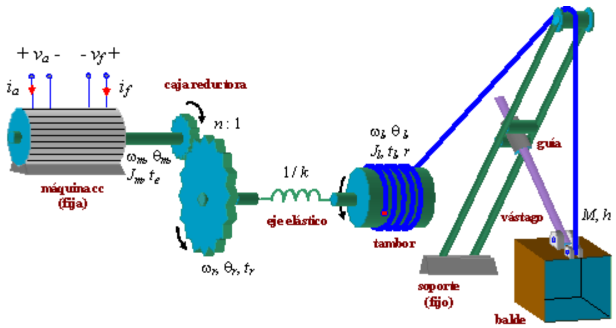
Solución Tarea N°2

Problema Estudiar señales en el movimiento vertical de un balde accionado eléctricamente.



Parámetros y Matrices

$R_a := 100 \cdot 10^{-3}$ $L_a := 10 \cdot 10^{-3}$ $k_\phi := 200 \cdot 10^{-3}$ $I_f := 80$ $n := 50$ $M := 50 \cdot 10^{3}$ $r := 1.5$ $G := 10$

Parte A **Modelo**

$$A := \begin{pmatrix} \frac{-R_a}{L_a} & \frac{-k_\phi \cdot I_f}{L_a} & 0 \\ \frac{k_\phi \cdot I_f \cdot n^2}{M \cdot r^2} & 0 & 0 \\ 0 & \frac{r}{n} & 0 \end{pmatrix} \quad e := \begin{pmatrix} 0 \\ \frac{-n}{r} \\ 0 \end{pmatrix} \quad b := \begin{pmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{pmatrix} \quad c := \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$$

Voltajes de armadura.

$$\Omega_{m1} := 400 \cdot \frac{2 \cdot \pi}{60} \quad I_{a1} := M \cdot G \cdot \frac{r}{n \cdot k_\phi \cdot I_f} \quad V_{a1} := \Omega_{m1} \cdot k_\phi \cdot I_f + R_a \cdot \frac{M \cdot G \cdot r}{n \cdot k_\phi \cdot I_f} \quad I_{a1} = 937.5 \quad V_{a1} = 763.956$$

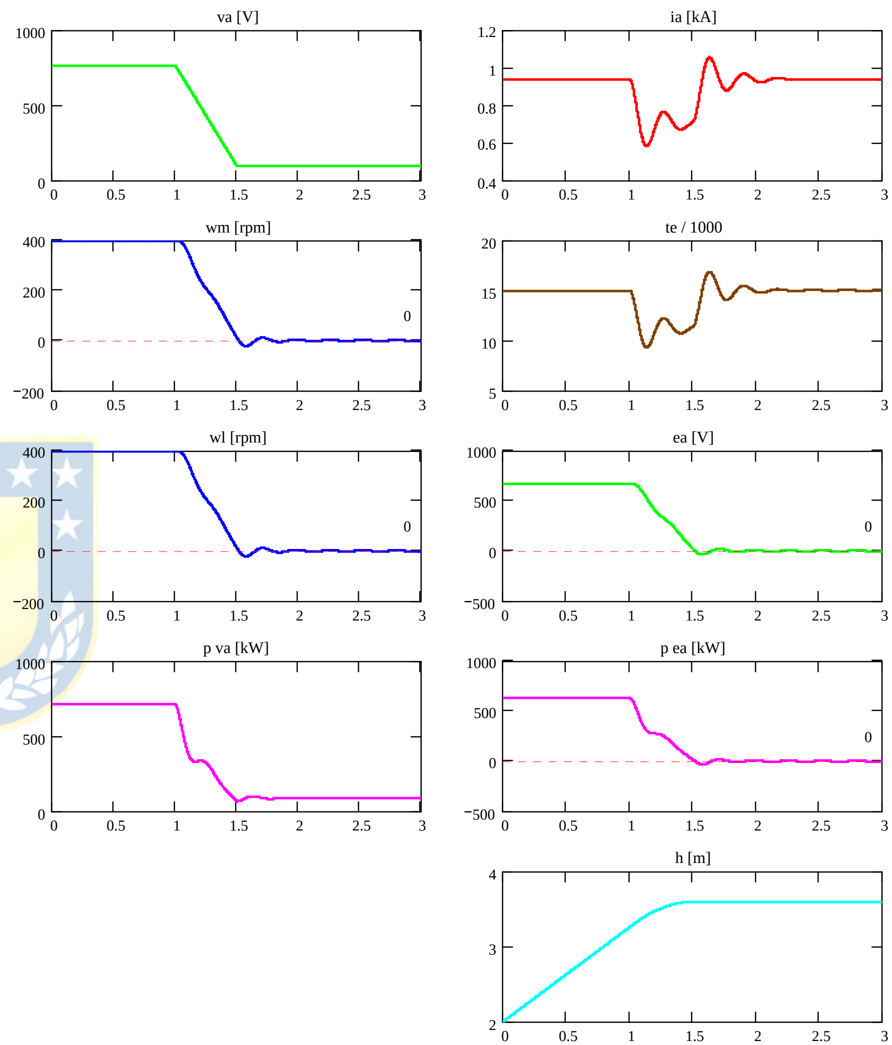
$$\Omega_{m2} := 000 \cdot \frac{2 \cdot \pi}{60} \quad I_{a2} := M \cdot G \cdot \frac{r}{n \cdot k_\phi \cdot I_f} \quad V_{a2} := \Omega_{m2} \cdot k_\phi \cdot I_f + R_a \cdot \frac{M \cdot G \cdot r}{n \cdot k_\phi \cdot I_f} \quad I_{a2} = 937.5 \quad V_{a2} = 93.75$$

