UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE INGENIERÍA AGRÍCOLA
DEPTO. DE AGROINDUSTRIAS
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PAUTA TEST Nº 4 CÁLCULO INTEGRAL + EDO
INGENIERÍA AGROINDUSTRIAL — INGENIERÍA AMBIENTAL

NOMBRE :____CARRERA. :________
TIEMPO MÁXIMO : 30 MINUTOS FECHA : Ju 18/04/19

Responda V (Verdadero) o F (Falso), **justificando su respuesta**.

__F__ $\int_{1/2}^{3/2} \sqrt{-x^2+x+\frac{3}{4}}\,dx = ln(\frac{1}{3}\sqrt{7}+1)$

(60 puntos).

__**Solución:**__
Completemos cuadrados.

$$-x^2+x+\frac{3}{4} = -(x^2-x-\frac{3}{4}) = -(x-\frac{1}{2})^2+1 = 1-(x-\frac{1}{2})^2$$

Usemos sustitución trigonométrica.

$$(x-\frac{1}{2}) = sen(\alpha) \Rightarrow x = sen(\alpha)+\frac{1}{2}$$

$$dx = cos(\alpha)\,d\alpha$$

$$(x-\frac{1}{2}) = sen(\alpha) \Rightarrow \alpha = Arcsen(x-\frac{1}{2})$$

$$sen(2\alpha) = 2\,sen(\alpha)\,cos(\alpha) = 2\,(x-\frac{1}{2})\sqrt{1-(x-\frac{1}{2})^2}$$

$$\sqrt{-x^2+x+\frac{3}{4}} = \sqrt{1-(x-\frac{1}{2})^2} = \sqrt{1-sen^2(\alpha)} = \sqrt{cos^2(\alpha)} = cos(\alpha)$$

$$\int \sqrt{-x^2+x+\frac{3}{4}}\,dx = \int cos(\alpha)\,cos(\alpha)\,d\alpha = \int cos^2(\alpha)\,d\alpha$$

$$= \frac{1}{2}\int\ (1+cos(2\alpha))\ d\alpha = \frac{1}{2}\int\ d\alpha + \frac{1}{2}\int cos(2\alpha)\ d\alpha = \frac{1}{2}\ \alpha + \frac{1}{4}\ sen(2\alpha) =$$

$$\frac{1}{2}Arcsen(x-\frac{1}{2}) + \frac{1}{4}2\,(x-\frac{1}{2})\ \sqrt{1-(x-\frac{1}{2})^2}$$

$$= \frac{1}{2}Arcsen(x-\frac{1}{2}) + \frac{1}{2}\,(x-\frac{1}{2})\ \sqrt{1-(x-\frac{1}{2})^2}$$

Finalmente,

$$\int_{1/2}^{3/2} \sqrt{-x^2+x+\frac{3}{4}}\,dx = \left[\frac{1}{2}Arcsen(x-\frac{1}{2}) + \frac{1}{2}\,(x-\frac{1}{2})\ \sqrt{1-(x-\frac{1}{2})^2}\right]_{1(2}^{3/2}$$

$$= \frac{1}{2}Arcsen(1) - \frac{1}{2}Arcsen(0) + 0 - 0 = \frac{1}{2}\frac{\pi}{2} = \frac{\pi}{4} \neq ln(\frac{1}{3}\ \sqrt{7}\ +1)\ \square$$