A Novel Iterative Image Restoration Algorithm using Nonstationary Image Priors

Esteban Vera[1*], **Miguel Vega**[2], **Rafael Molina** [3], **and Aggelos K. Katsaggelos**[4]

[1] Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85750
[2] Departamento de Lenguajes y Sistemas Informáticos, Universidad de Granada, 18071, Granada, Spain
[3] Departamento de Ciencias de la Computación e I.A., Universidad de Granada, 18071, Granada, Spain
[4] Department of Electrical Engineering and Computer Science, Northwestern University, Evanston, IL 60208-3118

Abstract: *We propose a novel algorithm for image restoration based on a combination of nonstationary edge-preserving priors. We develop a Bayesian modeling followed by an evidence approximation inference approach for deriving the iterative algorithm, making use of a diagonal covariance matrix approximation for a fast implementation.*

Introduction

Image restoration is the process of recovering a clean image $\mathbf{x}$ from a degraded blurry and noisy image $\mathbf{y}$. A classical linear model for image degradation is:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where $\mathbf{H}$ is the blurring kernel operator, and $\mathbf{n}$ is an additive noise component. Thus, image restoration is an ill-posed inverse problem, where $\mathbf{H}$ is typically given and often considered to be a space-invariant Point Spread Function (**PSF**).

Bayesian Modeling

Observation Model

If the noise is Gaussian the pdf of the observation model is:

$$p(\mathbf{y}|\mathbf{x},\beta) \propto \beta^{N/2} \exp\left\{-\frac{\beta}{2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2\right\}.$$

Prior Model

As image prior we define a zero-mean multivariate Gaussian distribution that combines the constraints given by a set of L filters $\mathbf{C}_i$ (e.g. high-pass first order differences) as follows:

$$p(\mathbf{x}|\mathbf{a}_1,\ldots,\mathbf{a}_L) \propto \left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i\right|^{1/2} \exp\left\{-\frac{1}{2}\sum_{i=1}^{L}\|\mathbf{A}_i^{1/2}\mathbf{C}_i\mathbf{x}\|^2\right\},$$

where $\mathbf{A}_i$ is the diagonal matrix of the hyperparameters a_i^j associated with the precision of the corresponding response of each filter operator $\mathbf{C}_i$ for any pixel j. Thus, $\mathbf{A}_i = \mathrm{DIAG}(\mathbf{a}_i)$, and $\mathbf{a}_i = [a_i^1, a_i^2, \ldots, a_i^N]^t$.

Bayesian Inference

The joint probability $p(\mathbf{y},\mathbf{x},\beta,\mathbf{a}_1,\ldots,\mathbf{a}_L)$ is proportional to:

$$\left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i\right|^{1/2} \beta^{N/2} \exp\left\{-\frac{1}{2}\sum_{i=1}^{L}\|\mathbf{A}_i^{1/2}\mathbf{C}_i\mathbf{x}\|^2 - \frac{\beta}{2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2\right\}.$$

Now, to perform the inference for the hyperparameters based on the evidence analysis, by marginalizing over $\mathbf{x}$ we have that the marginal distribution is as follows:

$$\begin{aligned} p(\mathbf{y}|\beta,\mathbf{a}_1,\ldots,\mathbf{a}_L) &= \int_{\mathbf{x}} p(\mathbf{y},\mathbf{x},\beta,\mathbf{a}_1,\ldots,\mathbf{a}_L)d\mathbf{x} \\ &\propto \left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i\right|^{1/2} \beta^{N/2} \left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i + \beta\mathbf{H}^t\mathbf{H}\right|^{-1/2} \\ &\quad \times \exp\left\{-\frac{1}{2}\sum_{i=1}^{L}\|\sqrt{\mathbf{A}_i}\mathbf{C}_i\bar{\mathbf{x}}\|^2 - \frac{\beta}{2}\|\mathbf{y}-\mathbf{H}\bar{\mathbf{x}}\|^2\right\}. \end{aligned}$$

Thus, the marginal logarithm is equal to:

$$\begin{aligned} \ln p(\mathbf{y}|\beta,\mathbf{a}_1,\ldots,\mathbf{a}_L,) &= \frac{1}{2}\ln\left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i\right| - \frac{\beta}{2}\|\mathbf{y}-\mathbf{H}\bar{\mathbf{x}}\|^2 \\ &+ \frac{N}{2}\ln\beta - \frac{1}{2}\sum_{i=1}^{L}\|\mathbf{A}_i^{1/2}\mathbf{C}_i\bar{\mathbf{x}}\|^2 - \frac{1}{2}\ln\left|\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i + \beta\mathbf{H}^t\mathbf{H}\right|. \end{aligned}$$

