[\mu + \frac{3}{2}\ell^2 k^2 - \frac{2r^4}{\ell^2} \right] + [\text{vanishing terms}]$$

Example, thermodynamics of 5D Schwarzschild-AdS

- Therefore, we have

$$I^E_{B_4} = \frac{1}{3}\beta M - \frac{\beta Vol\left(\Sigma_{k,3}\right)}{16\pi G}\lim_{r\to\infty}\left[\frac{2r^4}{\ell^2}\right] + \beta E_0$$

$$E_0 = \frac{Vol\left(\Sigma_{k,3}\right)}{16\pi G}\left(\frac{3}{2}\ell^2 k^2\right)$$

$$I^E_{ren} = I^E_{EH} + I^E_{B_4} = -S + \beta\left(M + E_0\right) = \beta F$$

- The extrinsically renormalized Euclidean action then reproduces the correct thermodynamics including the vacuum energy (Miskovic and Olea, [1012.4867])

Example, thermodynamics of 5D Schwarzschild-AdS

- Therefore, we have

$$I_{B_4}^E = \frac{1}{3}\beta M - \frac{\beta Vol\left(\Sigma_{k,3}\right)}{16\pi G}\lim_{r\to\infty}\left[\frac{2r^4}{\ell^2}\right] + \beta E_0$$

$$E_0 = \frac{Vol\left(\Sigma_{k,3}\right)}{16\pi G}\left(\frac{3}{2}\ell^2 k^2\right)$$

$$I_{ren}^E = I_{EH}^E + I_{B_4}^E = -S + \beta\left(M + E_0\right) = \beta F$$

- The extrinsically renormalized Euclidean action then reproduces the correct thermodynamics including the vacuum energy (Miskovic and Olea, [1012.4867])

Holographic Entanglement Entropy

- Using AdS/CFT, EE of spatial subregions in Holographic CFTs can be obtained by geometric computation on bulk gravity dual.
- Holographic EE for Einstein-AdS bulk gravity is computed using area prescription of Ryu-Takayanagi [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- Σ is minimal surface in AdS bulk. $\partial\Sigma$ at spacetime boundary B is required to be conformally cobordant to entangling surface ∂A at conformal boundary C.
- Covariant version of prescription constructed by Hubeni, Rangamani and Takayanagi [0705.0016].

Holographic Entanglement Entropy

- Using AdS/CFT, EE of spatial subregions in Holographic CFTs can be obtained by geometric computation on bulk gravity dual.
- Holographic EE for Einstein-AdS bulk gravity is computed using area prescription of Ryu-Takayanagi [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- Σ is minimal surface in AdS bulk. $\partial\Sigma$ at spacetime boundary B is required to be conformally cobordant to entangling surface ∂A at conformal boundary C.
- Covariant version of prescription constructed by Hubeni, Rangamani and Takayanagi [0705.0016].

Holographic Entanglement Entropy

- Using AdS/CFT, EE of spatial subregions in Holographic CFTs can be obtained by geometric computation on bulk gravity dual.
- Holographic EE for Einstein-AdS bulk gravity is computed using area prescription of Ryu-Takayanagi [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- Σ is minimal surface in AdS bulk. $\partial\Sigma$ at spacetime boundary B is required to be conformally cobordant to entangling surface ∂A at conformal boundary C.
- Covariant version of prescription constructed by Hubeni, Rangamani and Takayanagi [0705.0016].

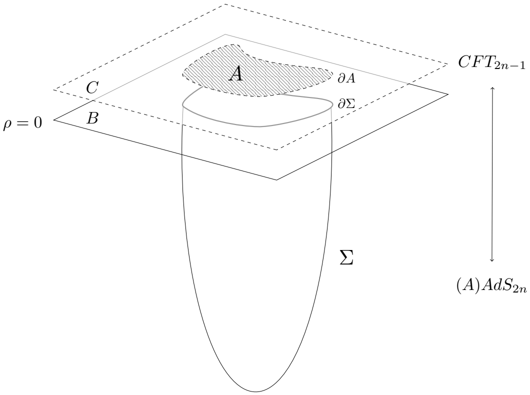
Holographic Entanglement Entropy

- Using AdS/CFT, EE of spatial subregions in Holographic CFTs can be obtained by geometric computation on bulk gravity dual.
- Holographic EE for Einstein-AdS bulk gravity is computed using area prescription of Ryu-Takayanagi [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- Σ is minimal surface in AdS bulk. $\partial\Sigma$ at spacetime boundary B is required to be conformally cobordant to entangling surface ∂A at conformal boundary C.
- Covariant version of prescription constructed by Hubeni, Rangamani and Takayanagi [0705.0016].

Ryu-Takayanagi Construction



Extremization of area functional

- Area functional given by integral over codimension-2 surface Σ of determinant of induced metric.
- Consider an embedding function for Σ . I.e, write one coordinate in terms of the others, at $t =$ const.
- Impose the extremization condition on the area functional and derive the corresponding E-L equation.
- Embedding has to satisfy resulting differential equation.
- Explicit computation in [1712.09099, Appendix A].

Extremization of area functional

- Area functional given by integral over codimension-2 surface Σ of determinant of induced metric.
- Consider an embedding function for Σ . I.e, write one coordinate in terms of the others, at $t =$ const.
- Impose the extremization condition on the area functional and derive the corresponding E-L equation.
- Embedding has to satisfy resulting differential equation.
- Explicit computation in [1712.09099, Appendix A].

Extremization of area functional

- Area functional given by integral over codimension-2 surface Σ of determinant of induced metric.
- Consider an embedding function for Σ . I.e, write one coordinate in terms of the others, at $t =$ const.
- Impose the extremization condition on the area functional and derive the corresponding E-L equation.
- Embedding has to satisfy resulting differential equation.
- Explicit computation in [1712.09099, Appendix A].

