UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE INGENIERÍA AGRÍCOLA
DEPTO. DE AGROINDUSTRIAS
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PAUTA TAREA SUMATIVA (1 PUNTO) PRIMER CERTAMEN
CÁLCULO II - CÁLCULO INTEGRAL
INGENIERÍA AGROINDUSTRIAL - INGENIERÍA CIVIL AGRÍCOLA

Fecha: Ma 07 de Noviembre de 2006
Fecha Entrega: Lu 13 de Noviembre de 2006.

Calcule : $\int \frac{x^3-2x^2+x}{3x^4-2x^2}dx$

<u>Solución:</u>

Calculemos en primer lugar las raíces de $q(x) = 3x^4 - 2x^2$.

$$q(x) = 0 \Rightarrow 3x^4 - 2x^2 = 0 \Rightarrow x^2(3x^2-2) = 0 \Rightarrow x^2 = 0 \ \vee 3x^2 - 2 = 0 \Rightarrow$$

$$x_1 = 0 \ \vee x_2 = 0 \vee 3x^2 - 2 = 0 \Rightarrow x_1 = 0 \ \vee x_2 = 0 \vee x^2 = \frac{2}{3} \Rightarrow$$

$$x_1 = 0 \ \vee x_2 = 0 \vee x_3 = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \vee x_4 = -\sqrt{\frac{2}{3}} = -\frac{\sqrt{2}}{\sqrt{3}}$$

Por lo tanto,

$$3x^4 - 2x^2 = x^2(\sqrt{3}\,x + \sqrt{2}\,)(\sqrt{3}\,x - \sqrt{2}\,)$$

Usando la factorización en suma de fracciones parciales, se tiene que :

$$\frac{x^3-2x^2+x}{3x^4-2x^2} = \frac{x^3-2x^2+x}{x^2(\sqrt{3}\,x+\sqrt{2}\,)(\sqrt{3}\,x-\sqrt{2}\,)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{\sqrt{3}\,x+\sqrt{2}} + \frac{A_4}{\sqrt{3}\,x-\sqrt{2}} \Rightarrow$$

$$\frac{x^3-2x^2+x}{x^2(\sqrt{3}\,x+\sqrt{2}\,)(\sqrt{3}\,x-\sqrt{2}\,)} = \frac{A_1x(3x^2-2)+A_2(3x^2-2)+A_3x^2(\sqrt{3}\,x-\sqrt{2}\,)+A_4x^2(\sqrt{3}\,x+\sqrt{2}\,)}{x^2(\sqrt{3}\,x+\sqrt{2}\,)(\sqrt{3}\,x-\sqrt{2}\,)} \Rightarrow$$

$$x^3 - 2x^2 + x = A_1x(3x^2-2) + A_2(3x^2-2) + A_3x^2(\sqrt{3}\,x - \sqrt{2}\,) +$$

$$A_4x^2(\sqrt{3}x + \sqrt{2}\,)$$

Para $x = 0$: $0^3 - 2(0)^2 + 0 = A_2(3(0)^2 - 2) \Rightarrow -2A_2 = 0 \Rightarrow A_2 = 0$

Para $x = \frac{\sqrt{2}}{\sqrt{3}}$: $\frac{2\sqrt{2}}{3\sqrt{3}} - \frac{4}{3} + \frac{\sqrt{2}}{\sqrt{3}} = A_4 \frac{2}{3}(\sqrt{2} + \sqrt{2}\,) \Rightarrow$

$\frac{2\sqrt{2} - 4\sqrt{3} + 3\sqrt{2}}{3\sqrt{3}} = A_4 \frac{4\sqrt{2}}{3} \Rightarrow A_4 = \frac{5\sqrt{2} - 4\sqrt{3}}{4\sqrt{3}\,\sqrt{2}} \Rightarrow A_4 = \frac{5\sqrt{3} - 6\sqrt{2}}{12}$

Para $x = -\frac{\sqrt{2}}{\sqrt{3}}$: $-\frac{2\sqrt{2}}{3\sqrt{3}} - \frac{4}{3} - \frac{\sqrt{2}}{\sqrt{3}} = A_3 \left[\frac{2}{3}\right](-\sqrt{2} - \sqrt{2}\,) \Rightarrow$

$\frac{-2\sqrt{2} - 4\sqrt{3} - 3\sqrt{2}}{3\sqrt{3}} = A_3 \frac{-4\sqrt{2}}{3} \Rightarrow A_3 = \frac{-5\sqrt{2} - 4\sqrt{3}}{-4\sqrt{3}\,\sqrt{2}} \Rightarrow A_3 = \frac{5\sqrt{3} + 6\sqrt{2}}{12}$

Para $x = 1$: $1^3 - 2(1)^2 + 1 = A_1 + A_2 + A_3(\sqrt{3} - \sqrt{2}\,) + A_4(\sqrt{3} + \sqrt{2}\,) \Rightarrow$

$0 = A_1 + A_2 + A_3(\sqrt{3} - \sqrt{2}\,) + A_4(\sqrt{3} + \sqrt{2}\,) \Rightarrow$

$0 = A_1 + 0 + \frac{5\sqrt{3} + 6\sqrt{2}}{12}(\sqrt{3} - \sqrt{2}\,) + \frac{5\sqrt{3} - 6\sqrt{2}}{12}(\sqrt{3} + \sqrt{2}\,) \Rightarrow$

$0 = A_1 + 0 + \frac{\sqrt{6} + 3}{12} + \frac{3 - \sqrt{6}}{12} \Rightarrow A_1 = -\frac{1}{2}$

Luego

$\frac{x^3 - 2x^2 + x}{3x^4 - 2x^2} = -\frac{1}{2}\frac{1}{x} + \frac{11\sqrt{2}}{12}\frac{1}{\sqrt{3}\,x + \sqrt{2}} - \frac{\sqrt{2}}{12}\frac{1}{\sqrt{3}\,x - \sqrt{2}}$

Por lo tanto :

$\int \frac{x^3 - 2x^2 + x}{3x^4 - 2x^2} dx = -\frac{1}{2}\int \frac{1}{x} dx + \frac{5\sqrt{3} + 6\sqrt{2}}{12}\int \frac{1}{\sqrt{3}\,x + \sqrt{2}} dx + \frac{5\sqrt{3} - 6\sqrt{2}}{12}\int \frac{1}{\sqrt{3}\,x - \sqrt{2}} dx$

$= -\frac{1}{2}\, ln|x| + \frac{5\sqrt{3} + 6\sqrt{2}}{12\sqrt{3}}\, ln\left|\sqrt{3}\,x + \sqrt{2}\right| + \frac{5\sqrt{3} - 6\sqrt{2}}{12\sqrt{3}} ln\left|\sqrt{3}\,x - \sqrt{2}\right| + c =$

$-\frac{1}{2}\, ln|x| + \frac{5 + 2\sqrt{6}}{12}\, ln\left|\sqrt{3}\,x + \sqrt{2}\right| + \frac{5 - 2\sqrt{6}}{12} ln\left|\sqrt{3}\,x - \sqrt{2}\right| + c$ ❑