Simulación.

$$v_a(t) := V_{a1} \cdot \Phi(t) + \frac{V_{a2} - V_{a1}}{0.5} \cdot (t-1) \cdot \Phi(t-1) - \frac{V_{a2} - V_{a1}}{0.5} \cdot (t-1-0.5) \cdot \Phi(t-1-0.5)$$

$t_f := 3$ $m_f := 2000$ $m := 0 .. m_f$ $u(t) := v_a(t)$ $p(t) := G$ $H := 2$

$$D(t,x) := A \cdot \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T + b \cdot u(t) + e \cdot p(t) \quad Z_a := \text{rkfixed}\left(\begin{pmatrix} I_{a1} & \Omega_{m1} & H \end{pmatrix}^T, 0, t_f, m_f, D\right) \quad rpm := \frac{60}{2 \cdot \pi}$$



Parte B **Respuesta Gráfica a Impulso en la Entrada $u(t) = \delta(t)$.**

Simulación.

$$del_T(t,T) := \frac{1}{T}\cdot(\Phi(t) - \Phi(t-T))$$

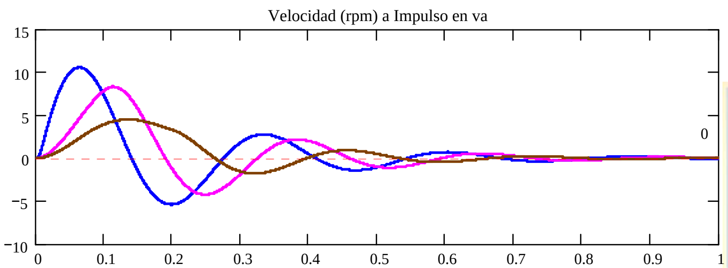
$$u(t,T) := del_T(t,T) \qquad p(t,T) := 0$$

$$t_f := 1 \qquad m_f := 4000 \qquad m := 0..m_f$$

$$D_1(t,x) := A\cdot\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}^T + b\cdot u(t,0.20) + e\cdot p(t,0.20) \qquad CI := \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \qquad Z_{u1} := \text{rkfixed}(CI,0,t_f,m_f,D_1)$$

$$D_2(t,x) := A\cdot\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}^T + b\cdot u(t,0.10) + e\cdot p(t,0.10) \qquad CI := \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \qquad Z_{u2} := \text{rkfixed}(CI,0,t_f,m_f,D_2)$$

$$D_3(t,x) := A\cdot\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}^T + b\cdot u(t,0.01) + e\cdot p(t,0.01) \qquad CI := \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \qquad Z_{u3} := \text{rkfixed}(CI,0,t_f,m_f,D_3)$$