Extremization of area functional

- Area functional given by integral over codimension-2 surface Σ of determinant of induced metric.
- Consider an embedding function for Σ . I.e, write one coordinate in terms of the others, at $t =$ const.
- Impose the extremization condition on the area functional and derive the corresponding E-L equation.
- Embedding has to satisfy resulting differential equation.
- Explicit computation in [1712.09099, Appendix A].

Extremization of area functional

- Area functional given by integral over codimension-2 surface Σ of determinant of induced metric.
- Consider an embedding function for Σ . I.e, write one coordinate in terms of the others, at $t =$ const.
- Impose the extremization condition on the area functional and derive the corresponding E-L equation.
- Embedding has to satisfy resulting differential equation.
- Explicit computation in [1712.09099, Appendix A].

Example: HEE of an interval in AdS_3/CFT_2

- We consider global AdS_3 in Poincaré coordinates which is dual to a CFT_2 in flat Minkowski spacetime, and an interval of length L as the entangling region A:

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\left(-dt^2 + dx^2\right)$$
$$A : \left\{\left(t = \text{const}, x \in \left[-\frac{L}{2}, \frac{L}{2}\right]\right)\right\}$$

- From the RT formula, EE of L is given by length of minimal arc cobordant to A:

$$S_{EE} = \frac{1}{4G}\int_{\Sigma} d\xi \sqrt{\frac{1}{z^2}\left(z'^2 + x'^2\right)}$$
$$z' = \frac{dz}{d\xi} \; ; \; x' = \frac{dx}{d\xi}$$

Example: HEE of an interval in AdS_3/CFT_2

- We consider global AdS_3 in Poincaré coordinates which is dual to a CFT_2 in flat Minkowski spacetime, and an interval of length L as the entangling region A:

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\left(-dt^2 + dx^2\right)$$

$$A : \left\{ \left(t = \text{const}, x \in \left[-\frac{L}{2}, \frac{L}{2} \right] \right) \right\}$$

- From the RT formula, EE of L is given by length of minimal arc cobordant to A:

$$S_{EE} = \frac{1}{4G} \int_{\Sigma} d\xi \sqrt{\frac{1}{z^2}\left(z'^2 + x'^2\right)}$$

$$z' = \frac{dz}{d\xi} \; ; \; x' = \frac{dx}{d\xi}$$

Example: HEE of an interval in AdS_3/CFT_2

- Minimal arc given by geodesic in the bulk. Parametrized by:

$$\Sigma : \{(x(\xi), z(\xi))\}_{\xi \in [0,\pi]}$$
$$x(\xi) = \frac{L}{2}\cos\xi \ ; \ z(\xi) = \frac{L}{2}\sin\xi$$

- Computing the length, we obtain

$$S_{EE} = \lim_{\epsilon \to 0} \frac{1}{4G} 2 \int_{\frac{2\epsilon}{L}}^{\frac{\pi}{2}} \frac{d\xi}{\left(\frac{L}{2}\right)\sin\xi} \left(\frac{L}{2}\right) \sqrt{\sin^2\xi + \cos^2\xi}$$
$$= \lim_{\epsilon \to 0} \frac{1}{2G} \int_{\frac{2\epsilon}{L}}^{\frac{\pi}{2}} \frac{d\xi}{\sin\xi} = \lim_{\epsilon \to 0} \frac{1}{2G} \ln\left| \frac{1 + \cos\left(\frac{2\epsilon}{L}\right)}{\sin\left(\frac{2\epsilon}{L}\right)} \right| = \lim_{\epsilon \to 0} \frac{1}{2G} \ln\left|\frac{L}{\epsilon}\right|$$

Example: HEE of an interval in AdS_3/CFT_2

- Minimal arc given by geodesic in the bulk. Parametrized by:

$$\Sigma : \{(x(\xi), z(\xi))\}_{\xi \in [0,\pi]}$$
$$x(\xi) = \frac{L}{2}\cos\xi \ ; \ z(\xi) = \frac{L}{2}\sin\xi$$

- Computing the length, we obtain

$$S_{EE} = \lim_{\epsilon\to 0}\frac{1}{4G}2\int_{\frac{2\epsilon}{L}}^{\frac{\pi}{2}}\frac{d\xi}{\left(\frac{L}{2}\right)\sin\xi}\left(\frac{L}{2}\right)\sqrt{\sin^2\xi+\cos^2\xi}$$
$$= \lim_{\epsilon\to 0}\frac{1}{2G}\int_{\frac{2\epsilon}{L}}^{\frac{\pi}{2}}\frac{d\xi}{\sin\xi} = \lim_{\epsilon\to 0}\frac{1}{2G}\ln\left|\frac{1+\cos\left(\frac{2\epsilon}{L}\right)}{\sin\left(\frac{2\epsilon}{L}\right)}\right| = \lim_{\epsilon\to 0}\frac{1}{2G}\ln\left|\frac{L}{\epsilon}\right|$$

Example: HEE of an interval in AdS_3/CFT_2

- The holographic dictionary for AdS_3/CFT_2 relates the central charge of the CFT to the 3D Newton's constant in the bulk. The previous result for the EE computed in the CFT is recovered:

$$c = \frac{3}{2G}$$

$$S_{EE} = \frac{c}{3} \ln \left| \frac{L}{\epsilon} \right|$$

- Computation reduced to a simple geometry problem!

