UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE INGENIERÍA AGRÍCOLA
DEPTO. DE AGROINDUSTRIAS
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PAUTA RECUPERACIÓN CERTAMEN N° 1
CÁLCULO AVANZADO
INGENIERÍA AMBIENTAL

NOMBRE :____
TIEMPO MÁXIMO : 1HORA 40 MINUTOS **FECHA : Vi 07/10/22**

1) Resuelva la integral $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$

(15 puntos)

Solución:

Sea $P^2 = x$

$P^2 = x \Rightarrow 2P\,dP = dx$

$P^2 = x \Rightarrow P = \sqrt{x} \Rightarrow 1+\sqrt{x} = 1+P$

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = \int \frac{P}{1+P}\,2P\,dP = 2\int \frac{P^2}{1+P}\,dP$$

$$\begin{array}{l} \quad P^2 : P+1 = P-1 \\ {}^{(-)}P^2 + {}^{(-)}P \\ ------\ -- \\ \qquad\qquad -P \\ \qquad\quad -{}^{(+)}P - {}^{(+)}1 \\ \qquad\quad --------- \\ \qquad\qquad\qquad 1 \end{array}$$

$$\frac{P^2}{1+P} = P - 1 + \frac{1}{1+P}$$

$$2\int \frac{P^2}{1+P}\,dP = 2\int \left(P - 1 + \frac{1}{1+P}\right) dP = 2\int P\,dP - 2\int dP + 2\int \frac{1}{1+P}\,dP =$$

$$P^2 - 2P + 2\,ln(1+P) + c = x - 2\sqrt{x} + 2\,ln(1+\sqrt{x}\,) + c$$

Finalmente, $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = x - 2\sqrt{x} + 2\,ln(1+\sqrt{x}\,) + c$ ❑

2) Muestre que : $\displaystyle\int_0^{\frac{1}{2}} t^2 Arcsen(t)\,dt = \frac{\pi}{144} + \frac{\sqrt{3}}{8} - \frac{2}{9}$

(15 puntos)

Solución:

Usemos integración por partes

$p = Arcsen(t) \Rightarrow\ p' = \frac{1}{\sqrt{1-t^2}}$

$q' = t^2 \Rightarrow q = \frac{t^3}{3}$

$$\int_0^{\frac{1}{2}} t^2 Arcsen(t)\,dt = \frac{t^3}{3} Arcsen(t)\Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{t^3}{3}\frac{dt}{\sqrt{1-t^2}}$$
$$= \frac{t^3}{3} Arcsen(t)\Big|_0^{\frac{1}{2}} - \frac{1}{3}\int_0^{\frac{1}{2}} \frac{t^3 dt}{\sqrt{1-t^2}} = \left(\frac{1}{24}\right)\left(\frac{\pi}{6}\right) - \frac{1}{3}\int_0^{\frac{1}{2}} \frac{t^3 dt}{\sqrt{1-t^2}}$$
$$= \frac{\pi}{144} - \frac{1}{3}\int_0^{\frac{1}{2}} \frac{t^3 dt}{\sqrt{1-t^2}}$$

Para la última integral usamos la sustitución $u = 1 - t^2$, de donde $u = 1 - t^2$, $du = -2tdt$ y $t^2 = 1 - u$. Luego

$$\int \frac{t^3 dx}{\sqrt{1-t^2}} = \int \frac{t^2 t\,dt}{\sqrt{1-t^2}} = \int \frac{(1-u)\,\frac{-du}{2}}{\sqrt{u}} = -\frac{1}{2}\int \frac{(1-u)\,du}{(u)^{\frac{1}{2}}}$$
$$= -\frac{1}{2}\int (u^{-\frac{1}{2}} - u^{\frac{1}{2}})du = -\frac{1}{2}\left[2\,u^{\frac{1}{2}} - \frac{2}{3}\,u^{\frac{3}{2}}\right]$$
$$= -u^{\frac{1}{2}} + \frac{1}{3}u^{\frac{3}{2}} = -(1-t^2)^{\frac{1}{2}} + \frac{1}{3}(1-t^2)^{\frac{3}{2}}$$

Luego,

$$\int_0^{\frac{1}{2}} \frac{t^3 dt}{\sqrt{1-t^2}} = \left[-(1-t^2)^{\frac{1}{2}} + \frac{1}{3}(1-t^2)^{\frac{3}{2}}\right]\Bigg|_0^{\frac{1}{2}}$$
$$= \left[-\left(1-\frac{1}{4}\right)^{\frac{1}{2}} + \frac{1}{3}\left(1-\frac{1}{4}\right)^{\frac{3}{2}}\right] - \left((-1) + \frac{1}{3}\right)$$
$$= -\frac{\sqrt{3}}{2} + \frac{1}{3}\left(\frac{3}{4}\right)^{\frac{3}{2}} + 1 - \frac{1}{3} = -\frac{\sqrt{3}}{2} + \frac{1}{3}\frac{3\sqrt{3}}{8} + \frac{2}{3} = \frac{2}{3} - \frac{3}{8}\sqrt{3}$$

