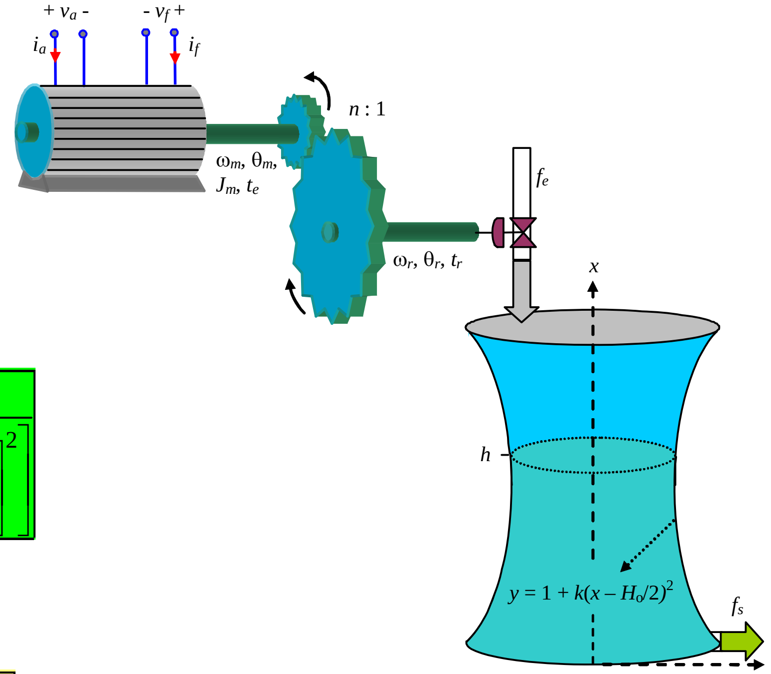
Solución Tarea N°3



Problema Estudiar la respuesta en frecuencia del llenado/vaciado de un estanque con válvula electrónica.

Ecuaciones de estado eliminando la dinámica de la corriente.

$$\frac{d\omega_m}{dt} = \frac{k_\phi \cdot I_f}{J_m \cdot R_a} \cdot v_a - \frac{k_\phi^2 \cdot I_f^2}{J_m \cdot R_a} \cdot \omega_m - \frac{1}{n \cdot J_m} \cdot k_t \cdot \theta_r$$

$$\frac{d\theta_r}{dt} = \frac{1}{n} \cdot \omega_m$$

$$\frac{dh}{dt} = k_e \cdot \frac{\theta_r}{\left[\pi \cdot \left[1 + k \cdot \left(h - \frac{H_o}{2}\right)^2\right]^2\right]} - k_v \cdot \frac{\sqrt{g \cdot h}}{\left[\pi \cdot \left[1 + k \cdot \left(h - \frac{H_o}{2}\right)^2\right]^2\right]}$$

Parámetros y Matrices

$R_a := 250$ $L_a := 1$ $k_\phi := 200$ $I_f := 10 \cdot 10^{-3}$ $n := 50$ $k_t := 25$ $k_v := 1$

$g := 10$ $k_e := 35$ $J_m := 5 \cdot 10^{-3}$ $H_o := 4$ $k := 0.25$

$H_1 := 3$ $\Omega_{m1} := 0$ $\Omega_{m1} = 0$

$$V_{a1} := (g \cdot H_1)^{\frac{1}{2}} \cdot R_a \cdot k_t \cdot \frac{k_v}{I_f \cdot k_\phi \cdot n \cdot k_e} \qquad V_{a1} = 9.781$$

$$\Theta_{r1} := n \cdot \frac{k_\phi}{k_t} \cdot I_f \cdot \frac{V_{a1}}{R_a} \qquad \Theta_{r1} \cdot \frac{180}{\pi} = 8.966$$

Parte A **Modelo**

$$A(H) := \begin{bmatrix} -\frac{k_\phi^2 \cdot I_f^2}{J_m \cdot R_a} & -\frac{1}{n \cdot J_m} \cdot k_t & 0 \\ \frac{1}{n} & 0 & 0 \\ 0 & \frac{k_e}{\text{Area}(H)} & \frac{-1}{2} \cdot \frac{k_v}{(g \cdot H)^{\frac{1}{2}}} \cdot \frac{g}{\text{Area}(H)} \end{bmatrix}$$

$$b := \begin{pmatrix} \frac{k_\phi \cdot I_f}{J_m \cdot R_a} \\ 0 \\ 0 \end{pmatrix} \qquad c := (0 \;\; 0 \;\; 1)$$

$$\text{Area}(H) := \pi \cdot \left[1 + k \cdot \left(H - \frac{H_o}{2}\right)^2\right]^2$$

Parte B Simulación Sistema.

$$T_o := 32 \qquad t_0 := T_o\cdot\frac{45}{360} \qquad t_1 := \frac{T_o}{2} - T_o\cdot\frac{45}{360} \qquad t_2 := \frac{T_o}{2} + T_o\cdot\frac{45}{360} \qquad t_3 := T_o - T_o\cdot\frac{45}{360} \qquad Am := 9 \qquad rpm := \frac{60}{2\cdot\pi}$$

$$\Delta u_c(t) := Am\cdot\left[\Phi(t-t_0) - \Phi(t-t_1) - \Phi(t-t_2) + \frac{4\cdot(t-t_2)}{T_o}\cdot\Phi(t-t_2) - \frac{4\cdot(t-t_3)}{T_o}\cdot\Phi(t-t_3)\right] \qquad \Delta v_a(t) := \Delta u_c(t) \qquad rad := \frac{180}{\pi}$$

$$F_o := \frac{1}{T_o} \qquad l_f := 2000 \qquad l := 0..l_f \qquad t_i := 0 \qquad t_f := 2\cdot T_o \qquad t := t_i, t_i + \frac{t_f - t_i}{l_f}..t_f$$

