UNIVERSIDAD DE CONCEPCIÓN
FACULTAD DE INGENIERÍA AGRÍCOLA
DEPTO. DE AGROINDUSTRIAS
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PAUTA PRUEBA Nº 1 CÁLCULO INTEGRAL + EDO

INGENIERÍA AGROINDUSTRIAL — INGENIERÍA AMBIENTAL

NOMBRE :____ CARRERA :________
TIEMPO MÁXIMO : 1 HORA 30 MINUTOS FECHA : Ju 02/01/18

1) Resuelva la integral $\int t^2 Arctg(t)\ dt$

(15 puntos).

Solución:

Usemos integración por partes.

$p' = t^2 \Rightarrow p = \frac{1}{3}\,t^3$

$q = Arctg(t) \Rightarrow q' = \frac{1}{1+t^2}$

$$\int t^2 Arctg(t)\ dt = \frac{1}{3}\,t^3\,Arctg(t) - \frac{1}{3}\int \frac{t^3}{1+t^2}\ dt$$

Para resolver la integral $\int \frac{t^3}{1+t^2}\ dt$ usaremos la sustitución simple $P = 1 + t^2$

$P = 1 + t^2 \Rightarrow dP = 2\,t\,dt \Rightarrow t\,dt = \frac{1}{2}\,dP$

$P = 1 + t^2 \Rightarrow t^2 = P - 1$

$$\int \frac{t^3}{1+t^2}\ dt = \int \frac{t^2}{1+t^2}\ t\,dt = \frac{1}{2}\int \frac{P-1}{P}\ dP = \frac{1}{2}\int\ dP - \frac{1}{2}\int \frac{1}{P}\ dP$$

$$= \frac{1}{2}P - \frac{1}{2}ln|\,P\,| = \frac{1}{2}(1+t^2) - \frac{1}{2}ln|\,1+t^2\,|$$

Volviendo a la integral original:

$$\int t^2 Arctg(t)\ dt = \frac{1}{3}\,t^3\,Arctg(t) - \frac{1}{3}\int \frac{t^3}{1+t^2}\ dt$$

$$= \frac{1}{3}\,t^3\,Arctg(t) - \frac{1}{6}(1+t^2) + \frac{1}{6}ln|\,1+t^2\,| + c \quad \square$$

2) Muestre que: $\int_1^3 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx = \frac{4\sqrt{2}}{3} - \frac{16\sqrt{3}}{27}$

(15 puntos).

Solución:

Resolvamos la integral indefinida $\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

Consideremos la sustición $P = 1 + \frac{1}{x}$

$P = 1 + \frac{1}{x} \Rightarrow dP = -\frac{1}{x^2}\,dx \Rightarrow \frac{1}{x^2}\,dx = -\,dP$

$\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx = -\int \sqrt{P}\,dP = -\int P^{1/2}\,dP = -\frac{2}{3}P^{3/2}$

$= -\frac{2}{3}\left(1+\frac{1}{x}\right)^{3/2}$

Luego

$$\int_1^3 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx = -\frac{2}{3}\left(1+\frac{1}{x}\right)^{3/2}\Big|_1^3 = -\frac{2}{3}\left(1+\frac{1}{3}\right)^{3/2} + \frac{2}{3}\left(1+\frac{1}{1}\right)^{3/2}$$

$$= -\frac{2}{3}\left(\frac{4}{3}\right)^{3/2} + \frac{2}{3}\left(2\right)^{3/2} = \frac{2}{3}\sqrt{2^3} - \frac{2}{3}\left(\frac{2^2}{3}\right)^{3/2} = \frac{2}{3}\sqrt{2^2\,2} - \frac{2}{3}\,\frac{(2^2)^{3/2}}{\sqrt{3^3}}$$

$$= \frac{2}{3}\,2\sqrt{2} - \frac{2}{3}\,\frac{2^3}{\sqrt{3^2\,3}} = \frac{4}{3}\sqrt{2} - \frac{16}{9}\,\frac{1}{\sqrt{3}} = \frac{4}{3}\sqrt{2} - \frac{16}{9}\,\frac{1}{\sqrt{3}}\,\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{4}{3}\sqrt{2} - \frac{16}{9}\,\frac{\sqrt{3}}{3} = \frac{4}{3}\sqrt{2} - \frac{16}{27}\sqrt{3} = \frac{4\sqrt{2}}{3} - \frac{16\sqrt{3}}{27} \quad \square$$

3) Calcule $\int_0^1 \frac{x^2}{(3+x^2)^{5/2}}\,dx$

(15 puntos).

