

PAUTA TEST N° 2 CÁLCULO AVANZADO
INGENIERÍA AMBIENTAL

NOMBRE : _____
TIEMPO MÁXIMO : 30 MINUTOS **FECHA : Lu 31/03/24**

1) Resuelva la integral $\int \frac{x^3}{\sqrt{1+x^2}} dx$

(30 puntos)

Solución:

$$\int \frac{x^3}{\sqrt{1+x^2}} dx = \int \frac{x^2}{\sqrt{1+x^2}} x dx$$

$$V = 1 + x^2 \Rightarrow dV = 2x dx \Rightarrow x dx = \frac{1}{2} dV$$

$$V = 1 + x^2 \Rightarrow x^2 = V - 1$$

Luego

$$\int \frac{x^3}{\sqrt{1+x^2}} dx = \int \frac{x^2}{\sqrt{1+x^2}} x dx = \frac{1}{2} \int \frac{V-1}{\sqrt{V}} dV = \frac{1}{2} \int \frac{V-1}{V^{1/2}} dV =$$

$$\frac{1}{2} \int \frac{V}{V^{1/2}} dV - \frac{1}{2} \int \frac{1}{V^{1/2}} dV = \frac{1}{2} \int V^{1/2} dV - \frac{1}{2} \int V^{-1/2} dV =$$

$$\frac{1}{2} \cdot \frac{2}{3} V^{3/2} - \frac{1}{2} \cdot 2 V^{1/2} + c = \frac{1}{3} V^{3/2} - V^{1/2} + c = \frac{1}{3} (1+x^2)^{3/2} - (1+x^2)^{1/2} + c$$

$$\text{Finalmente, } \int \frac{x^3}{\sqrt{1+x^2}} dx = \frac{1}{3} (1+x^2)^{3/2} - (1+x^2)^{1/2} + c \quad \square$$

2) Resuelva la integral $\int_0^1 \left[e^{x+7} - \frac{1}{1+x^2} + \cos(2-x) \right] dx$

(30 puntos)

Solución:

$$\begin{aligned} \int_0^1 \left[e^{x+7} - \frac{1}{1+x^2} + \cos(2-x) \right] dx &= \int_0^1 e^{x+7} dx - \int_0^1 \frac{1}{1+x^2} dx + \int_0^1 \cos(2-x) dx \\ &= e^{x+7} \Big|_0^1 - \text{Arctg}(x) \Big|_0^1 - \text{sen}(2-x) \Big|_0^1 = e^8 - e^7 - \frac{\pi}{4} - \text{sen}(1) + \text{sen}(2) \quad \square \end{aligned}$$