Así,

$$\int_0^{\frac{1}{2}} t^2 Arcsen(t)\,dt = \frac{\pi}{144} - \frac{1}{3}\int_0^{\frac{1}{2}} \frac{t^3 dt}{\sqrt{1-t^2}} = \frac{\pi}{144} - \frac{1}{3}\left[\frac{2}{3} - \frac{3}{8}\sqrt{3}\right]$$
$$= \frac{\pi}{144} - \frac{2}{9} + \frac{1}{8}\sqrt{3}$$

Finalmente, $\displaystyle\int_0^{\frac{1}{2}} t^2 Arcsen(t)\,dt = \frac{\pi}{144} - \frac{2}{9} + \frac{1}{8}\sqrt{3}$ ❑

3) Resuelva $\displaystyle\int cos^2(q)\, sen^3(q)\, dq$

(15 puntos)

Solución:

$$\int cos^2(q)\, sen^3(q)\, dq = \int cos^2(q)\, sen^2(q)\, sen(q)\, dq =$$

$$\int cos^2(q)\, (1 - cos^2(q))\, sen(q)\, dq$$

Sea $m = cos(q)$

$m = cos(q) \Rightarrow dm = \, - sen(q)\, dq \Rightarrow sen(q)\, dq = \, - dm$

$$\int cos^2(q)\, (1 - cos^2(q))\, sen(q)\, dq = \, - \int m^2\, (1 - m^2)\, dm =$$

$$- \int m^2\, dm + \int m^4\, dm = \, - \tfrac{1}{3}\, m^3 + \tfrac{1}{5}\, m^5 + c = \, - \tfrac{1}{3}\, cos^3(q) + \tfrac{1}{5}\, cos^5(q) + c$$

Finalmente, $\int cos^2(q)\, sen^3(q)\, dq = \, - \tfrac{1}{3}\, cos^3(q) + \tfrac{1}{5}\, cos^5(q) + c$ ☐

4) Calcule $\int_0^{\sqrt{2}} \frac{x^2}{2+x^2} dx$

(15 puntos)

Solución:

$x^2 : x^2 + 2 = 1$

${}^{(-)}x^2 + {}^{(-)}2$

$---------$

$-\,2$

$\frac{x^2}{2+x^2} = 1 - \frac{2}{x^2+2}$

$$\int \frac{x^2}{2+x^2} dx = \int \left(1 - \frac{2}{x^2+2}\right) dx = \int dx - \int \frac{2}{x^2+2}\, dx = x - 2 \int \frac{1}{x^2+2}\, dx$$

Obtengamos la integral $\int \frac{1}{x^2+2}\, dx$ usando la sustitución trigonométrica

$x = \sqrt{2}\, tg(\alpha)$

$x = \sqrt{2}\, tg(\alpha) \Rightarrow dx = \sqrt{2}\, sec^2(\alpha)\, d\alpha$

$x = \sqrt{2}\, tg(\alpha) \Rightarrow \alpha = Arctg(\frac{x}{\sqrt{2}})$

$x^2 + 2 = 2\, tg^2(\alpha) + 2 = 2(tg^2(\alpha) + 1) = 2\, sec^2(\alpha)$

$$\int \frac{1}{x^2+2}\, dx = \int \frac{1}{2\, sec^2(\alpha)}\, \sqrt{2}\, sec^2(\alpha)\, d\alpha = \frac{\sqrt{2}}{2} \int d\alpha = \frac{\sqrt{2}}{2}\, \alpha = \frac{\sqrt{2}}{2}\, Arctg(\frac{x}{\sqrt{2}})$$

$$\int_0^{\sqrt{2}} \frac{x^2}{2+x^2} dx = x - 2 \int \frac{1}{x^2+2}\, dx = \left[x - \sqrt{2}\, Arctg(\frac{x}{\sqrt{2}})\right]_0^{\sqrt{2}} =$$

$$\sqrt{2} - \sqrt{2}\, Arctg(1) = \sqrt{2} - \tfrac{\pi}{4}\sqrt{2}$$

Finalmente, $\int_0^{\sqrt{2}} \frac{x^2}{2+x^2} dx = \sqrt{2} - \frac{\pi}{4}\sqrt{2}$ ☐