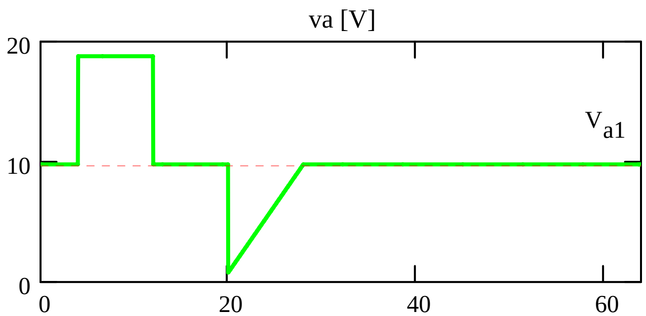
$$D(t,x) := A(H_1)\cdot(x_1\ \ x_2\ \ x_3)^T + b\cdot\Delta u_c(t) \qquad CI := (0\ \ 0\ \ 0)^T \qquad Z_p := \text{rkfixed}(CI, 0, t_f, l_f, D) \qquad \frac{4}{T_o} = 0.125$$

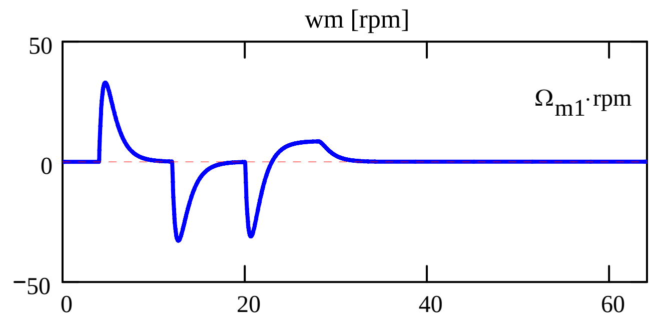
$t_0 = 4$

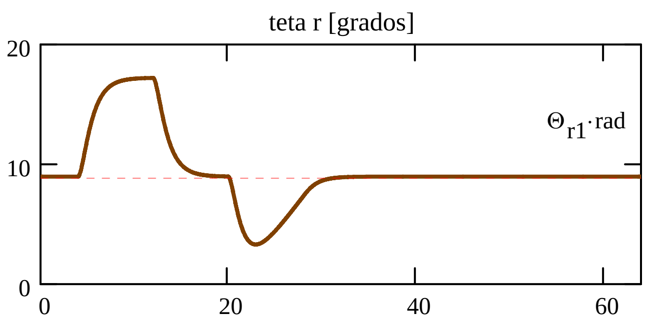
$t_1 = 12$

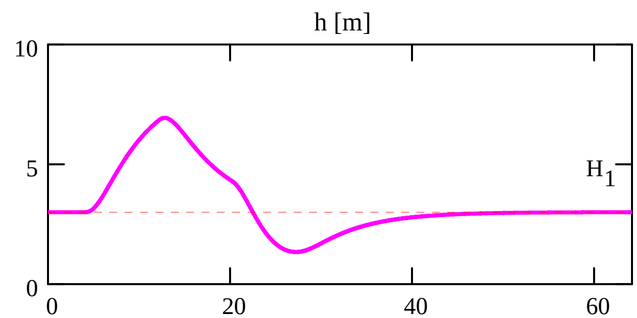
$t_2 = 20$

$t_3 = 28$









Parte C Ecuaciones de Estado Discretas Equivalentes.

$$\Phi_c(t) := \text{eigenvecs}(A(H_1))\cdot\begin{pmatrix} \exp(\text{eigenvals}(A(H_1))_1\cdot t) & 0 & 0 \\ 0 & \exp(\text{eigenvals}(A(H_1))_2\cdot t) & 0 \\ 0 & 0 & \exp(\text{eigenvals}(A(H_1))_3\cdot t) \end{pmatrix}\cdot\text{eigenvecs}(A(H_1))^{-1} \qquad T_m := 2$$

$A_d := \Phi_c(T_m)$

$$A_d = \begin{pmatrix} -0.089 & -11.556 & 0 \\ 2.311\times 10^{-3} & 0.28 & 0 \\ 0.043 & 7.249 & 0.689 \end{pmatrix}$$

$$b_d := \left[\int_0^{T_m} \left(\Phi_c(T_m-\tau)\cdot b\right)_1 d\tau \quad \int_0^{T_m} \left(\Phi_c(T_m-\tau)\cdot b\right)_2 d\tau \quad \int_0^{T_m} \left(\Phi_c(T_m-\tau)\cdot b\right)_3 d\tau \right]^T$$

$$b_d = \begin{pmatrix} 0.185 \\ 0.012 \\ 0.075 \end{pmatrix}$$

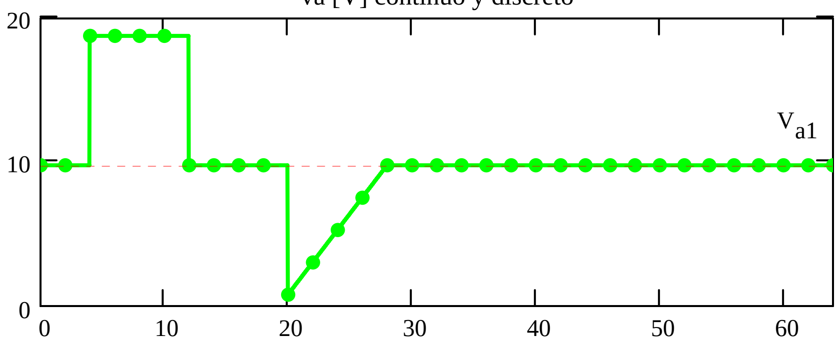
$c_d := c \qquad c_d = (0 \;\; 0 \;\; 1)$

Simulación de Ecuaciones de Estado Discretas Equivalentes y Comparación.