Example: HEE of an interval in AdS_3/CFT_2

- The holographic dictionary for AdS_3/CFT_2 relates the central charge of the CFT to the 3D Newton's constant in the bulk. The previous result for the EE computed in the CFT is recovered:

$$c = \frac{3}{2G}$$

$$S_{EE} = \frac{c}{3} \ln \left| \frac{L}{\epsilon} \right|$$

- Computation reduced to a simple geometry problem!

Example: Spherical entangling surface

- When $\partial\Sigma$ is a sphere, embedding given by:

$$\Sigma : \left\{ t = const \; ; \; r^2 + \ell^2 \rho = R^2 \right\}$$

- Substitute in extremization condition. Verify that it is satisfied.
- Compute area integral explicitly. Split integral into radial part and boundary part.
- Integral diverges. Expand in powers of the radial coordinate. For d=odd, universal part is finite. For d=even, universal part is log-divergent.
- Later: How to obtain the universal part directly (Kounterterm renormalization).

Example: Spherical entangling surface

- When $\partial\Sigma$ is a sphere, embedding given by:

$$\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$$

- Substitute in extremization condition. Verify that it is satisfied.
- Compute area integral explicitly. Split integral into radial part and boundary part.
- Integral diverges. Expand in powers of the radial coordinate. For d=odd, universal part is finite. For d=even, universal part is log-divergent.
- Later: How to obtain the universal part directly (Kounterterm renormalization).

Example: Spherical entangling surface

- When $\partial\Sigma$ is a sphere, embedding given by:

$$\Sigma : \left\{ t = const \; ; \; r^2 + \ell^2 \rho = R^2 \right\}$$

- Substitute in extremization condition. Verify that it is satisfied.
- Compute area integral explicitly. Split integral into radial part and boundary part.
- Integral diverges. Expand in powers of the radial coordinate. For d=odd, universal part is finite. For d=even, universal part is log-divergent.
- Later: How to obtain the universal part directly (Kounterterm renormalization).

Example: Spherical entangling surface

- When $\partial\Sigma$ is a sphere, embedding given by:

$$\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$$

- Substitute in extremization condition. Verify that it is satisfied.
- Compute area integral explicitly. Split integral into radial part and boundary part.
- Integral diverges. Expand in powers of the radial coordinate. For d=odd, universal part is finite. For d=even, universal part is log-divergent.
- Later: How to obtain the universal part directly (Kounterterm renormalization).

Example: Spherical entangling surface

- When $\partial\Sigma$ is a sphere, embedding given by:

$$\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$$

- Substitute in extremization condition. Verify that it is satisfied.
- Compute area integral explicitly. Split integral into radial part and boundary part.
- Integral diverges. Expand in powers of the radial coordinate. For d=odd, universal part is finite. For d=even, universal part is log-divergent.
- Later: How to obtain the universal part directly (Kounterterm renormalization).

HEE and RT prescription

- We had the RT formula (for Einstein-AdS) [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- We motivate this formula by using the Replica Trick, considering conically singular manifolds and following Lewkowycz and Maldacena [1304.4926].
- S_{EE} can be expressed in terms of derivatives on on-shell gravity actions. Useful for considering other theories of gravity and for renormalizing the EE.

HEE and RT prescription

- We had the RT formula (for Einstein-AdS) [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- We motivate this formula by using the Replica Trick, considering conically singular manifolds and following Lewkowycz and Maldacena [1304.4926].
- S_{EE} can be expressed in terms of derivatives on on-shell gravity actions. Useful for considering other theories of gravity and for renormalizing the EE.

HEE and RT prescription

- We had the RT formula (for Einstein-AdS) [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- We motivate this formula by using the Replica Trick, considering conically singular manifolds and following Lewkowycz and Maldacena [1304.4926].
- S_{EE} can be expressed in terms of derivatives on on-shell gravity actions. Useful for considering other theories of gravity and for renormalizing the EE.

HEE and RT prescription

- We had the RT formula (for Einstein-AdS) [hep-th/0603001]:

$$S_{EE} = \frac{Vol(\Sigma)}{4G}.$$

- We motivate this formula by using the Replica Trick, considering conically singular manifolds and following Lewkowycz and Maldacena [1304.4926].
- S_{EE} can be expressed in terms of derivatives on on-shell gravity actions. Useful for considering other theories of gravity and for renormalizing the EE.

Replica Trick

- EE can be computed as

$$S_{EE} = \lim_{m \to 1} -\frac{1}{m-1} \ln(tr\,(\widehat{\rho}_A^m)).$$

- For evaluating $tr\,(\widehat{\rho}_A^m)$, construct branched cover manifold C_m from m copies of CFT, cyclically permuted by 2π rotations along complexified time. Define orbifold $\widehat{C}_m = C_m/Z_m$ as quotient of cover manifold by the permutation symmetry. (Cardy and Calabrese [0905.4013])
- In saddle-point approximation, AdS bulk orbifold $\widehat{M}_m$ is constructed from boundary orbifold $\widehat{C}_m$ on-shell. Then, $ln\,(tr\,(\widehat{\rho}_A^m)) = m\left(ln(Z(\widehat{C}_m)) - ln(Z(\widehat{C}_1))\right)$, and $ln(Z(\widehat{C}_m)) = -I_E(\widehat{M}_m)$.