Next, by taking the derivative with respect to the hyperparameter a_i^j corresponding to the element j of the diagonal matrix $\mathbf{A}_i$, we have that:

$$\begin{aligned} \frac{\delta \ln p(\mathbf{y}|\beta,\mathbf{a}_1,\ldots,\mathbf{a}_L)}{\delta a_i^j} = \frac{1}{2}\Bigg(&\mathrm{trace}[(\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i)^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i] \\ &-\bar{\mathbf{x}}^t\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i\bar{\mathbf{x}} - \mathrm{trace}[(\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i + \beta\mathbf{H}^t\mathbf{H})^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i]\Bigg), \end{aligned}$$

where $\mathbf{J}^{ij}$ is the single-entry matrix which is zero everywhere except at the entry (i,j), and $\bar{\mathbf{x}}$ is the **MAP** estimate for the unknown image to be recovered.

Parameter Estimation

By setting the marginal derivative equal to zero, and defining $\Sigma_P = \sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i$ and $\Sigma_T = \sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i\mathbf{C}_i + \beta\mathbf{H}^t\mathbf{H}$, then:

$$\mathrm{trace}[\Sigma_P^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i] = \bar{\mathbf{x}}^t\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i\bar{\mathbf{x}} + \mathrm{trace}[\Sigma_T^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i].$$

Using an **EM** approach the hyperparameters are found as:

$$a_i^{j(k+1)} = \frac{a_i^{j(k)}\mathrm{trace}[\Sigma_P^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i]}{\bar{\mathbf{x}}^{t(k)}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i\bar{\mathbf{x}}^{(k)} + \mathrm{trace}[\Sigma_T^{-1}\mathbf{C}_i^t\mathbf{J}^{jj}\mathbf{C}_i]}, \quad (1)$$

where $\bar{\mathbf{x}}^{(k)}$ is the **MAP** estimate computed as follows:

$$\bar{\mathbf{x}}^{(k)} = \left(\sum_{i=1}^{L}\mathbf{C}_i^t\mathbf{A}_i^{(k)}\mathbf{C}_i + \beta\mathbf{H}^t\mathbf{H}\right)^{-1}\beta\mathbf{H}^t\mathbf{y}, \quad (2)$$

Implementation

To simplify their inversion, we use a diagonal approximation for the covariance matrices in Eq. 1, Σ_P and Σ_T, leading to the proposed iterative update formula as follows:

$$\begin{aligned} \mathbf{a}_i^{(k+1)} = \mathrm{diag}\Bigg[&\mathrm{DIAG}\left(\mathbf{B}\,\mathrm{diag}(\Sigma_P^{(k)^{-1}})\right) \quad (3) \\ &\times \mathrm{DIAG}\left(\mathrm{diag}((\mathbf{C}_i\bar{\mathbf{x}}^{(k)})(\mathbf{C}_i\bar{\mathbf{x}}^{(k)})^t) + \mathbf{B}\,\mathrm{diag}(\Sigma_T^{(k)^{-1}})\right)^{-1}\Bigg], \end{aligned}$$