Parte C **Respuesta Matemática a Impulso en la Entrada $u(t) = d(t)$.**

$$\frac{di_a}{dt} = \frac{-R_a}{L_a}\cdot i_a - \frac{k_\phi\cdot I_f}{L_a}\cdot\omega_m + \frac{1}{L_a}\cdot v_a \qquad \frac{d\omega_m}{dt} = \frac{n^2\cdot k_\phi\cdot I_f}{M\cdot r^2}\cdot i_a - \frac{n}{r}\cdot g \qquad \frac{dh}{dt} = \frac{r}{n}\cdot\omega_m$$

Derivando la 2 da Ecuación:

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{n^2\cdot k_\phi\cdot I_f}{M\cdot r^2}\cdot\frac{d}{dt}i_a - \frac{n}{r}\cdot\frac{d}{dt}g$$

Reemplazando la 1 ra ec. en la anterior

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{n^2\cdot k_\phi\cdot I_f}{M\cdot r^2}\cdot\left(\frac{-R_a}{L_a}\cdot i_a - \frac{k_\phi\cdot I_f}{L_a}\cdot\omega_m + \frac{1}{L_a}\cdot v_a\right) - \frac{n}{r}\cdot\frac{d}{dt}g$$

Reemplazando i_a de la 2 da ec. del modelo en la ec. anterior

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{-R_a\cdot n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot\left[\frac{M\cdot r^2}{n^2\cdot k_\phi\cdot I_f}\cdot\left(\frac{d\omega_m}{dt} + \frac{n}{r}\cdot g\right)\right] - \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m + \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot\frac{d}{dt}g$$

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{-R_a\cdot n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot\left(\frac{M\cdot r^2}{n^2\cdot k_\phi\cdot I_f}\cdot\frac{d\omega_m}{dt} + \frac{M\cdot r^2}{n^2\cdot k_\phi\cdot I_f}\cdot\frac{n}{r}\cdot g\right) - \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m + \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot\frac{d}{dt}g$$

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{-R_a}{L_a}\cdot\left(\frac{d\omega_m}{dt} + \frac{n}{r}\cdot g\right) - \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m + \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot\frac{d}{dt}g$$

$$\frac{d}{dt}\frac{d}{dt}\omega_m = \frac{-R_a}{L_a}\cdot\frac{d\omega_m}{dt} + \frac{-R_a\cdot n}{L_a\cdot r}\cdot g - \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m + \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot\frac{d}{dt}g$$

$$\frac{d}{dt}\frac{d}{dt}\omega_m + \frac{R_a}{L_a}\cdot\frac{d\omega_m}{dt} + \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m = \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot\frac{d}{dt}g - \frac{R_a\cdot n}{L_a\cdot r}\cdot g$$

$$s^2\cdot\omega_m + \frac{R_a}{L_a}\cdot s\cdot\omega_m + \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m = \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a - \frac{n}{r}\cdot s\cdot g - \frac{R_a\cdot n}{L_a\cdot r}\cdot g$$

Con $u = \delta(t)$ y $\Delta p = 0$

$$s^2\cdot\omega_m + \frac{R_a}{L_a}\cdot s\cdot\omega_m + \frac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}\cdot\omega_m = \frac{n^2\cdot k_\phi\cdot I_f}{L_a\cdot M\cdot r^2}\cdot v_a$$

$$h(s) = \omega_m(s) = \frac{1}{k_\phi\cdot I_f}\cdot\frac{\dfrac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}}{s^2 + \dfrac{R_a}{L_a}\cdot s + \dfrac{n^2\cdot k_\phi^2\cdot I_f^2}{L_a\cdot M\cdot r^2}}$$