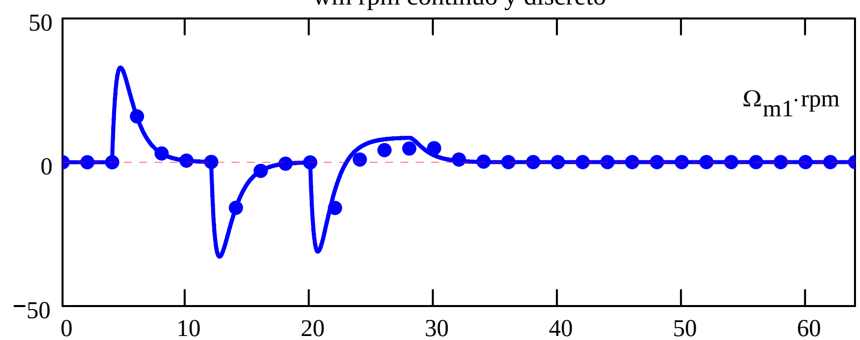
$$\Delta u_d(k) := Am\cdot\left[\Phi(k\cdot T_m - t_0) - \Phi(k\cdot T_m - t_1) - \Phi(k\cdot T_m - t_2) + \frac{4\cdot(k\cdot T_m - t_2)}{T_o}\cdot\Phi(k\cdot T_m - t_2) - \frac{4\cdot(k\cdot T_m - t_3)}{T_o}\cdot\Phi(k\cdot T_m - t_3)\right] \qquad k_f := \frac{t_f}{T_m} \qquad k := 0 .. k_f$$

$$x_o := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad x_d(k) := \mathrm{if}\left(k = 0, x_o, A_d^{\,k}\cdot x_o + \sum_{j=0}^{k-1} A_d^{\,k-j-1}\cdot b_d\cdot\Delta u_d(j)\right)$$

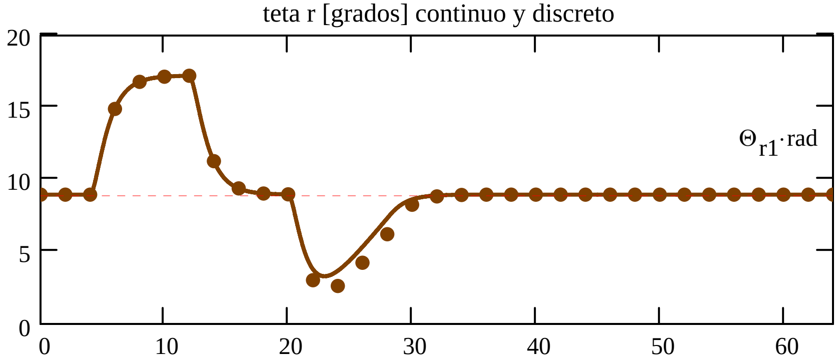
va [V] continuo y discreto



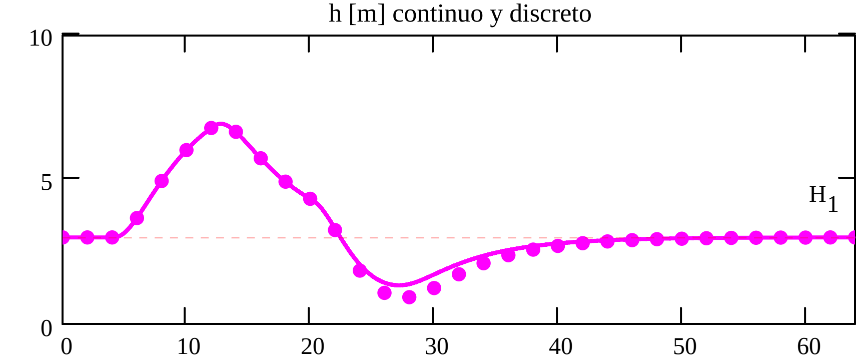
wm rpm continuo y discreto



teta r [grados] continuo y discreto



h [m] continuo y discreto

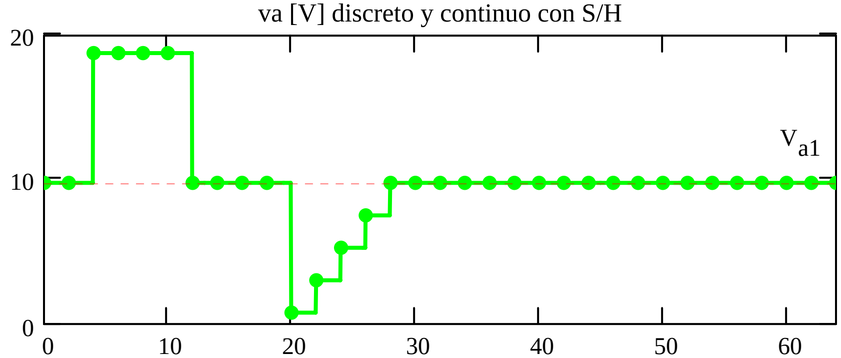


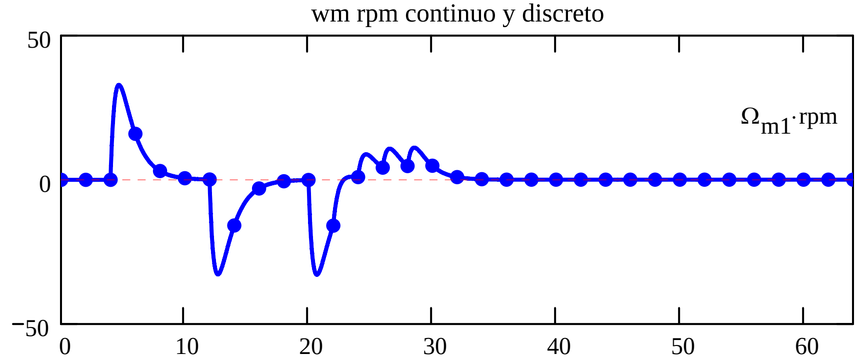
Parte D — Entrada continua con Sample/Hold.