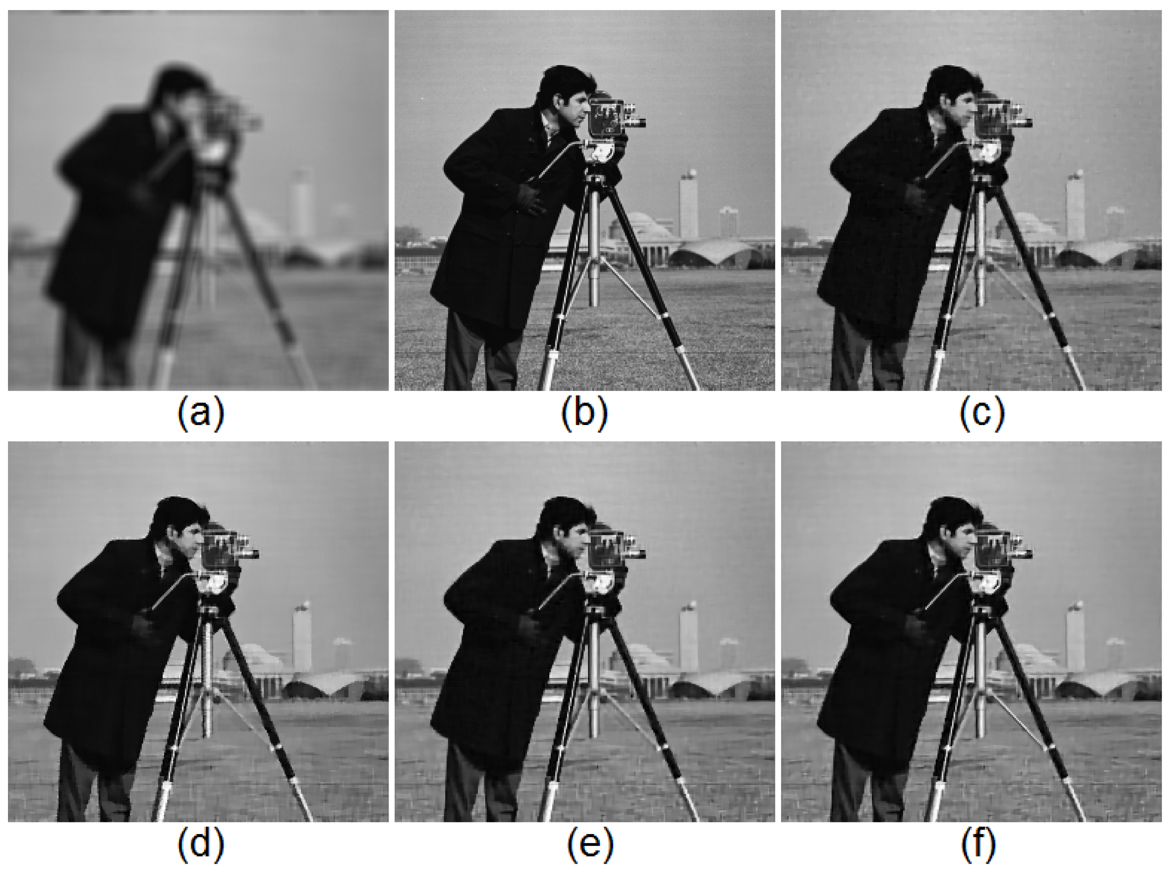
which can be implemented in the Fourier domain, and where $\Sigma_P^{(k)} = \sum_{i=1}^{L}\mathrm{DIAG}(\mathbf{D}_i^t\mathbf{a}_i^{(k)})$ and $\Sigma_T^{(k)} = \sum_{i=1}^{L}\mathrm{DIAG}(\mathbf{D}_i^t\mathbf{a}_i^{(k)}) + \beta h^2\mathbf{I}$, $\mathbf{B}$ is a 4-connected filter, $\mathbf{D}_i$ is a filter related to each filter $\mathbf{C}_i$, and h is a constant related to the blurring matrix $\mathbf{H}$.

Simulation Results

We implemented two variants of the proposed algorithm: **NF2** uses only the first order horizontal and vertical difference filters, whereas **NF4** also add the first order diagonal filters. We compared the restoration performance against **BTV** [1] and **BST** [2] using a variety of **PSF** kernels and noise levels. We used four standard images: Cameraman (CAM, 256×256), Lena (LEN, 256×256), Shepp-Logan Phantom (PHA, 256×256) and Barbara (BAR, 512×512. The noise variance $\sigma^2 = 1/\beta$ is known, and the **ISNR** is defined as $20\log_{10}(\|\mathbf{x}-\mathbf{y}\|/\|\mathbf{x}-\hat{\mathbf{x}}\||)$.

Comparative Restoration Performance - ISNR (dB):

		Uniform Kernel (9×9)				Gaussian Kernel ($\sigma^2=9$)			
BSNR	Method	CAM	LEN	PHA	BAR	CAM	LEN	PHA	BAR
40 dB	BTV	8.60	8.51	17.74	3.22	3.49	**4.85**	7.87	**1.61**
	BST	8.80	8.13	19.02	3.16	3.25	4.34	**10.34**	1.36
	NF2	9.34	9.13	20.03	**3.74**	3.56	3.87	5.39	1.54
	NF4	**9.75**	**9.63**	**23.05**	3.65	**3.71**	4.03	8.06	1.52
30 dB	BTV	5.08	5.89	11.00	1.71	2.73	**3.96**	5.29	**1.27**
	BST	5.89	5.50	**12.51**	1.61	2.69	3.08	**7.97**	1.04
	NF2	6.38	6.40	10.44	**2.01**	3.03	3.37	5.71	1.18
	NF4	**6.61**	**6.86**	12.14	1.93	**3.17**	3.49	6.43	1.20
20 dB	BTV	2.42	3.59	5.52	1.15	1.81	2.84	2.79	**1.14**
	BST	3.18	2.65	**8.25**	0.73	2.03	1.93	5.16	0.79
	NF2	**3.85**	4.22	6.83	**1.35**	2.36	**3.18**	5.10	1.11
	NF4	3.70	**4.45**	7.84	1.33	**2.41**	3.11	**5.70**	1.09

