UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE INGENIERÍA AGRÍCOLA
DEPTO. DE AGROINDUSTRIAS
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PAUTA CERTAMEN N° 1 CÁLCULO AVANZADO
INGENIERÍA AMBIENTAL

NOMBRE :____
TIEMPO MÁXIMO : 1 HORA 40 MINUTOS **FECHA : Mi 03/04/24**

1) Resuelva la EDO : $y' = 2\,tg(3t) + e^{\sqrt{2}\ t} - ln(t+1) + \frac{5}{1+t^2}$

(15 puntos)

Solución:

$y' = 2\,tg(3t) + e^{\sqrt{2}\ t} - ln(t+1) + \frac{5}{1+t^2} \Rightarrow$

$\int y'\,dt = 2\int tg(3t)\,dt + \int e^{\sqrt{2}\ t}\,dt - \int ln(t+1)\,dt + 5\int \frac{1}{1+t^2}dt \Rightarrow$

$y(t) = \,-\,2\,\frac{ln|cos(3t)|}{3} + \frac{e^{\sqrt{2}\ t}}{\sqrt{2}} - (t+1)\,ln|t+1| + (t+1) + 5\,Arctg(t) + c$ ❑

2) Calcule el valor medio de $tg^3(x)\,sec^3(x)$ en el intervalo $[0, \frac{\pi}{4}]$

(15 puntos)

Solución:

Recordemos que el valor medio de f en el intervalo $[a, b]$ es $\frac{1}{b-a}\int_a^b f(x)\,dx$

Calculemos $\int f(x)\,dx = \int tg^3(x)\,sec^3(x)\,dx$

$A = \int tg^3(x)\,sec^3(x)\,dx = \int tg^2(x)\,sec^2(x)\,tg(x)\,sec(x)\,dx =$

$\int (sec^2(x) - 1)\,sec^2(x)\,tg(x)\,sec(x)\,dx$

Consideremos el cambio de variable $V = sec(x)$

$V = sec(x) \Rightarrow dV = sec(x)\,tg(x)\,dx$

Luego

$A = \int (sec^2(x) - 1)\,sec^2(x)\,tg(x)\,sec(x)\,dx =$

$\int (V^2 - 1)\,V^2\,dV = \int (V^4 - V^2)\,dV = \int V^4\,dV - \int V^2\,dV = \frac{1}{5}\,V^5 - \frac{1}{3}\,V^3 =$

$\frac{1}{5}\,sec^5(x) - \frac{1}{3}\,sec^3(x) = \frac{1}{5}\,\frac{1}{cos^5(x)} - \frac{1}{3}\,\frac{1}{cos^3(x)}$

Ahora $\int_a^b f(x)\,dx = \int_0^{\frac{\pi}{4}} tg^3(x)\,sec^3(x)\,dx = \frac{1}{5}\ \frac{1}{cos^5(x)}\Big|_0^{\frac{\pi}{4}} - \frac{1}{3}\ \frac{1}{cos^3(x)}\Big|_0^{\frac{\pi}{4}} =$

$\frac{1}{5}\ \frac{1}{cos^5(\frac{\pi}{4})} - \frac{1}{5}\ \frac{1}{cos^5(0)} - \frac{1}{3}\ \frac{1}{cos^3(\frac{\pi}{4})} + \frac{1}{3}\ \frac{1}{cos^3(0)} =$

$\frac{1}{5}\ \frac{1}{(\frac{\sqrt{2}}{2})^5} - \frac{1}{5}\ \frac{1}{1} - \frac{1}{3}\ \frac{1}{(\frac{\sqrt{2}}{2})^3} + \frac{1}{3}\ \frac{1}{1} = \frac{1}{5}\ \frac{1}{\frac{4\sqrt{2}}{32}} - \frac{1}{5} - \frac{1}{3}\ \frac{1}{\frac{2\sqrt{2}}{8}} + \frac{1}{3} =$

$\frac{1}{5}\ \frac{1}{\frac{\sqrt{2}}{8}} - \frac{1}{3}\ \frac{1}{\frac{\sqrt{2}}{4}} + \frac{2}{15} = \frac{8}{5\sqrt{2}} - \frac{4}{3\sqrt{2}} + \frac{2}{15} = \frac{4}{15\sqrt{2}} + \frac{2}{15}$

Finalmente, el valor medio de $tg^3(x)\,sec^3(x)$ en el intervalo $[0, \frac{\pi}{4}]$ es

$\frac{1}{\frac{\pi}{4}-0} \int_0^{\frac{\pi}{4}} tg^3(x)\,sec^3(x)\,dx = \frac{4}{\pi}\left(\frac{4}{15\sqrt{2}} + \frac{2}{15}\right)$ ❑

3) Resuelva $\int x\ 5^x\,dx$

(15 puntos)

Solución:

$I = \int x\ 5^x\,dx$

Usemos integración por partes

$f = x \Rightarrow f' = 1$

$g' = 5^x \Rightarrow g = \frac{1}{ln(5)}5^x$

Luego, $I = \frac{x}{ln(5)}5^x - \frac{1}{ln(5)}\int 5^x\,dx = \frac{x}{ln(5)}5^x - \frac{1}{ln(5)}\frac{1}{ln(5)}5^x + c =$

$\frac{x}{ln(5)}5^x - \frac{1}{(ln(5))^2}5^x + c$ ❑

4) Obtenga $\int \frac{e^x}{e^x+1}\,dx$

(15 puntos)

Solución:

Para resolver la integral $J = \int \frac{e^x}{e^x+1}\,dx$ consideremos la sustitución simple $M = e^x + 1$

$M = e^x + 1 \Rightarrow dM = e^x\,dx$

$J = \int \frac{dM}{M} = ln|\,M\,| + c = ln|\,e^x + 1\,| + c$ ❑