Replica Trick

- EE can be computed as

$$S_{EE} = \lim_{m \to 1} -\frac{1}{m-1} \ln(tr\,(\widehat{\rho}_A^m)).$$

- For evaluating $tr\,(\widehat{\rho}_A^m)$, construct branched cover manifold C_m from m copies of CFT, cyclically permuted by 2π rotations along complexified time. Define orbifold $\widehat{C}_m = C_m / Z_m$ as quotient of cover manifold by the permutation symmetry. (Cardy and Calabrese [0905.4013])
- In saddle-point approximation, AdS bulk orbifold $\widehat{M}_m$ is constructed from boundary orbifold $\widehat{C}_m$ on-shell. Then, $ln\,(tr\,(\widehat{\rho}_A^m)) = m\left(ln(Z(\widehat{C}_m)) - ln(Z(\widehat{C}_1))\right)$, and $ln(Z(\widehat{C}_m)) = -I_E(\widehat{M}_m)$.

Replica Trick

- EE can be computed as

$$S_{EE} = \lim_{m \to 1} -\frac{1}{m-1} \ln(tr\,(\widehat{\rho}_A^m)).$$

- For evaluating $tr\,(\widehat{\rho}_A^m)$, construct branched cover manifold C_m from m copies of CFT, cyclically permuted by 2π rotations along complexified time. Define orbifold $\widehat{C}_m = C_m / Z_m$ as quotient of cover manifold by the permutation symmetry. (Cardy and Calabrese [0905.4013])
- In saddle-point approximation, AdS bulk orbifold $\widehat{M}_m$ is constructed from boundary orbifold $\widehat{C}_m$ on-shell. Then, $ln\,(tr\,(\widehat{\rho}_A^m)) = m\left(ln(Z(\widehat{C}_m)) - ln(Z(\widehat{C}_1))\right)$, and $ln(Z(\widehat{C}_m)) = -I_E(\widehat{M}_m)$.

Replica Trick

- Therefore,
$$S_{EE} = \lim_{m \to 1} \frac{m}{m-1}(I_E(\widehat{M}_m) - I_E(\widehat{M}_1)) = m^2 \partial_m I_E\left(\widehat{M}_m\right)\Big|_{m=1}.$$
- Orbifold $\widehat{M}_m$ has conical singularity at fixed point set of Z_m. It is a squashed-cone (no U(1) isometry). Angular deficit of $2\pi(1-\alpha) = 2\pi(1-\frac{1}{m})$.
- $\widehat{M}_m$ sourced by codimension-2 cosmic brane with tension $T(m) = \frac{(m-1)}{4mG}$ coupled through NG action (Xi Dong [1601.06788]; Lewkowycz and Maldacena [1304.4926]).
- We consider
$$S_{EE} = -\partial_\alpha I_E\left(\widehat{M}_D^{(\alpha)}\right)\Big|_{\alpha=1}.$$
- Computation of S_{EE} reduced to evaluating a derivative of Euclidean action $I_E\left(\widehat{M}_D^{(\alpha)}\right)$.

Replica Trick

- Therefore,

$$S_{EE} = \lim_{m\to 1} \frac{m}{m-1}(I_E(\widehat{M}_m) - I_E(\widehat{M}_1)) = m^2 \partial_m I_E\left(\widehat{M}_m\right)\Big|_{m=1}.$$

- Orbifold $\widehat{M}_m$ has conical singularity at fixed point set of Z_m. It is a squashed-cone (no U(1) isometry). Angular deficit of $2\pi(1-\alpha) = 2\pi(1-\frac{1}{m})$.
- $\widehat{M}_m$ sourced by codimension-2 cosmic brane with tension $T(m) = \frac{(m-1)}{4mG}$ coupled through NG action (Xi Dong [1601.06788]; Lewkowycz and Maldacena [1304.4926]).
- We consider

$$S_{EE} = -\partial_\alpha I_E\left(\widehat{M}_D^{(\alpha)}\right)\Big|_{\alpha=1}.$$

- Computation of S_{EE} reduced to evaluating a derivative of Euclidean action $I_E\left(\widehat{M}_D^{(\alpha)}\right)$.

Replica Trick

- Therefore,
$$S_{EE} = \lim_{m \to 1} \frac{m}{m-1}\left(I_E(\widehat{M}_m) - I_E(\widehat{M}_1)\right) = m^2 \partial_m I_E\left(\widehat{M}_m\right)\Big|_{m=1}.$$
- Orbifold $\widehat{M}_m$ has conical singularity at fixed point set of Z_m. It is a squashed-cone (no U(1) isometry). Angular deficit of $2\pi(1-\alpha) = 2\pi(1-\frac{1}{m})$.
- $\widehat{M}_m$ sourced by codimension-2 cosmic brane with tension $T(m) = \frac{(m-1)}{4mG}$ coupled through NG action (Xi Dong [1601.06788]; Lewkowycz and Maldacena [1304.4926]).
- We consider
$$S_{EE} = -\partial_\alpha I_E\left(\widehat{M}_D^{(\alpha)}\right)\Big|_{\alpha=1}.$$
- Computation of S_{EE} reduced to evaluating a derivative of Euclidean action $I_E\left(\widehat{M}_D^{(\alpha)}\right)$.

Replica Trick

- Therefore,

$$S_{EE} = \lim_{m \to 1} \frac{m}{m-1}(I_E(\widehat{M}_m) - I_E(\widehat{M}_1)) = m^2 \partial_m I_E\left(\widehat{M}_m\right)\Big|_{m=1}.$$

- Orbifold $\widehat{M}_m$ has conical singularity at fixed point set of Z_m. It is a squashed-cone (no U(1) isometry). Angular deficit of $2\pi(1-\alpha) = 2\pi(1-\frac{1}{m})$.
- $\widehat{M}_m$ sourced by codimension-2 cosmic brane with tension $T(m) = \frac{(m-1)}{4mG}$ coupled through NG action (Xi Dong [1601.06788]; Lewkowycz and Maldacena [1304.4926]).
- We consider

$$S_{EE} = -\partial_\alpha I_E\left(\widehat{M}_D^{(\alpha)}\right)\Big|_{\alpha=1}.$$

- Computation of S_{EE} reduced to evaluating a derivative of Euclidean action $I_E\left(\widehat{M}_D^{(\alpha)}\right)$.

