

PAUTA TEST N° 4 CÁLCULO II
INGENIERÍA AMBIENTAL

NOMBRE : _____

TIEMPO MÁXIMO : 1 HORA 20 MINUTOS

FECHA : Vi 21/11/25

1) Obtenga el radio y el intervalo de convergencia para las siguientes series:

a) $\sum_{n=0}^{\infty} \frac{n!}{10^n} (x - \pi)^n$

Solución:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)!}{10^{n+1}} (x - \pi)^{n+1}}{\frac{n!}{10^n} (x - \pi)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n!(n+1)}{10^{n+1}} (x - \pi)^n (x - \pi)}{\frac{n!}{10^n} (x - \pi)^n} \right| =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)(x - \pi)}{10} \right| = \left| \frac{x - \pi}{10} \right| \lim_{n \rightarrow \infty} (n + 1) = \infty$$

Luego, el radio de convergencia es $R = \frac{1}{\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|} = \frac{1}{\infty} = 0$

El intervalo de convergencia es $\{\pi\}$, pues $x - \pi = 0$. $\square$

b) $\sum_{n=0}^{\infty} \frac{n}{4^n} (2x - 1)^n$

Solución:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n+1}{4^{n+1}} (2x-1)^{n+1}}{\frac{n}{4^n} (2x-1)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n+1}{4^{n+1}} (2x-1)^n (2x-1)}{\frac{n}{4^n} (2x-1)^n} \right| =$$

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)(2x-1)}{4n} \right| = \left| \frac{2x-1}{4} \right| \lim_{n \rightarrow \infty} \frac{n+1}{n} = \left| \frac{1}{2}x - \frac{1}{4} \right| \lim_{n \rightarrow \infty} \frac{n+1}{n} =$$

$$\frac{1}{2} \left| x - \frac{1}{2} \right| \lim_{n \rightarrow \infty} \frac{\frac{n+1}{n}}{\frac{n}{n}} = \frac{1}{2} \left| x - \frac{1}{2} \right| \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right) = \frac{1}{2} \left| x - \frac{1}{2} \right| < 1$$

$$\Rightarrow \left| x - \frac{1}{2} \right| < 2 \Rightarrow -2 < x - \frac{1}{2} < 2 \Rightarrow -\frac{3}{2} < x < \frac{5}{2}$$

Observamos que el radio de convergencia es 2 y el intervalo de convergencia es $(-\frac{3}{2}, \frac{5}{2})$ $\square$

(20 puntos)

2) Obtenga la serie de Maclaurin de $f(x) = \frac{x}{1-x}$

Solución:

Recordemos que: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Luego : $\frac{x}{1-x} = x \frac{1}{1-x} = x \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} x \cdot x^n = \sum_{n=0}^{\infty} x^{n+1}$ $\square$

(10 puntos)

3) Obtenga, si es posible, la suma de las siguientes series:

a) $\frac{1}{2^6} + \frac{1}{2^8} + \frac{1}{2^{10}} + \frac{1}{2^{12}} + \dots$

Solución:

$$\begin{aligned} \frac{1}{2^6} + \frac{1}{2^8} + \frac{1}{2^{10}} + \frac{1}{2^{12}} + \dots &= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n+4} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} \left(\frac{1}{2}\right)^4 = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^2\right]^n \left(\frac{1}{2}\right)^4 \\ &= \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^2\right]^n \left(\frac{1}{2}\right)^4 = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^2\right]^{n-1} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4 = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{n-1} \left(\frac{1}{2}\right)^6 \\ &= \sum_{n=1}^{\infty} \frac{1}{64} \left(\frac{1}{4}\right)^{n-1} = \frac{\frac{1}{64}}{1 - \frac{1}{4}} = \frac{\frac{1}{64}}{\frac{3}{4}} = \frac{1}{64} \cdot \frac{4}{3} = \frac{1}{16} \cdot \frac{1}{3} = \frac{1}{48} \quad \square \end{aligned}$$

b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

Solución:

Notemos que :

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \Rightarrow 1 = A(n+1) + Bn$$

$$n = 0 : 1 = A \Rightarrow A = 1$$

$$n = -1 : 1 = -B \Rightarrow B = -1$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\text{Luego: } \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots = 1 \quad \square$$

(20 puntos)

4) Obtenga la serie de Taylor de $f(x) = \sqrt{x}$ alrededor de 4

Solución:

La serie de Taylor de $f(x)$ alrededor de 4 es

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(4)}{n!} (x-4)^n$$

$$f(x) = \sqrt{x} \Rightarrow f(4) = \sqrt{4} = 2$$

$$f'(x) = (x^{1/2})' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$f^{(2)}(x) = \left(\frac{1}{2}x^{-1/2}\right)' = -\frac{1}{4}x^{-3/2} = -\frac{1}{4\sqrt{x^3}} \Rightarrow f^{(2)}(4) = -\frac{1}{4\sqrt{4^3}} = -\frac{1}{32}$$

$$f^{(3)}(x) = \left(-\frac{1}{4}x^{-3/2}\right)' = \frac{3}{8}x^{-5/2} = \frac{3}{8\sqrt{x^5}} \Rightarrow f^{(3)}(4) = \frac{3}{8\sqrt{4^5}} = \frac{3}{256}$$

$$f^{(4)}(x) = \left(\frac{3}{8}x^{-5/2}\right)' = -\frac{15}{16}x^{-7/2} = -\frac{15}{16\sqrt{x^7}} \Rightarrow f^{(4)}(4) = -\frac{15}{16\sqrt{4^7}} = -\frac{15}{2048}$$

Reemplazando en la serie de Taylor se tiene:

$$\sqrt{x} = \sum_{n=0}^{\infty} \frac{f^{(n)}(4)}{n!} (x-4)^n =$$

$$f(4) + \frac{f'(4)}{1!} (x-4) + \frac{f^{(2)}(4)}{2!} (x-4)^2 + \frac{f^{(3)}(4)}{3!} (x-4)^3 +$$

$$\frac{f^{(4)}(4)}{4!} (x-4)^4 + \dots =$$

$$2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3 - \frac{5}{16384}(x-4)^4 + \dots \quad \square$$

(10 puntos)