

PAUTA PRUEBA N° 2 SEGUNDA PARTE
CÁLCULO II - CÁLCULO INTEGRAL
INGENIERÍA AGROINDUSTRIAL - INGENIERÍA CIVIL AGRÍCOLA

NOMBRE1 : _____

NOMBRE2 : _____

TIEMPO MÁXIMO : 1 HORA 10 MINUTOS

FECHA : Vi 17/11/06

PTOS. : _____

(2) Calcule $\int \frac{dx}{x^3-8}$

(30 puntos).

Solución:

$$\frac{1}{x^3-8} = \frac{1}{(x-2)(x^2+2x+4)} = \frac{A_1}{x-2} + \frac{A_2x+B_2}{x^2+2x+4} = \frac{A_1(x^2+2x+4)+(A_2x+B_2)(x-2)}{(x-2)(x^2+2x+4)} \Rightarrow$$

$$1 = A_1(x^2 + 2x + 4) + (A_2x + B_2)(x - 2) \Rightarrow$$

$$1 = A_1x^2 + 2A_1x + 4A_1 + A_2x^2 - 2A_2x + B_2x - 2B_2 \Rightarrow$$

$$1 = (A_1 + A_2)x^2 + (2A_1 - 2A_2 + B_2)x + (4A_1 - 2B_2) \Rightarrow$$

$$A_1 + A_2 = 0 \Rightarrow A_1 = -A_2$$

$$2A_1 - 2A_2 + B_2 = 0 \Rightarrow -4A_2 + B_2 = 0 \Rightarrow B_2 = 4A_2$$

$$4A_1 - 2B_2 = 1 \Rightarrow -4A_2 - 8A_2 = 1 \Rightarrow -12A_2 = 1 \Rightarrow A_2 = -\frac{1}{12}$$

$$A_1 = -A_2 \Rightarrow A_1 = \frac{1}{12}$$

$$B_2 = 4A_2 \Rightarrow B_2 = -\frac{4}{12}$$

$$\therefore \frac{1}{x^3-8} = \frac{\frac{1}{12}}{x-2} + \frac{-\frac{1}{12}x - \frac{4}{12}}{x^2+2x+4} = \frac{1}{12} \frac{1}{x-2} - \frac{1}{12} \frac{x+4}{x^2+2x+4}$$

Ahora

$$\int \frac{dx}{x^3-8} = \frac{1}{12} \int \frac{1}{x-2} dx - \frac{1}{12} \int \frac{x+4}{x^2+2x+4} dx$$

Para calcular la integral $\int \frac{x+4}{x^2+2x+4} dx$ debemos completar cuadrados.

$$x^2 + 2x + 4 = (x^2 + 2x + 1) + 3 = (x + 1)^2 + 3$$

Haciendo $u = x + 1$, se tiene que $du = dx$ y $x = u - 1$

$$\int \frac{x+4}{x^2+2x+4} dx = \int \frac{u-1+4}{u^2+3} du = \int \frac{u+3}{u^2+3} du = \int \frac{u}{u^2+3} du + \int \frac{3}{u^2+3} du$$

Resolvamos $\int \frac{u}{u^2+3} du$:

$$\int \frac{u}{u^2+3} du = \int_{z=u^2+3} \frac{1}{z} dz = \frac{1}{2} \int \frac{1}{z} dz = \frac{1}{2} \ln|z| = \frac{1}{2} \ln|u^2 + 3| = \frac{1}{2} \ln|(x + 1)^2 + 3|$$

Resolvamos $\int \frac{3}{u^2+3} du$:

$$\int \frac{3}{u^2+3} du = \text{Fórmula 17} \quad \frac{3}{\sqrt{3}} \arctg\left(\frac{u}{\sqrt{3}}\right) = \frac{3}{\sqrt{3}} \arctg\left(\frac{x+1}{\sqrt{3}}\right)$$

Finalmente :

$$\int \frac{dx}{x^3-8} = \frac{1}{12} \int \frac{1}{x-2} dx - \frac{1}{12} \int \frac{x+4}{x^2+2x+4} dx =$$

$$\frac{1}{12} \ln|x - 2| - \frac{1}{12} \frac{1}{2} \ln|(x + 1)^2 + 3| - \frac{1}{12} \frac{3}{\sqrt{3}} \arctg\left(\frac{x+1}{\sqrt{3}}\right) + c =$$

$$\frac{1}{12} \ln|x - 2| - \frac{1}{24} \ln|(x + 1)^2 + 3| - \frac{1}{4\sqrt{3}} \arctg\left(\frac{x+1}{\sqrt{3}}\right) + c =$$

$$\frac{1}{12} \ln|x - 2| - \frac{1}{24} \ln|x^2 + 2x + 4| - \frac{\sqrt{3}}{12} \arctg\left(\frac{(x+1)\sqrt{3}}{3}\right) + c \quad \square$$