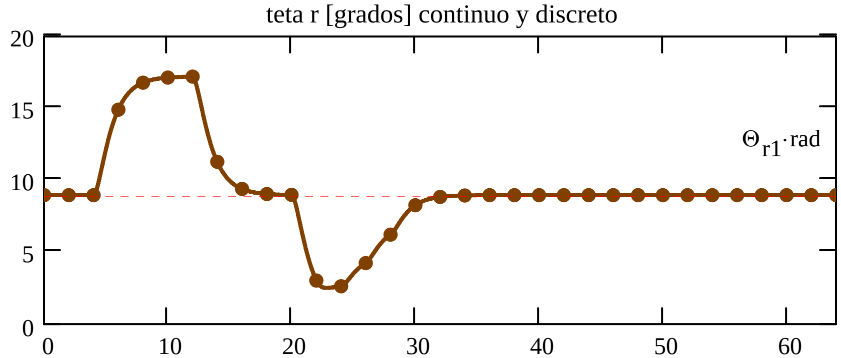
$$del_{Tm}(t) := \frac{1}{T_m}\cdot\left(\Phi(t) - \Phi\left(t - T_m\right)\right) \quad T_m = 2 \qquad N := \frac{t_f}{T_m} \qquad \Delta u_r(t) := \sum_{i=0}^{N} \Delta u_c\left(i\cdot T_m\right) del_{Tm}\left(t - i\cdot T_m\right)\cdot T_m$$

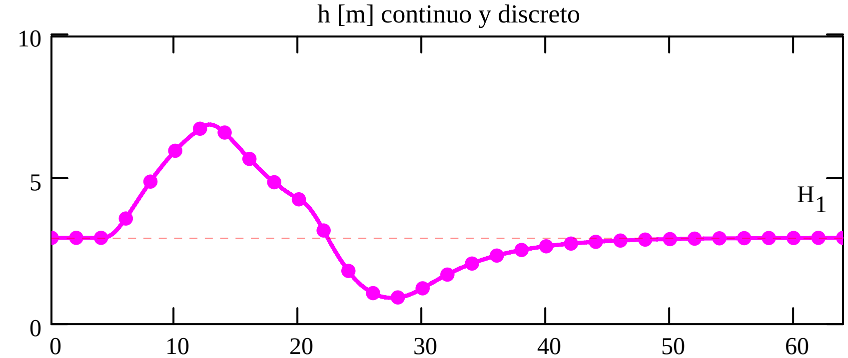
Simulación para encontrar la C.I.

$$D(t,x) := A\left(H_1\right)\cdot\left(x_1 \quad x_2 \quad x_3\right)^T + b\cdot\Delta u_r(t) \qquad CI := \left(0 \quad 0 \quad 0\right)^T \quad Z_p := \text{rkfixed}\left(CI, 0, t_f, l_f, D\right)$$









Parte E — Función de Transferencia

$$\frac{di_a}{dt} = \frac{-R_a}{L_a}\cdot i_a - \frac{k_\phi\cdot I_f}{L_a}\cdot\omega_m + \frac{1}{L_a}\cdot v_a \qquad \frac{d\omega_m}{dt} = \frac{n^2\cdot k_\phi\cdot I_f}{M\cdot r^2}\cdot i_a - \frac{n}{r}\cdot g \qquad \frac{dh}{dt} = \frac{r}{n}\cdot\omega_m$$

$$\Delta h_{hva}(s) = \frac{2\cdot\left(g\cdot H_1\right)^{\frac{1}{2}}\cdot k_\phi\cdot I_f}{n\cdot J_m\cdot R_a}\cdot\frac{1}{k_v\cdot g}\cdot\frac{k_e}{\left[2\cdot s\cdot\left(g\cdot H_1\right)^{\frac{1}{2}}\cdot\frac{Area\left(H_1\right)}{k_v\cdot g}+1\right]\cdot\left(s^2+s\cdot k_\phi{}^2\cdot\frac{I_f{}^2}{J_m\cdot R_a}+k_t\cdot\frac{1}{n^2\cdot J_m}\right)}$$

$$\Delta h_{hva}(s) = k_p\cdot\frac{\omega_n{}^2}{(s\cdot\tau+1)\cdot\left(s^2+s\cdot 2\cdot\xi\cdot\omega_n+\omega_n{}^2\right)}$$

$$k_p := \frac{2\cdot\left(g\cdot H_1\right)^{\frac{1}{2}}\cdot k_\phi\cdot I_f}{R_a}\cdot\frac{k_e\cdot n}{k_v\cdot k_t\cdot g} \qquad \omega_n := \sqrt{k_t\cdot\frac{1}{n^2\cdot J_m}} \qquad \tau := 2\cdot\left(g\cdot H_1\right)^{\frac{1}{2}}\cdot\frac{Area\left(H_1\right)}{k_v\cdot g} \qquad \xi := \frac{k_\phi{}^2}{2\cdot\sqrt{k_t\cdot\frac{1}{n^2\cdot J_m}}}\cdot\frac{I_f{}^2}{J_m\cdot R_a}$$

$$\Delta h_c(s) = k_p\cdot\frac{\omega_n{}^2}{(s\cdot\tau+1)\cdot\left(s^2+s\cdot 2\cdot\xi\cdot\omega_n+\omega_n{}^2\right)}$$

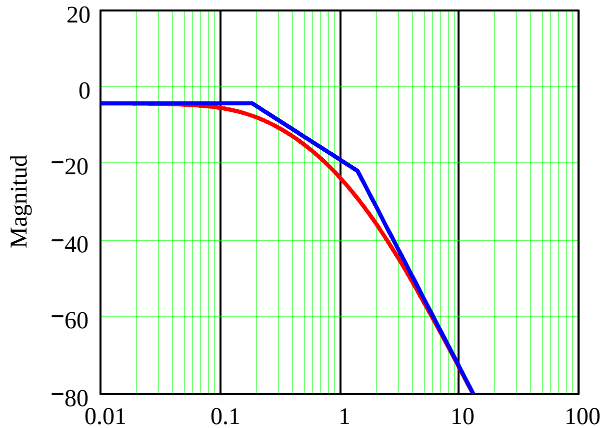
Parte F — Diagramas de Bode Asintótico y Exacto del Sistema Continuo.