Definiendo

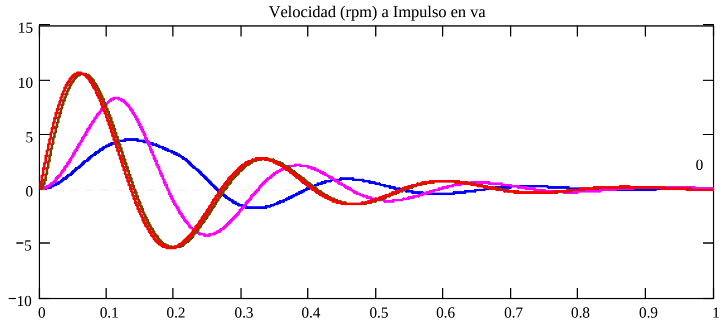
$$\omega_n := \frac{n\cdot k_\phi\cdot I_f}{\sqrt{L_a\cdot M\cdot r}} \qquad k_p := \frac{1}{k_\phi\cdot I_f}$$

$$\zeta := \frac{R_a}{2\cdot L_a\cdot\left(\dfrac{n\cdot k_\phi\cdot I_f}{\sqrt{L_a\cdot M\cdot r}}\right)}$$

$$h_{\omega m}(s) = \omega_m(s) = k_p\cdot\frac{\omega_n^2}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2}$$

Tomando Laplace Inversa.

$$h_{\omega m}(t) := k_p\cdot\frac{\omega_n}{\sqrt{1-\zeta^2}}\cdot\exp(-\zeta\cdot\omega_n\cdot t)\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot t\right)\cdot\Phi(t)$$



Parte D **Respuesta Matemática a una entrada arbitraria u(t) en la entrada.**

$T_o := 4 \qquad t_0 := T_o\cdot\frac{60}{360} \qquad t_1 := \frac{T_o}{2} - T_o\cdot\frac{60}{360} \qquad t_2 := \frac{T_o}{2} + T_o\cdot\frac{60}{360} \qquad t_3 := T_o - T_o\cdot\frac{60}{360}$

$u(t) := \Phi(t-t_0) - \Phi(t-t_1) - \Phi(t-t_2) + \Phi(t-t_3)$ La entrada es la suma de escalones.

$y(t) = h(t) \oplus u(t)$

$y(t) = h(t) \oplus \left(\Phi(t-t_0) - \Phi(t-t_1) - \Phi(t-t_2) + \Phi(t-t_3)\right)$

$y(t) = h(t) \oplus \Phi(t-t_0) - h(t) \oplus \Phi(t-t_1) - h(t) \oplus \Phi(t-t_2) + h(t) \oplus \Phi(t-t_3)$

La respuesta a entrada escalón $y_{esc}(t)$ es la integral de la respuesta a impulso. Por lo tanto,

$$y(t) = \int_0^{t-t_0} h(\tau)\,d\tau - \int_0^{t-t_1} h(\tau)\,d\tau - \int_0^{t-t_2} h(\tau)\,d\tau + \int_0^{t-t_3} h(\tau)\,d\tau$$

definiendo, para una entrada en la tensión va,

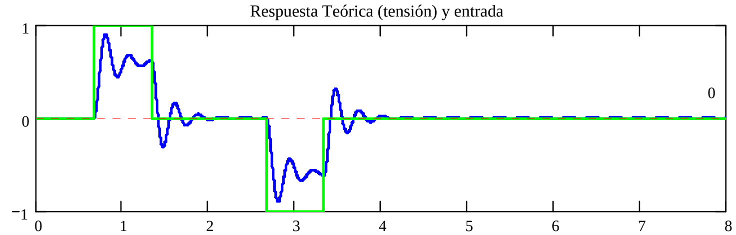
$$y_{esc}(t) := \int_0^t h_{\omega m}(\tau)\,d\tau\cdot\Phi(t)$$

$$y_{esc}(t) := k_p\cdot\left(1 - \exp(-\zeta\cdot\omega_n\cdot t)\cdot\frac{1}{\sqrt{1-\zeta^2}}\cdot\sin\left(\omega_n\cdot\sqrt{1-\zeta^2}\cdot t + \operatorname{atan}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)\right)\cdot\Phi(t)$$

la respuesta $y_{esc\_va}(t)$ a la entrada dada es,

$$y(t) := y_{esc}(t-t_0) - y_{esc}(t-t_1) - y_{esc}(t-t_2) + y_{esc}(t-t_3)$$

$t_f := T_o\cdot 2 \qquad t_i := 0 \qquad t := t_i, t_i + \frac{t_f - t_i}{m_f} .. t_f$

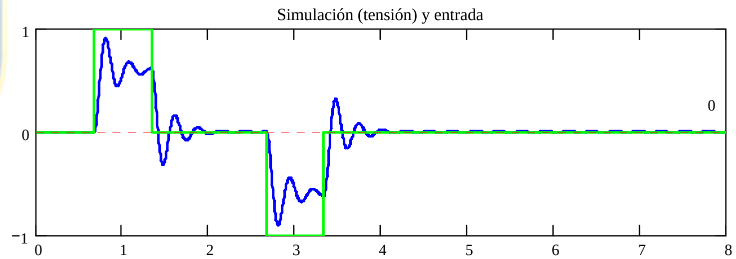


Simulación del sistema normalizado para la entrada dada (para comparar).