Euclidean Einstein-Hilbert Action and Ryu-Takayanagi

- Consider Euclidean EH action evaluated in orbifold $\widehat{M}_D^{(\alpha)}$,

$$I_E^{EH} = \frac{1}{16\pi G}\left(\int_{\hat{M}_D^{(\alpha)}} d^D x\sqrt{G}\left(R^{(\alpha)} - 2\Lambda\right)\right).$$

- Using that

$$R^{(\alpha)} = R + 4\pi\left(1-\alpha\right)\delta_\Sigma$$

(Fursaev, Patrushev and Solodukhin [1306.4000]), S_{EE} is then given by area prescription of RT.

- S_{EE} is divergent. Have to use renormalized action to obtain universal part.

Euclidean Einstein-Hilbert Action and Ryu-Takayanagi

- Consider Euclidean EH action evaluated in orbifold $\widehat{M}_D^{(\alpha)}$,

$$I_E^{EH} = \frac{1}{16\pi G}\left(\int_{\hat{M}_D^{(\alpha)}} d^D x \sqrt{G}\left(R^{(\alpha)} - 2\Lambda\right)\right).$$

- Using that

$$R^{(\alpha)} = R + 4\pi\left(1-\alpha\right)\delta_\Sigma$$

(Fursaev, Patrushev and Solodukhin [1306.4000]), S_{EE} is then given by area prescription of RT.

- S_{EE} is divergent. Have to use renormalized action to obtain universal part.

Euclidean Einstein-Hilbert Action and Ryu-Takayanagi

- Consider Euclidean EH action evaluated in orbifold $\widehat{M}_D^{(\alpha)}$,

$$I_E^{EH} = \frac{1}{16\pi G}\left(\int_{\hat{M}_D^{(\alpha)}} d^D x\sqrt{G}\left(R^{(\alpha)} - 2\Lambda\right)\right).$$

- Using that

$$R^{(\alpha)} = R + 4\pi\left(1-\alpha\right)\delta_\Sigma$$

(Fursaev, Patrushev and Solodukhin [1306.4000]), S_{EE} is then given by area prescription of RT.

- S_{EE} is divergent. Have to use renormalized action to obtain universal part.

Euler density and Chern form for 4D squashed cones

- For manifolds with a squashed-cone singularity (no $U(1)$ isometry), in $D = 4$, we have (Fursaev, Patrushev and Solodukhin [1306.4000])

$$\int_{M_4^{(\alpha)}} \mathcal{E}_4^{(\alpha)} = \int_{M_4} \mathcal{E}_4^{(r)} + 8\pi (1-\alpha) \int_{\Sigma} \mathcal{E}_2 .$$

- Then, by the Euler theorem, we also have

$$\int_{\partial M_4^{(\alpha)}} B_3^{(\alpha)} = \int_{\partial M_4} B_3^{(r)} + 8\pi (1-\alpha) \int_{\partial \Sigma} B_1 .$$

Euler density and Chern form for 4D squashed cones

- For manifolds with a squashed-cone singularity (no $U(1)$ isometry), in $D = 4$, we have (Fursaev, Patrushev and Solodukhin [1306.4000])

$$\int_{M_4^{(\alpha)}} \mathcal{E}_4^{(\alpha)} = \int_{M_4} \mathcal{E}_4^{(r)} + 8\pi\,(1-\alpha) \int_{\Sigma} \mathcal{E}_2 .$$

- Then, by the Euler theorem, we also have

$$\int_{\partial M_4^{(\alpha)}} B_3^{(\alpha)} = \int_{\partial M_4} B_3^{(r)} + 8\pi\,(1-\alpha) \int_{\partial\Sigma} B_1 .$$

Generalization to higher even-dimensional manifolds

- We assume that, for arbitrary even-dimensional squashed-cones,

$$\int\limits_{M_{2n}^{(\alpha)}} \mathcal{E}_{2n}^{(\alpha)} = \int\limits_{M_{2n}} \mathcal{E}_{2n}^{(r)} + 4n\pi\,(1-\alpha)\int\limits_{\Sigma} \mathcal{E}_{2n-2},$$

and

$$\int\limits_{\partial M_{2n}^{(\alpha)}} B_{2n-1}^{(\alpha)} = \int\limits_{\partial M_{2n}} B_{2n-1}^{(r)} + 4n\pi\,(1-\alpha)\int\limits_{\partial\Sigma} B_{2n-3}.$$

Generalization to odd-dimensional manifolds

- By analogy with the even-dimensional case, we also assume that

$$\int_{\partial M_{2n+1}^{(\alpha)}} B_{2n}^{(\alpha)} = \int_{\partial M_{2n+1}} B_{2n}^{(r)} + 4\pi n \left(1-\alpha\right) \int_{\partial \Sigma} B_{2n-2}$$

- Then, the extrinsically renormalized Euclidean action on the Replica orbifold can be easily evaluated.

Generalization to odd-dimensional manifolds

- By analogy with the even-dimensional case, we also assume that

$$\int_{\partial M_{2n+1}^{(\alpha)}} B_{2n}^{(\alpha)} = \int_{\partial M_{2n+1}} B_{2n}^{(r)} + 4\pi n\,(1-\alpha) \int_{\partial\Sigma} B_{2n-2}$$

- Then, the extrinsically renormalized Euclidean action on the Replica orbifold can be easily evaluated.