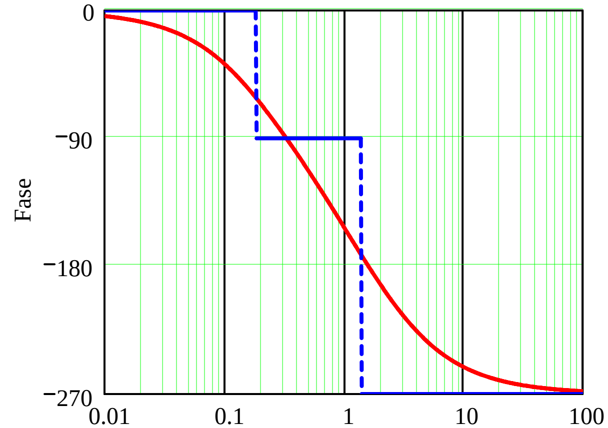
$$m_{max} := 500 \quad m := 1..m_{max} \quad \omega_{min} := 10^{-2} \quad \omega_{max} := 10^2 \quad ratio := \log\left(\frac{\omega_{max}}{\omega_{min}}\right)\cdot\frac{1}{m_{max}} \quad \omega(m) := \omega_{min}\cdot 10^{m\cdot ratio}$$

$$\Delta h_c(s) := k_p\cdot\frac{\omega_n{}^2}{(s\cdot\tau+1)\cdot\left(s^2+s\cdot 2\cdot\xi\cdot\omega_n+\omega_n{}^2\right)} \qquad g_a(s) := k_p\cdot \mathrm{if}\left(|s|<\frac{1}{\tau},\frac{1}{1},\mathrm{if}\left(|s|<\omega_n,\frac{1}{\tau\cdot s},\frac{\omega_n{}^2}{s^3\cdot\tau}\right)\right)$$

$$M_a(m) := 20\cdot\log\left(\left|g_a(j\cdot\omega(m))\right|\right) \qquad F_a(m) := \frac{180}{\pi}\cdot\left(\mathrm{if}\left(\arg\left(g_a(j\cdot\omega(m))\right)>0,\arg\left(g_a(j\cdot\omega(m))\right)-2\pi,\arg\left(g_a(j\cdot\omega(m))\right)\right)\right)$$

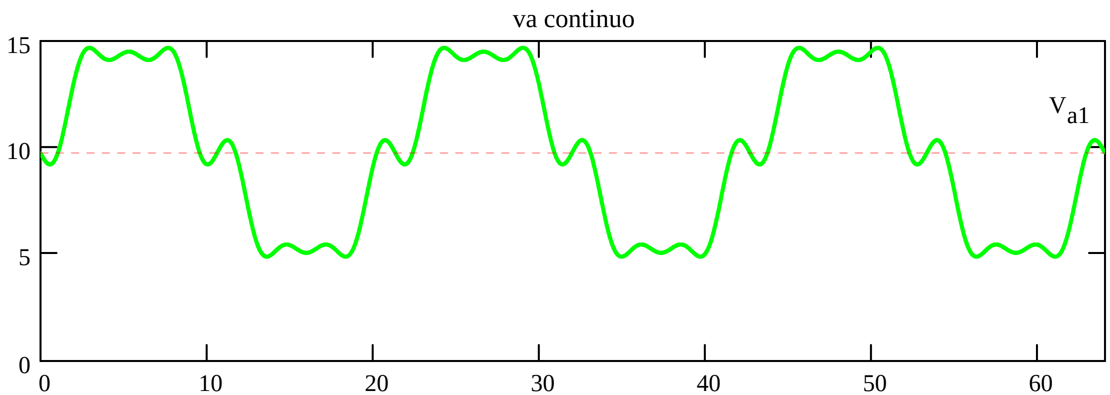
$$M_c(m) := 20\cdot\log\left(\left|\Delta h_c(j\cdot\omega(m))\right|\right) \qquad F_c(m) := \frac{180}{\pi}\cdot\left(\mathrm{if}\left(\arg\left(\Delta h_c(j\cdot\omega(m))\right)>0,\arg\left(\Delta h_c(j\cdot\omega(m))\right)-2\pi,\arg\left(\Delta h_c(j\cdot\omega(m))\right)\right)\right)$$





Parte G **Entrada Sinusoidal Continua.** $f_o := \frac{3}{t_f}$ $am := 5$ $f_o^{-1} = 21.333$ $t_f = 64$

$$\Delta u_c(t) := am \cdot \left(\frac{1}{1} \cdot \sin(1 \cdot 2 \cdot \pi \cdot f_o \cdot t) + \frac{-1}{5} \cdot \sin(5 \cdot 2 \cdot \pi \cdot f_o \cdot t) + \frac{-1}{7} \cdot \sin(7 \cdot 2 \cdot \pi \cdot f_o \cdot t) \right)$$

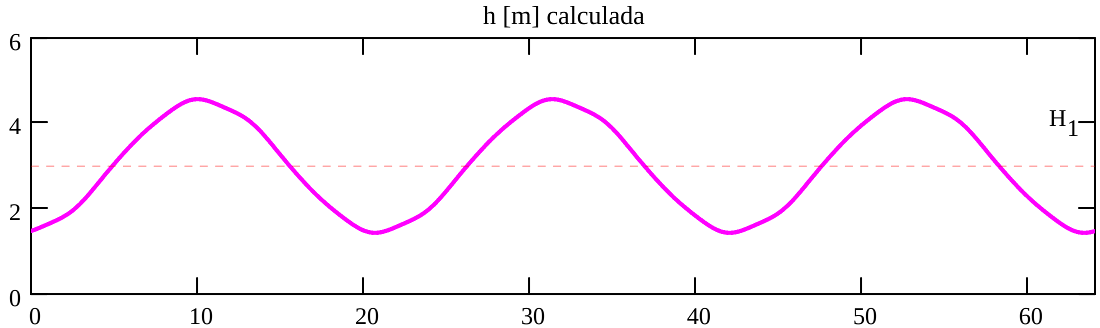


$\Delta x_1 := |\Delta h_c(j \cdot 2 \cdot 1 \cdot \pi \cdot f_o)|$ $\Delta x_1 = 0.307$ $\Delta x_{f1} := F_c\left(\log\left(\frac{2 \cdot 1 \cdot \pi \cdot f_o}{\omega_{min}}\right) \cdot \frac{1}{ratio}\right)$ $\Delta x_{f1} = -83.956$