$u(t) := \Phi(t-t_0) - \Phi(t-t_1) - \Phi(t-t_2) + \Phi(t-t_3) \qquad p(t) := 0$

$t_f := T_o\cdot 2 \qquad m_f := 2000 \qquad m := 0.. m_f$

$D_1(t,x) := A\cdot\begin{pmatrix} x_1 & x_2 & x_3\end{pmatrix}^T + b\cdot u(t) + e\cdot p(t) \qquad CI := \begin{pmatrix}0 & 0 & 0\end{pmatrix}^T \qquad Z_u := \mathrm{rkfixed}(CI, 0, t_f, m_f, D_1)$



Parte E **Transformada de Fourier de la entrada u(t), de la respuesta a impulso h(t) y la salida y(t).**

Cálculo de la T. de L. de la entrada ut),

$$\int_{-\infty}^{\infty} \left(\Phi(t-t_0) - \Phi(t-t_1) - \Phi(t-t_2) + \Phi(t-t_3)\right)\cdot\exp(-s\cdot t)\,dt \;\Big|\begin{smallmatrix}\text{simplify}\\ \text{float}, 6\end{smallmatrix} \rightarrow \exp(-.666667s)\cdot\frac{1. - 1.\cdot\exp(-.666}{}$$

Laplace de la entrada u(t) que se denotará por $u_s(s)$ es,

$$u_s(s) := \frac{\exp(-t_0\cdot s) - 1.\cdot\exp(-t_1\cdot s) - 1.\cdot\exp(-t_2\cdot s) + \exp(-t_3\cdot s)}{s}$$

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T. de F. de u(t) que se denotará por $u_{\omega}(\omega)$ es,

$$u_{\omega}(\omega) := \frac{\exp(-t_0\cdot j\cdot\omega) - 1\cdot\exp(-t_1\cdot j\cdot\omega) - 1\cdot\exp(-t_2\cdot j\cdot\omega) + \exp(-t_3\cdot j\cdot\omega)}{j\cdot\omega}$$

Laplace de la respuesta a impulso $h_{\omega mva}(t)$ que se denotará por $h_{\omega mvas}(s)$ es,

$$h_{\omega ms}(s) := k_p\cdot\frac{\omega_n^2}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2}$$

T. de F. de $h_{\omega mva}(t)$ que se denotará por $h_{\omega mva_{\omega}}(\omega)$ es,

$$h_{\omega m\omega}(\omega) := k_p\cdot\frac{\omega_n^2}{(j\cdot\omega)^2 + 2\cdot\zeta\cdot\omega_n\cdot(j\cdot\omega) + \omega_n^2}$$

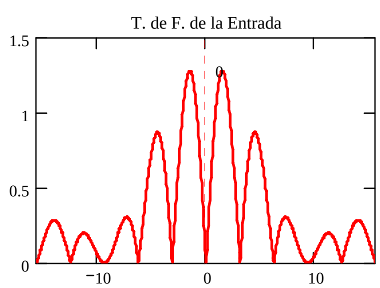
Laplace de la salida $y_{va}(t)$ que se denotará por $y_{vas}(s)$ es,

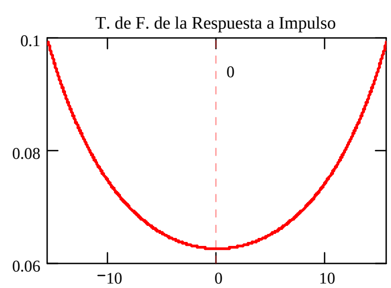
$$y_s(s) := k_p\cdot\frac{\omega_n^2}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2}\cdot\frac{\exp(-t_0\cdot s) - 1\cdot\exp(-t_1\cdot s) - 1\cdot\exp(-t_2\cdot s) + \exp(-t_3\cdot s)}{s}$$