Euclidean action on replica orbifold

- In particular, we find that

$$I_E^{ren}\left[\widehat{M}_{2n}^{(\alpha)}\right] = I_E^{ren}\left[\widehat{M}_{2n}^{(\alpha)} \setminus \Sigma\right] + \frac{(1-\alpha)}{4G} Area_{ren}\left[\Sigma\right]$$

$$Area_{ren}\left[\Sigma\right] = \begin{cases} Area\left[\Sigma\right] + \frac{(-1)^n \ell^{2(n-1)}}{[2(n-1)!]} \int\limits_{\partial\Sigma} B_{2n-3} \ ; \ D = 2n \\ Area\left[\Sigma\right] + \frac{(-1)^n \ell^{2(n-1)}}{2^{2(n-1)}[(n-1)!]^2} \int\limits_{\partial\Sigma} B_{2n-2} \ ; \ D = 2n+1 \end{cases}$$

- $Area_{ren}\left[\Sigma\right]$ is the renormalized area of the cosmic brane with tension T (the RT surface for $T \to 0$).

Euclidean action on replica orbifold

- In particular, we find that

$$I_E^{ren}\left[\widehat{M}_{2n}^{(\alpha)}\right] = I_E^{ren}\left[\widehat{M}_{2n}^{(\alpha)} \setminus \Sigma\right] + \frac{(1-\alpha)}{4G} Area_{ren}\left[\Sigma\right]$$

$$Area_{ren}\left[\Sigma\right] = \begin{cases} Area\left[\Sigma\right] + \frac{(-1)^n \ell^{2(n-1)}}{[2(n-1)!]} \int\limits_{\partial\Sigma} B_{2n-3} \ ; \ D = 2n \\ Area\left[\Sigma\right] + \frac{(-1)^n \ell^{2(n-1)}}{2^{2(n-1)}[(n-1)!]^2} \int\limits_{\partial\Sigma} B_{2n-2} \ ; \ D = 2n+1 \end{cases}$$

- $Area_{ren}\left[\Sigma\right]$ is the renormalized area of the cosmic brane with tension T (the RT surface for $T \to 0$).

EE as the renormalized area of the RT surface

- We considering the replica definition of renormalized EE given by

$$S_{EE}^{ren} = -\partial_\alpha I_E^{ren}\left(\widehat{M}_{2n}^{(\alpha)}\right)\Big|_{\alpha=1}.$$

- Then, we obtain

$$S_{EE}^{ren} = \frac{Area_{ren}(\Sigma)}{4G}.$$

- Renormalized EE is then obtained from the RT formula but considering the renormalized area of the extremal surface.

EE as the renormalized area of the RT surface

- We considering the replica definition of renormalized EE given by

$$S_{EE}^{ren} = -\partial_\alpha I_E^{ren}\left(\widehat{M}_{2n}^{(\alpha)}\right)\Big|_{\alpha=1}.$$

- Then, we obtain

$$S_{EE}^{ren} = \frac{Area_{ren}(\Sigma)}{4G}.$$

- Renormalized EE is then obtained from the RT formula but considering the renormalized area of the extremal surface.

EE as the renormalized area of the RT surface

- We considering the replica definition of renormalized EE given by

$$S_{EE}^{ren} = -\partial_\alpha I_E^{ren}\left(\widehat{M}_{2n}^{(\alpha)}\right)\Big|_{\alpha=1}.$$

- Then, we obtain

$$S_{EE}^{ren} = \frac{Area_{ren}(\Sigma)}{4G}.$$

- Renormalized EE is then obtained from the RT formula but considering the renormalized area of the extremal surface.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE can be written as

$$S_{EE}^{ren} = -\frac{\ell^2}{8G(2n-3)}\left(\int_{\Sigma} d^{2n-2}y\sqrt{\gamma}\,\ell^{2(n-2)}P_{2n-2}\left[\mathcal{F}\right] - c_{2n-2}\left(4\pi\right)^{n-1}(n-1)!\,\chi\left[\Sigma\right]\right).$$

- For $D=4$, the renormalized EE entropy is given by

$$\widetilde{S}_m^{ren} = \frac{\ell^2}{16G}\int_{\Sigma_T} d^2y\sqrt{\gamma}\,\delta_{[a_1a_2]}^{[b_1b_2]}\,\mathcal{F}_{AdS}{}^{a_1a_2}_{b_1b_2} - \frac{\pi\ell^2}{2G}\chi\left[\Sigma_T\right],$$

in agreement with Alexakis and Mazzeo's formula [math/0504161] for renormalized area of extremal surfaces.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE can be written as

$$S_{EE}^{ren} = -\tfrac{\ell^2}{8G(2n-3)} \left(\int_{\Sigma} d^{2n-2}y \sqrt{\gamma} \ell^{2(n-2)} P_{2n-2}[\mathcal{F}] - c_{2n-2} (4\pi)^{n-1} (n-1)! \chi[\Sigma] \right).$$

- For $D=4$, the renormalized EE entropy is given by

$$\widetilde{S}_m^{ren} = \frac{\ell^2}{16G} \int_{\Sigma_T} d^2y \sqrt{\gamma} \delta^{[b_1 b_2]}_{[a_1 a_2]} \mathcal{F}_{AdS}{}^{a_1 a_2}_{b_1 b_2} - \frac{\pi \ell^2}{2G} \chi[\Sigma_T],$$

in agreement with Alexakis and Mazzeo's formula [math/0504161] for renormalized area of extremal surfaces.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE is separated into a geometric part ($\int P_{2n-2}[\mathcal{F}]$) and a purely topological part ($\chi[\Sigma]$).
- Geometric part is zero when extremal surface has constant curvature.
- Topological part is robust against continuous deformations of the entangling surface.
- Power of result illustrated in case of renormalized EE for spherical entangling surface.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE is separated into a geometric part ($\int P_{2n-2}[\mathcal{F}]$) and a purely topological part ($\chi[\Sigma]$).
- Geometric part is zero when extremal surface has constant curvature.
- Topological part is robust against continuous deformations of the entangling surface.
- Power of result illustrated in case of renormalized EE for spherical entangling surface.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE is separated into a geometric part ($\int P_{2n-2}[\mathcal{F}]$) and a purely topological part ($\chi[\Sigma]$).
- Geometric part is zero when extremal surface has constant curvature.
- Topological part is robust against continuous deformations of the entangling surface.
- Power of result illustrated in case of renormalized EE for spherical entangling surface.