$\Delta x_5 := |\Delta h_c(j \cdot 2 \cdot 5 \cdot \pi \cdot f_o)|$ $\Delta x_5 = 0.033$ $\Delta x_{f5} := F_c\left(\log\left(\frac{2 \cdot 5 \cdot \pi \cdot f_o}{\omega_{min}}\right) \cdot \frac{1}{ratio}\right)$ $\Delta x_{f5} = -174.852$

$\Delta x_7 := |\Delta h_c(j \cdot 2 \cdot 7 \cdot \pi \cdot f_o)|$ $\Delta x_7 = 0.016$ $\Delta x_{f7} := F_c\left(\log\left(\frac{2 \cdot 7 \cdot \pi \cdot f_o}{\omega_{min}}\right) \cdot \frac{1}{ratio}\right)$ $\Delta x_{f7} = -193.681$

$$\Delta h_{cc}(t) := am\cdot\left(\Delta x_1\cdot\sin\left(2\cdot\pi\cdot f_o\cdot t + \Delta x_{f1}\cdot\frac{\pi}{180}\right) + \frac{-\Delta x_5}{5}\cdot\sin\left(5\cdot 2\cdot\pi\cdot f_o\cdot t + \Delta x_{f5}\cdot\frac{\pi}{180}\right) \ldots + \frac{-\Delta x_7}{7}\cdot\sin\left(7\cdot 2\cdot\pi\cdot f_o\cdot t + \Delta x_{f7}\cdot\frac{\pi}{180}\right)\right)$$



Parte H **Entrada Periódica - Simulación.**

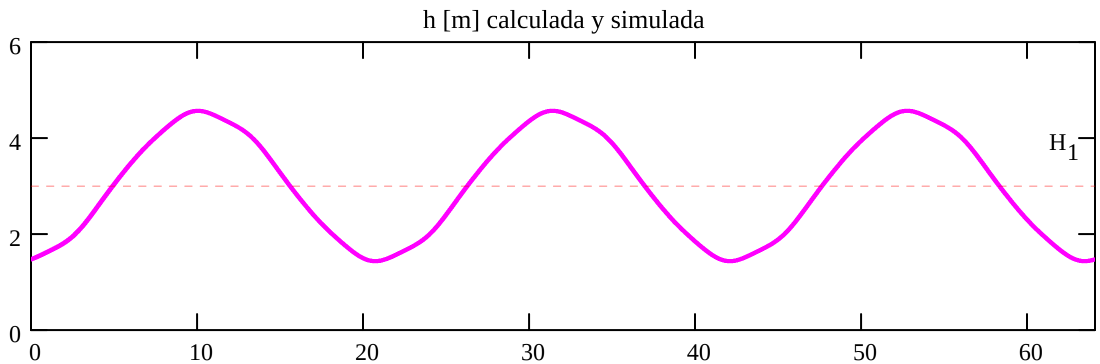
$$D(t,x) := A(H_1)\cdot(x_1\ \ x_2\ \ x_3)^T + b\cdot\Delta u_c(t)$$

$$CI := (0\ \ 0\ \ 0)^T \qquad Z_p := \text{rkfixed}(CI, 0, t_f, l_f, D)$$

$$D(t,x) := A(H_1)\cdot(x_1\ \ x_2\ \ x_3)^T + b\cdot\Delta u_c(t) \qquad CI := \begin{pmatrix} Zp_{l_f,2} \\ Zp_{l_f,3} \\ Zp_{l_f,4} \end{pmatrix}$$

La CI se calcula para estar en S.S. en t = 0.

$$Z_p := \text{rkfixed}(CI, 0, t_f, l_f, D)$$

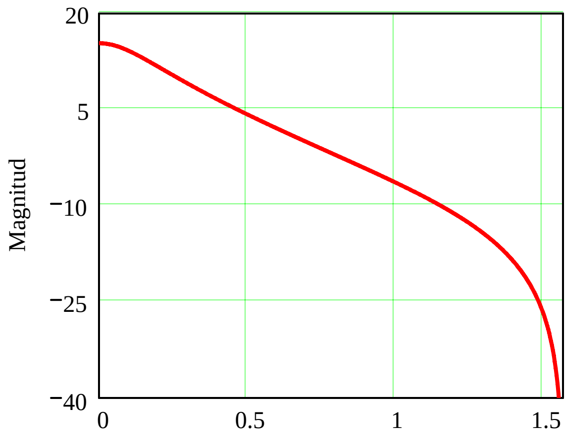


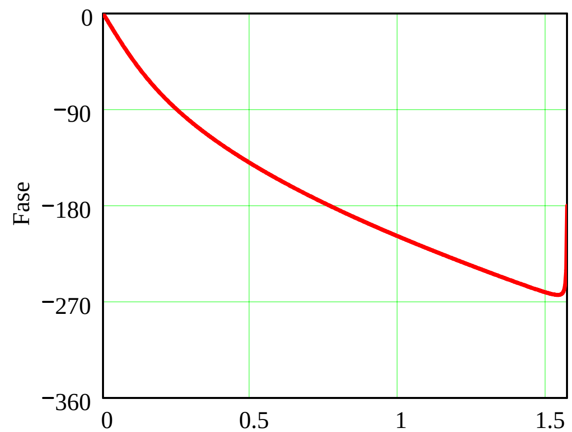
Parte I Diagramas de Bode Sistema Discreto.