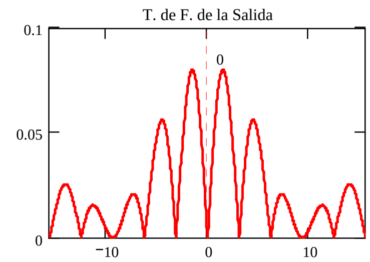
T. de F. de $y_{va}(t)$ que se denotará por $y_{va\omega}(\omega)$ es,

$$y_{\omega}(\omega) := k_p\cdot\frac{\omega_n^2}{(j\cdot\omega)^2 + 2\cdot\zeta\cdot\omega_n\cdot(j\cdot\omega) + \omega_n^2}\cdot\frac{\exp(-t_0\cdot j\cdot\omega) - 1\cdot\exp(-t_1\cdot j\cdot\omega) - 1\cdot\exp(-t_2\cdot j\cdot\omega) + \exp(-t_3\cdot j\cdot\omega)}{j\cdot\omega}$$

$\omega_i := -5\cdot\pi$ $\quad\omega_f := 5\cdot\pi$ $\quad n_{\omega} := 1000$ $\quad\omega := \omega_i, \omega_i + \frac{\omega_f - \omega_i}{n_{\omega}} .. \omega_f$



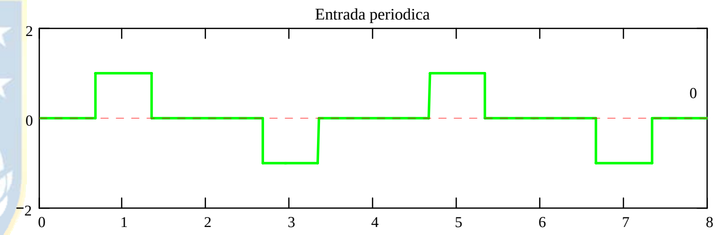




Parte F **Transformada de Fourier de la salida y(t) para una entrada periódica u(t) dada.**

$T_o := T_o$ $\quad u(t) := \Phi(t - t_0) - \Phi(t - t_1) - \Phi(t - t_2) + \Phi(t - t_3)$ $\quad u_p(t) := \sum_{m=0}^{2} u(t - m\cdot T_o)$

$t_f := 2\cdot T_o$ $\quad t_i := 0$ $\quad t := t_i, t_i + \frac{t_f - t_i}{m_f} .. t_f$ $\quad F_o := \frac{1}{T_o}$

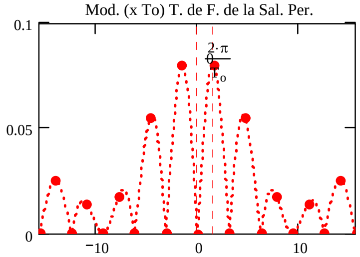


T. de F. de la salida periodica $\Delta y_{np}(t)$ que se denotará por $\Delta y_{npn}(n)$ es,

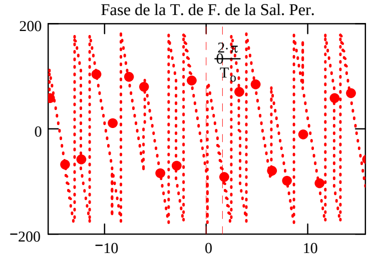
$$y_{pn}(n) := \frac{1}{T_o}\cdot\left[k_p\cdot\frac{\omega_n^2}{[j\cdot(2\cdot n\cdot\pi\cdot F_o)]^2 + 2\cdot\zeta\cdot\omega_n\cdot[j\cdot(2\cdot n\cdot\pi\cdot F_o)] + \omega_n^2}\cdot\frac{\exp[-t_0\cdot j\cdot(2\cdot n\cdot\pi\cdot F_o)] + -1\cdot\exp[-t_1\cdot j\cdot(2\cdot n\cdot\pi\cdot F_o)] + -1\cdot\exp[-t_2\cdot j\cdot(2\cdot n\cdot\pi\cdot F_o)] + \exp[-t_3\cdot j\cdot(2\cdot n\cdot\pi\cdot F_o)]}{j\cdot(2\cdot n\cdot\pi\cdot F_o)}\right]$$

$n_i := -10$ $\quad n_f := 10$ $\quad n := n_i, n_i + \frac{n_f - n_i}{n_f - n_i} .. n_f$

Mod. (x To) T. de F. de la Sal. Per.

