

**PAUTA TEST N° 4 ECUACIONES DIFERENCIALES
INGENIERÍA CIVIL AGRÍCOLA**

NOMBRE : _____
TIEMPO MÁXIMO : 1 HORA 30 MINUTOS **FECHA : Ju 13/11/25**

1) Resuelva

$$2x' + y' - 2x = 1$$

$$x' + y' - 3x - 3y = 2$$

$$x(0) = 0, y(0) = 0$$

(15 puntos)

Solución:

$$\begin{aligned} 2x' + y' - 2x = 1 &\Rightarrow \mathcal{L}[2x' + y' - 2x] = \mathcal{L}[1] \Rightarrow 2\mathcal{L}[x'] + \mathcal{L}[y'] - 2\mathcal{L}[x] = \mathcal{L}[1] \Rightarrow \\ 2(sX(s) - x(0)) + sY(s) - y(0) - 2X(s) &= \frac{1}{s} \Rightarrow 2sX(s) + sY(s) - 2X(s) = \frac{1}{s} \Rightarrow \\ (2s - 2)X(s) + sY(s) &= \frac{1}{s} \end{aligned}$$

$$\begin{aligned} x' + y' - 3x - 3y = 2 &\Rightarrow \mathcal{L}[x' + y' - 3x - 3y] = \mathcal{L}[2] \Rightarrow \\ \mathcal{L}[x'] + \mathcal{L}[y'] - 3\mathcal{L}[x] - 3\mathcal{L}[y] &= 2\mathcal{L}[1] \Rightarrow \\ sX(s) - x(0) + sY(s) - y(0) - 3X(s) - 3Y(s) &= \frac{2}{s} \Rightarrow \\ (s - 3)X(s) + (s - 3)Y(s) &= \frac{2}{s} \end{aligned}$$

El sistema a resolver es:

$$(2s - 2)X(s) + sY(s) = \frac{1}{s}$$

$$(s - 3)X(s) + (s - 3)Y(s) = \frac{2}{s}$$

$$\begin{aligned} \text{Calculemos } \begin{vmatrix} 2s - 2 & s \\ s - 3 & s - 3 \end{vmatrix} &= (2s - 2)(s - 3) - s(s - 3) = (s - 3)(2s - 2 - s) = \\ (s - 3)(s - 2) &\neq 0 \text{ (Asumimos que } s \neq 2 \text{ y } s \neq 3) \end{aligned}$$

$$X(s) = \frac{\begin{vmatrix} \frac{1}{s} & s \\ \frac{2}{s} & s - 3 \end{vmatrix}}{(s - 3)(s - 2)} = \frac{\frac{s - 3}{s} - 2}{(s - 3)(s - 2)} = \frac{\frac{s - 3 - 2s}{s}}{(s - 3)(s - 2)} = \frac{-s - 3}{s(s - 2)(s - 3)} \Rightarrow \text{GeoGebra}$$

$$x(t) = \mathcal{L}^{-1} \left[\frac{-s - 3}{s(s - 2)(s - 3)} \right] \Rightarrow x(t) = \frac{5}{2} e^{2t} - 2e^{3t} - \frac{1}{2}$$

$$Y(s) = \left| \begin{array}{cc} 2s-2 & \frac{1}{s} \\ s-3 & \frac{2}{s} \end{array} \right| = \frac{\frac{2(2s-2)}{s} - \frac{s-3}{s}}{(s-3)(s-2)} = \frac{\frac{4s-4-s+3}{s}}{(s-3)(s-2)} = \frac{3s-1}{s(s-2)(s-3)} \Rightarrow \text{GeoGebra}$$

$$y(t) = \mathcal{L}^{-1} \left[\frac{3s-1}{s(s-2)(s-3)} \right] \Rightarrow y(t) = -\frac{5}{2} e^{2t} + \frac{8}{3} e^{3t} - \frac{1}{6} \quad \square$$

2) Obtenga la serie de Maclaurin de $f(x) = \frac{1}{1+x^2}$

(15 puntos)

Solución:

Sabemos que : $\sum_{n=0}^{\infty} B^n = \frac{1}{1-B}$

Si hacemos $B = -x^2$, entonces

$$\sum_{n=0}^{\infty} B^n = \frac{1}{1-B} \Rightarrow \sum_{n=0}^{\infty} (-x^2)^n = \frac{1}{1-(-x^2)} \Rightarrow$$

$$f(x) = \frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad \square$$

3) Resuelva $y' + 2y = 3\delta(t)$, con $y(0) = 0$

(15 puntos)

Solución:

$$y' + 2y = 3\delta(t) \Rightarrow \mathcal{L}[y' + 2y] = \mathcal{L}[3\delta(t)] \Rightarrow \mathcal{L}[y'] + 2\mathcal{L}[y] = 3\mathcal{L}[\delta(t)] \Rightarrow$$

$$sY(s) - y(0) + 2Y(s) = 3e^{-s \cdot 0} \Rightarrow sY(s) + 2Y(s) = 3 \Rightarrow$$

$$(s+2)Y(s) = 3 \Rightarrow Y(s) = \frac{3}{s+2} \Rightarrow \text{GeoGebra } y(t) = \mathcal{L}^{-1} \left[\frac{3}{s+2} \right] = 3e^{-2t} \quad \square$$

4) Resuelva $x'' + x = f(t)$, $x(0) = 0$, $x'(0) = 0$, con $f(t) = \begin{cases} 0 & , \text{ si } t < \pi \\ \text{sen}(t) & , \text{ si } t \geq \pi \end{cases}$

(Indicación: $\text{Heaviside}(t-a) = u(t-a) = \begin{cases} 0 & , \text{ si } t < a \\ 1 & , \text{ si } t \geq a \end{cases}$)

(15 puntos)

Solución:

$$x'' + x = f(t) \Rightarrow \mathcal{L}[x'' + x] = \mathcal{L}[f(t)] \Rightarrow \mathcal{L}[x''] + \mathcal{L}[x] = \mathcal{L}[f(t)] \Rightarrow$$

$$s^2 X(s) - s x(0) - x'(0) + X(s) = -\frac{e^{-s\pi}}{s^2+1} \Rightarrow s^2 X(s) + X(s) = -\frac{e^{-s\pi}}{s^2+1} \Rightarrow$$

$$(s^2+1)X(s) = -\frac{e^{-s\pi}}{s^2+1} \Rightarrow X(s) = -\frac{e^{-s\pi}}{(s^2+1)^2} \Rightarrow \text{GeoGebra}$$

$$x(t) = \mathcal{L}^{-1} \left[-\frac{e^{-s\pi}}{(s^2+1)^2} \right] = \left(\frac{1}{2} \text{sen}(t) + \frac{\pi-t}{2} \cos(t) \right) \text{Heaviside}(t-\pi) \quad \square$$