$n_{max} := 500 \quad n := 0 .. n_{max} \quad \Omega_{min} := 0 \quad \Omega_{max} := \frac{1 \cdot \pi}{T_m} \quad \Omega(n) := \frac{\Omega_{max} - \Omega_{min}}{n_{max}} \cdot n + \Omega_{min}$

$\Delta h_d(z) := c_d \cdot \left(z \cdot identity(3) - A_d\right)^{-1} \cdot b_d \qquad M_d(n) := 20 \cdot \log\left(\left| rpm \cdot \Delta h_d\left(\exp\left(j \cdot \Omega(n) \cdot T_m\right)\right)\right|\right)$

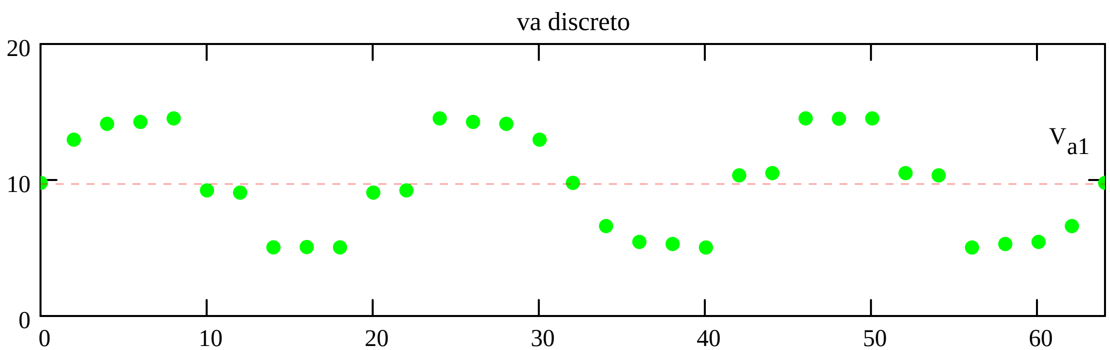
$F_d(n) := \frac{180}{\pi} \cdot \left(if\left(\arg\left(\Delta h_d\left(\exp\left(j \cdot \Omega(n) \cdot T_m\right)\right)\right) > 0, \arg\left(\Delta h_d\left(\exp\left(j \cdot \Omega(n) \cdot T_m\right)\right)\right) - 2\pi, \arg\left(\Delta h_d\left(\exp\left(j \cdot \Omega(n) \cdot T_m\right)\right)\right)\right)\right)$





Parte J Entrada Sinusoidal Discreta.

$\Delta u_d(k) := am \cdot \left[\sin\left(2 \cdot \pi \cdot f_o \cdot k \cdot T_m\right) + \frac{-1}{5} \cdot \sin\left[10 \cdot \pi \cdot f_o \cdot \left(k \cdot T_m\right)\right] + \frac{-1}{7} \cdot \sin\left[14 \cdot \pi \cdot f_o \cdot \left(k \cdot T_m\right)\right]\right]$

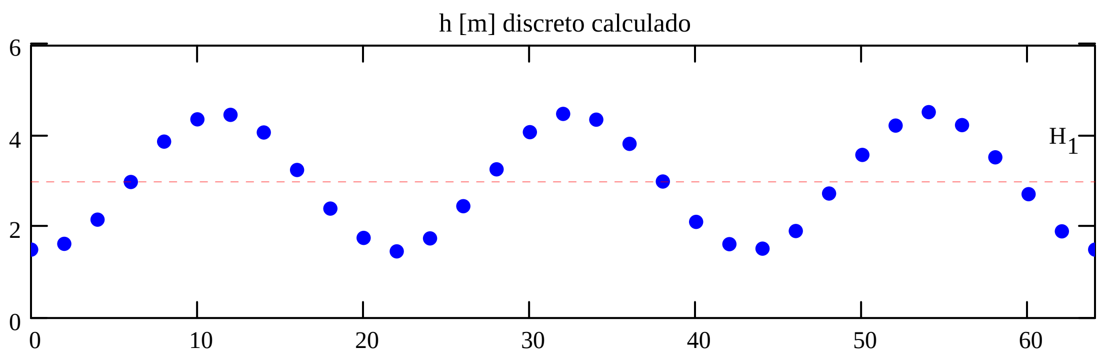


$\Delta x_1 := \left|\Delta h_d\left[\exp\left[j\cdot\left(2\cdot\pi\cdot f_o\right)\cdot T_m\right]\right]\right|$ $\quad \Delta x_1 = 0.302$ $\quad \Delta x_{f1} := F_d\left(\dfrac{2\cdot\pi\cdot f_o - \Omega_{min}}{\Omega_{max} - \Omega_{min}}\cdot n_{max}\right)$ $\quad \Delta x_{f1} = -100.75$

$\Delta x_5 := \left|\Delta h_d\left[\exp\left[j\cdot\left(2\cdot 5\cdot\pi\cdot f_o\right)\cdot T_m\right]\right]\right|$ $\quad \Delta x_5 = 7.025\times 10^{-3}$ $\quad \Delta x_{f5} := F_d\left(\dfrac{5\cdot 2\cdot\pi\cdot f_o - \Omega_{min}}{\Omega_{max} - \Omega_{min}}\cdot n_{max}\right)$ $\quad \Delta x_{f5} = -258.955$

$\Delta x_7 := \left|\Delta h_d\left[\exp\left[j\cdot\left(2\cdot 7\cdot\pi\cdot f_o\right)\cdot T_m\right]\right]\right|$ $\quad \Delta x_7 = 0.041$ $\quad \Delta x_{f7} := F_d\left(\dfrac{7\cdot 2\cdot\pi\cdot f_o - \Omega_{min}}{\Omega_{max} - \Omega_{min}}\cdot n_{max}\right)$ $\quad \Delta x_{f7} = -141.963$

$$\Delta h_{dc}(k) := am\cdot\left(\Delta x_1\cdot\sin\left(2\cdot\pi\cdot f_o\cdot k\cdot T_m + \Delta x_{f1}\cdot\frac{\pi}{180}\right) \ldots + \frac{\Delta x_5}{5}\cdot\sin\left(2\cdot 5\cdot\pi\cdot f_o\cdot k\cdot T_m + \Delta x_{f5}\cdot\frac{\pi}{180}\right) + \frac{\Delta x_7}{7}\cdot\sin\left(2\cdot 7\cdot\pi\cdot f_o\cdot k\cdot T_m + \Delta x_{f7}\cdot\frac{\pi}{180}\right)\right)$$