Topological interpretation of renormalized EE (for odd-D CFT)

- EE is separated into a geometric part ($\int P_{2n-2}[\mathcal{F}]$) and a purely topological part ($\chi[\Sigma]$).
- Geometric part is zero when extremal surface has constant curvature.
- Topological part is robust against continuous deformations of the entangling surface.
- Power of result illustrated in case of renormalized EE for spherical entangling surface.

Example: Ball-shaped Entangling Region in AdS_{2n}/CFT_{2n-1}

- Minimal RT surface Σ, parametrized by $\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$, is a constant curvature manifold $\rightarrow \mathcal{F}_{AdS} = 0$.
- Σ is topologically equivalent to a $(2n-2)$-ball $\rightarrow \chi\left[\Sigma\right] = 1$.
- Therefore,

$$S_{EE}^{ren} = \frac{(-1)^{n+1}(4\pi)^{(n-1)}(n-1)!\ell^{2(n-1)}}{4G(2n-2)!}.$$

- Result agrees with the calculation by Kawano, Nakaguchi and Nishioka [1410.5973].
- The renormalized Rényi entropy is computed in [1806.10708], using CHM map.

Example: Ball-shaped Entangling Region in AdS_{2n}/CFT_{2n-1}

- Minimal RT surface Σ, parametrized by $\Sigma : \left\{ t = const \; ; \; r^2 + \ell^2 \rho = R^2 \right\}$, is a constant curvature manifold $\to \mathcal{F}_{AdS} = 0$.
- Σ is topologically equivalent to a $(2n-2)$-ball $\to \chi[\Sigma] = 1$.
- Therefore,

$$S_{EE}^{ren} = \frac{(-1)^{n+1} (4\pi)^{(n-1)} (n-1)! \ell^{2(n-1)}}{4G (2n-2)!}.$$

- Result agrees with the calculation by Kawano, Nakaguchi and Nishioka [1410.5973].
- The renormalized Rényi entropy is computed in [1806.10708], using CHM map.

Example: Ball-shaped Entangling Region in $\mathrm{AdS}_{2n}/\mathrm{CFT}_{2n-1}$

- Minimal RT surface Σ, parametrized by $\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$, is a constant curvature manifold $\rightarrow \mathcal{F}_{AdS} = 0$.
- Σ is topologically equivalent to a $(2n-2)$-ball $\rightarrow \chi[\Sigma] = 1$.
- Therefore,

$$S_{EE}^{ren} = \frac{(-1)^{n+1} (4\pi)^{(n-1)} (n-1)! \ell^{2(n-1)}}{4G (2n-2)!}.$$

- Result agrees with the calculation by Kawano, Nakaguchi and Nishioka [1410.5973].
- The renormalized Rényi entropy is computed in [1806.10708], using CHM map.

Example: Ball-shaped Entangling Region in AdS_{2n}/CFT_{2n-1}

- Minimal RT surface Σ, parametrized by $\Sigma : \left\{ t = const \; ; \; r^2 + \ell^2 \rho = R^2 \right\}$, is a constant curvature manifold $\rightarrow \mathcal{F}_{AdS} = 0$.
- Σ is topologically equivalent to a $(2n-2)$-ball $\rightarrow \chi \left[\Sigma \right] = 1$.
- Therefore,

$$S_{EE}^{ren} = \frac{(-1)^{n+1} (4\pi)^{(n-1)} (n-1)! \ell^{2(n-1)}}{4G (2n-2)!}.$$

- Result agrees with the calculation by Kawano, Nakaguchi and Nishioka [1410.5973].
- The renormalized Rényi entropy is computed in [1806.10708], using CHM map.

Example: Ball-shaped Entangling Region in AdS_{2n}/CFT_{2n-1}

- Minimal RT surface Σ, parametrized by $\Sigma : \left\{ t = const \ ; \ r^2 + \ell^2 \rho = R^2 \right\}$, is a constant curvature manifold $\rightarrow \mathcal{F}_{AdS} = 0$.
- Σ is topologically equivalent to a $(2n-2)$-ball $\rightarrow \chi\left[\Sigma\right] = 1$.
- Therefore,

$$S_{EE}^{ren} = \frac{(-1)^{n+1} (4\pi)^{(n-1)} (n-1)! \ell^{2(n-1)}}{4G (2n-2)!}.$$

- Result agrees with the calculation by Kawano, Nakaguchi and Nishioka [1410.5973].
- The renormalized Rényi entropy is computed in [1806.10708], using CHM map.

Interpretation of renormalized EE (in even-dimensional CFT)

- For even-D CFTs, the renormalized EE is logarithmically divergent and it corresponds to the universal part.
- It contains the information about the conformal anomaly of the CFT.
- In particular, for ball-shaped entangling regions, we have

$$S_{EE}^{ren} = 2\,(-1)^n \log(\varepsilon)\, A$$
$$A = \frac{\ell^{(2n-1)}\pi^{(n-1)}}{8G\,(n-1)!},$$

in agreement with Myers and Sinha [1006.1263].