Solución:

Usaremos la sustitución trigonométrica $x = \sqrt{3}\,tg(\theta)$. Luego,
$dx = \sqrt{3}\,sec^2(\theta)\,d\theta$ y $x = 0 \Rightarrow 0 = \sqrt{3}\,tg(\theta) \Rightarrow tg(\theta) = 0 \Rightarrow \theta = 0$,
$x = 1 \Rightarrow 1 = \sqrt{3}\,tg(\theta) \Rightarrow tg(\theta) = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$. Así,

$$\int_0^{\frac{\pi}{6}} \frac{\left[\sqrt{3}\,tg(\theta)\right]^2 \sqrt{3}\,sec^2(\theta)\,d\theta}{\left[3+\left(\sqrt{3}\,tg(\theta)\right)^2\right]^{5/2}} = \frac{3\sqrt{3}}{(3)^{5/2}}\int_0^{\frac{\pi}{6}} \frac{tg^2(\theta)\;sec^2(\theta)\,d\theta}{\left[1+tg^2(\theta)\right]^{5/2}}$$

$$= \frac{(3)^{3/2}}{(3)^{5/2}}\int_0^{\frac{\pi}{6}} \frac{tg^2(\theta)\;sec^2(\theta)\,d\theta}{\left[sec^2(\theta)\right]^{5/2}} = \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{tg^2(\theta)\;sec^2(\theta)\,d\theta}{sec^5(\theta)}$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{tg^2(\theta)\;d\theta}{sec^3(\theta)} = \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{\left[\frac{sen^2(\theta)}{cos^2(\theta)}\right]\,d\theta}{\left[\frac{1}{cos^3(\theta)}\right]}$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{6}} \left[\frac{sen^2(\theta)}{cos^2(\theta)}\right] cos^3(\theta)d\theta = \frac{1}{3}\int_0^{\frac{\pi}{6}} sen^2(\theta)\;\;cos(\theta)d\theta$$

Ahora, para resolver esta última integral usaremos la sustitución simple $u = sen(\theta)$, de donde $du = cos(\theta)d\theta$. Luego,

$\int sen^2(\theta)cos(\theta)d\theta = \int u^2 du = \frac{1}{3}u^3 = \frac{1}{3}sen^3(\theta)$

Luego

$$\int_0^1 \frac{x^2\,dx}{(3+x^2)^{5/2}} = \frac{1}{3}\int_0^{\frac{\pi}{6}} sen^2(\theta)\ cos(\theta)d\theta = \frac{1}{3}\left[\frac{1}{3}sen^3(\theta)\right]\Bigg|_0^{\frac{\pi}{6}}$$

$$= \frac{1}{9}\left[sen^3(\tfrac{\pi}{6}) - sen^3(0)\right] = \frac{1}{9}\left(\frac{1}{2}\right)^3 = \frac{1}{72} \quad \square$$

4) Obtenga el valor exacto de $\int_1^2 \frac{x^2+1}{x^2+3x}\,dx$

(15 puntos).

Solución:

Dividiendo, tenemos

$$x^2+1 : x^2+3x = 1$$

$$(-)\ x^2+3x$$

$$\overline{\qquad\qquad}$$

$$-3x+1$$

Luego,

$$\frac{x^2+1}{x^2+3x} = 1 + \frac{-3x+1}{x^2+3x}$$

Así,

$$\int_1^2 \frac{x^2+1}{x^2+3x}dx = \int_1^2 \left[1 + \frac{-3x+1}{x^2+3x}\right]dx = \int_1^2 dx + \int_1^2 \frac{-3x+1}{x^2+3x}dx = x\Big|_1^2 + \int_1^2 \frac{-3x+1}{x^2+3x}dx$$

$$\frac{-3x+1}{x^2+3x} = \frac{-3x+1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} \;\Rightarrow\; -3x+1 = A(x+3)+Bx$$

$$\Rightarrow\; -3x+1 \equiv (A+B)x + 3A$$

Igualando los coeficientes correspondientes en la identidad anterior, se tiene $A+B = -3$, $3A = 1$, $A = \frac{1}{3}$ y $B = -\frac{10}{3}$

Por lo tanto, $\frac{-3x+1}{x(x+3)} = \frac{(\frac{1}{3})}{x} + \frac{(-\frac{10}{3})}{x+3}$

Así,

$$\int_1^2 \frac{x^2+1}{x^2+3x}dx = x\Big|_1^2 + \int_1^2 \left[\frac{(\frac{1}{3})}{x} + \frac{(-\frac{10}{3})}{x+3}\right]dx = x\Big|_1^2 + \frac{1}{3}\int_1^2 \left[\frac{1}{x} - \frac{10}{x+3}\right]dx$$

$$= (2-1) + \frac{1}{3}\left[ln|x| - 10\,ln|x+3|\right]\Big|_1^2$$

$$= 1 + \frac{1}{3}\left[ln(2) - 10\,ln(2+3)\right] - \frac{1}{3}\left[ln(1) - 10\,ln(1+3)\right]$$

$$= 1 + \frac{1}{3}ln(2) - \frac{10}{3}ln(5) - \frac{1}{3}ln(1) + \frac{10}{3}ln(4) = 1 + \frac{1}{3}ln(2) - \frac{10}{3}ln(5) + \frac{10}{3}ln(4)$$

$$= 1 + \frac{1}{3}ln(2) + \frac{10}{3}ln(\tfrac{4}{5}) \quad \square$$