Parte K **Entrada Sinusoidal Discreta - Simulación.**

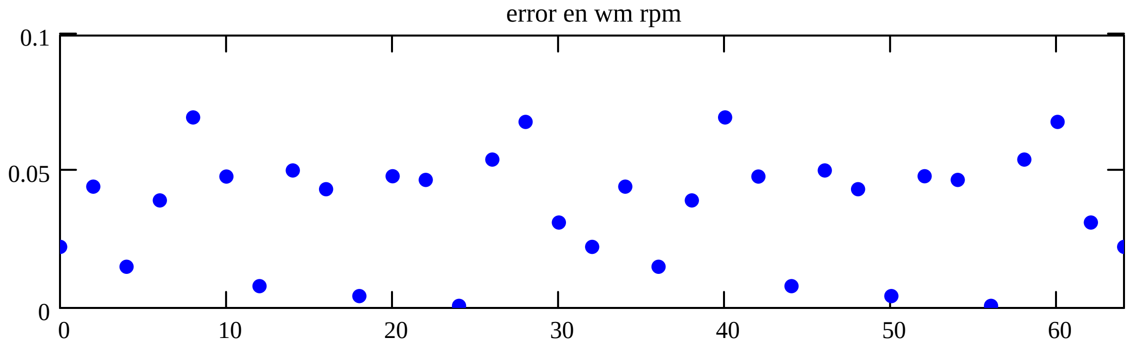
$$x_d(k) := \text{if}\left(k = 0, x_o, A_d{}^k\cdot x_o + \sum_{j=0}^{k-1} A_d{}^{k-j-1}\cdot b_d\cdot\Delta u_d(j)\right)$$

$x_o := x_d\left(k_f\right)$

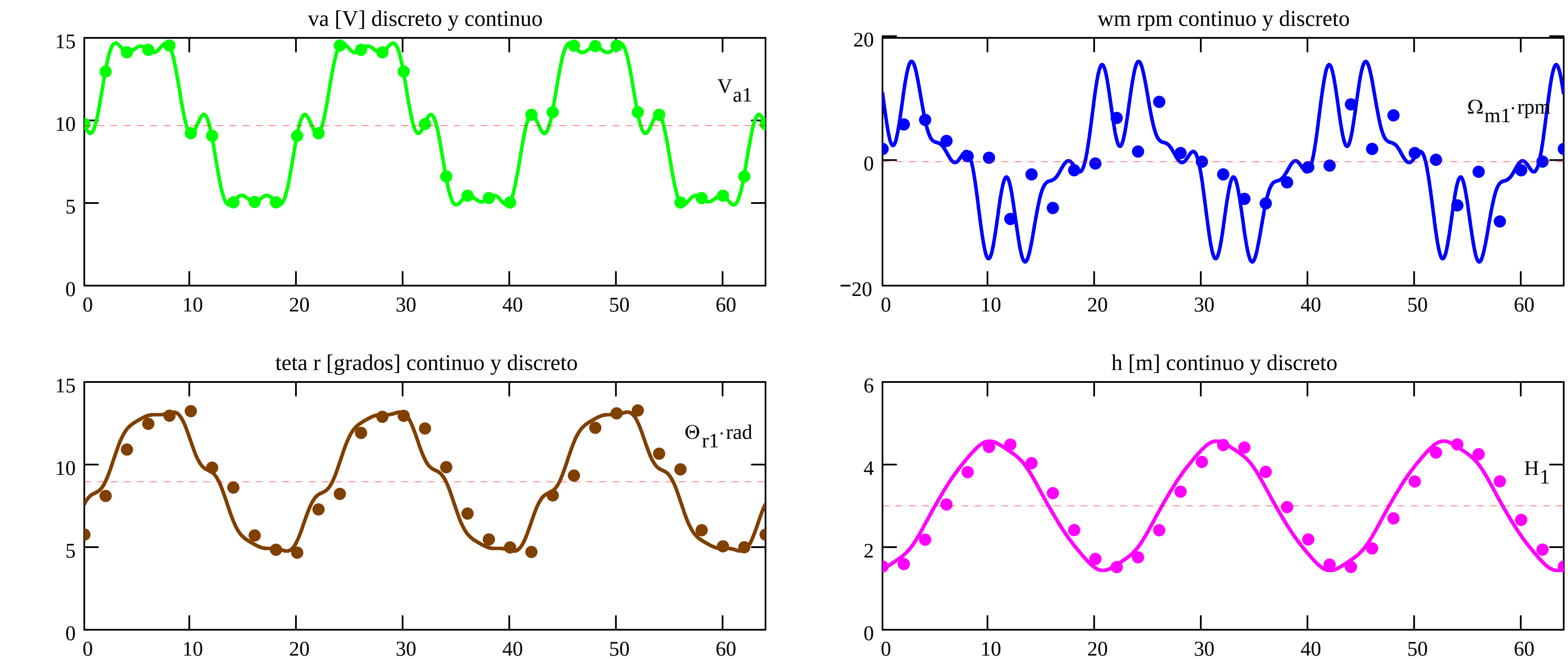
La CI se calcula para estar en S.S. en t = 0.

$$x_d(k) := \text{if}\left(k = 0, x_o, A_d{}^k\cdot x_o + \sum_{j=0}^{k-1} A_d{}^{k-j-1}\cdot b_d\cdot\Delta u_d(j)\right)$$

$$\text{error}(k) := \Delta h_{dc}(k) - x_d(k)_3$$



Parte L **Comparación Sistema Continuo y Discreto ante Entrada Periodica.**



Parte M **Entrada Sinusoidal Continua con Retentor de Orden Cero - Simulación.**

$$\Delta u_r(t) := \sum_{i=0}^{N} \Delta u_c(i \cdot T_m)\, del_{Tm}(t - i \cdot T_m) \cdot T_m$$

$$D(t,x) := A(H_1) \cdot (x_1 \;\; x_2 \;\; x_3)^T + b \cdot \Delta u_r(t) \qquad CI := (0 \;\; 0 \;\; 0)^T \qquad Z_p := rkfixed(CI, 0, t_f, l_f, D)$$

$$D(t,x) := A(H_1) \cdot (x_1 \;\; x_2 \;\; x_3)^T + b \cdot \Delta u_r(t) \qquad CI := \begin{pmatrix} Zp_{l_f,2} \\ Zp_{l_f,3} \\ Zp_{l_f,4} \end{pmatrix}$$

La CI se calcula para estar en S.S. en t = 0.

$$Z_p := rkfixed(CI, 0, t_f, l_f, D)$$