Interpretation of renormalized EE (in even-dimensional CFT)

- For even-D CFTs, the renormalized EE is logarithmically divergent and it corresponds to the universal part.
- It contains the information about the conformal anomaly of the CFT.
- In particular, for ball-shaped entangling regions, we have

$$S_{EE}^{ren} = 2\,(-1)^n \log(\varepsilon)\,A$$
$$A = \frac{\ell^{(2n-1)}\pi^{(n-1)}}{8G\,(n-1)!},$$

in agreement with Myers and Sinha [1006.1263].

Interpretation of renormalized EE (in even-dimensional CFT)

- For even-D CFTs, the renormalized EE is logarithmically divergent and it corresponds to the universal part.
- It contains the information about the conformal anomaly of the CFT.
- In particular, for ball-shaped entangling regions, we have

$$S_{EE}^{ren} = 2\left(-1\right)^{n}\log\left(\varepsilon\right)A$$
$$A = \frac{\ell^{(2n-1)}\pi^{(n-1)}}{8G\left(n-1\right)!},$$

in agreement with Myers and Sinha [1006.1263].

Interpretation of Results

- Renormalized EE equal to the universal part of EE. Related to parameters of CFT, e.g., a^*-charge (odd-d CFT) or A-anomaly coefficient (even-d CFT).
- a^* and the A-anomaly coefficient are conjectured to be C-function candidates (e.g., Myers and Sinha [1006.1263]).
- Renormalized EE is renormalized volume of codimension-2 RT surface. Einstein-AdS action is renormalized volume of bulk.
- Renormalized volume of the (even-D) bulk written as Euler characteristic plus (Einstein sector of) conformal invariant. May give hints for higher-dimensional conformal gravity.

Interpretation of Results

- Renormalized EE equal to the universal part of EE. Related to parameters of CFT, e.g., a^*-charge (odd-d CFT) or A-anomaly coefficient (even-d CFT).
- a^* and the A-anomaly coefficient are conjectured to be C-function candidates (e.g., Myers and Sinha [1006.1263]).
- Renormalized EE is renormalized volume of codimension-2 RT surface. Einstein-AdS action is renormalized volume of bulk.
- Renormalized volume of the (even-D) bulk written as Euler characteristic plus (Einstein sector of) conformal invariant. May give hints for higher-dimensional conformal gravity.

Interpretation of Results

- Renormalized EE equal to the universal part of EE. Related to parameters of CFT, e.g., a^*-charge (odd-d CFT) or A-anomaly coefficient (even-d CFT).
- a^* and the A-anomaly coefficient are conjectured to be C-function candidates (e.g., Myers and Sinha [1006.1263]).
- Renormalized EE is renormalized volume of codimension-2 RT surface. Einstein-AdS action is renormalized volume of bulk.
- Renormalized volume of the (even-D) bulk written as Euler characteristic plus (Einstein sector of) conformal invariant. May give hints for higher-dimensional conformal gravity.

Interpretation of Results

- Renormalized EE equal to the universal part of EE. Related to parameters of CFT, e.g., a^*-charge (odd-d CFT) or A-anomaly coefficient (even-d CFT).
- a^* and the A-anomaly coefficient are conjectured to be C-function candidates (e.g., Myers and Sinha [1006.1263]).
- Renormalized EE is renormalized volume of codimension-2 RT surface. Einstein-AdS action is renormalized volume of bulk.
- Renormalized volume of the (even-D) bulk written as Euler characteristic plus (Einstein sector of) conformal invariant. May give hints for higher-dimensional conformal gravity.

Future Work

- To finish and publish the analysis for even-D Einstein-AdS dual to odd-d CFTs.
- To generalize the method to higher curvature theories like Lovelock Gravity.
- To study the application of the procedure to other Quantum Information Theoretic (QIT) measures like complexity (see [1710.01327] for the particular case of AdS_3).
- To investigate the possible relation of the form of the renormalized volume with higher-dimensional conformal gravity.

This work is funded by FONDECYT grant No. 3180620 "Entanglement Entropy and AdS gravity".

Future Work

- To finish and publish the analysis for even-D Einstein-AdS dual to odd-d CFTs.
- To generalize the method to higher curvature theories like Lovelock Gravity.
- To study the application of the procedure to other Quantum Information Theoretic (QIT) measures like complexity (see [1710.01327] for the particular case of AdS_3).
- To investigate the possible relation of the form of the renormalized volume with higher-dimensional conformal gravity.

This work is funded by FONDECYT grant No. 3180620 "Entanglement Entropy and AdS gravity".

Future Work

- To finish and publish the analysis for even-D Einstein-AdS dual to odd-d CFTs.
- To generalize the method to higher curvature theories like Lovelock Gravity.
- To study the application of the procedure to other Quantum Information Theoretic (QIT) measures like complexity (see [1710.01327] for the particular case of AdS_3).
- To investigate the possible relation of the form of the renormalized volume with higher-dimensional conformal gravity.

This work is funded by FONDECYT grant No. 3180620 "Entanglement Entropy and AdS gravity".

Future Work

- To finish and publish the analysis for even-D Einstein-AdS dual to odd-d CFTs.
- To generalize the method to higher curvature theories like Lovelock Gravity.
- To study the application of the procedure to other Quantum Information Theoretic (QIT) measures like complexity (see [1710.01327] for the particular case of AdS_3).
- To investigate the possible relation of the form of the renormalized volume with higher-dimensional conformal gravity.

This work is funded by FONDECYT grant No. 3180620 "Entanglement Entropy and AdS gravity".