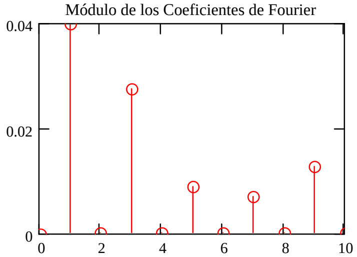


Fase de la T. de F. de la Sal. Per.

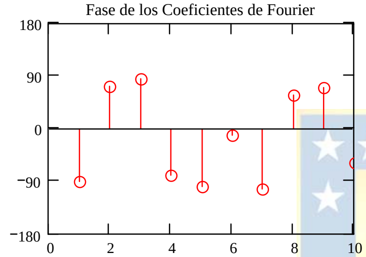


Parte G **Coeficientes de Fourier de la salida y(t) para una entrada periódica u(t) dada.**

Módulo de los Coeficientes de Fourier



Fase de los Coeficientes de Fourier

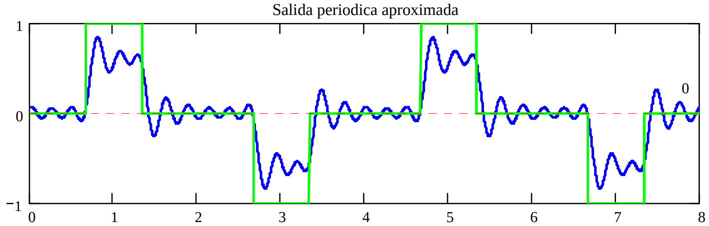


Parte H **Reconstrucción de la Salida en base a cuatro de los Coeficientes de Fourier.**

$$Sal_{157}(t) := 2\cdot\left|y_{pn}(1)\right|\cdot\cos\left(1\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(1)\right)\right) + 2\cdot\left|y_{pn}(3)\right|\cdot\cos\left(3\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(3)\right)\right) \ldots$$
$$+ 2\cdot\left|y_{pn}(5)\right|\cdot\cos\left(5\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(5)\right)\right) + 2\cdot\left|y_{pn}(7)\right|\cdot\cos\left(7\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(7)\right)\right) \ldots$$
$$+ 2\cdot\left|y_{pn}(9)\right|\cdot\cos\left(9\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(9)\right)\right) + 2\cdot\left|y_{pn}(11)\right|\cdot\cos\left(11\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(11)\right)\right) \ldots$$
$$+ 2\cdot\left|y_{pn}(13)\right|\cdot\cos\left(13\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(13)\right)\right) + 2\cdot\left|y_{pn}(15)\right|\cdot\cos\left(15\cdot 2\cdot\pi\cdot F_o\cdot t + \arg\left(y_{pn}(15)\right)\right)$$

$t_f := 2\cdot T_o$ $t_i := 0$ $m_f := 2000$ $t := t_i, t_i + \frac{t_f - t_i}{m_f} .. t_f$

Salida periodica aproximada



Parte I **Simulación del sistema con la Entrada Periodica.**

$u(t) := u_p(t)$ $p(t) := 0$

$t_f := T_o\cdot 2$ $m_f := 2000$ $m := 0 .. m_f$

Simulación para encontrar la C.I.

$D_1(t, x) := A\cdot\left(x_1 \; x_2 \; x_3\right)^T + b\cdot u(t) + e\cdot p(t)$ $CI := (0 \; 0 \; 0)^T$ $Z_{p1} := rkfixed\left(CI, 0, t_f, m_f, D_1\right)$

Simulación

$D_1(t, x) := A\cdot\left(x_1 \; x_2 \; x_3\right)^T + b\cdot u(t) + e\cdot p(t)$ $CI := \left(Zp1_{m_f,2} \; Zp1_{m_f,3} \; Zp1_{m_f,4}\right)^T$ $Z_{p1} := rkfixed\left(CI, 0, t_f, m_f, D_1\right)$

Salida periodica exacta y aproximada