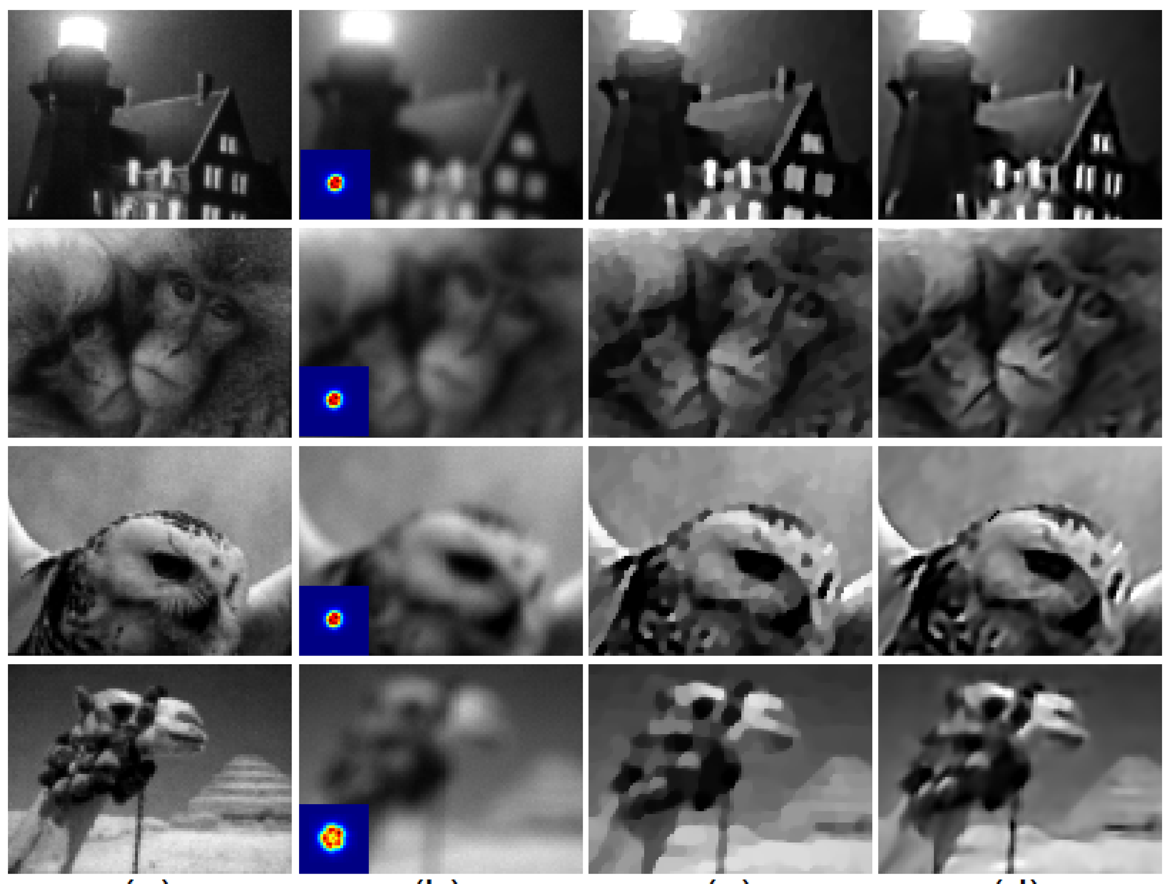


(a) (b) (c) (d) (e) (f)

Restoration example for Cameraman (CAM): (a) Degraded with a 9×9 uniform kernel and 40 dB BSNR:; (b) Original; (c) **BTV**, $\mathrm{ISNR} = 8.60$ dB; (d) **BST**, $\mathrm{ISNR} = 8.80$ dB; (e) **NF2**, $\mathrm{ISNR} = 9.34$ dB; (f) **NF4**, $\mathrm{ISNR} = 9.75$ dB.

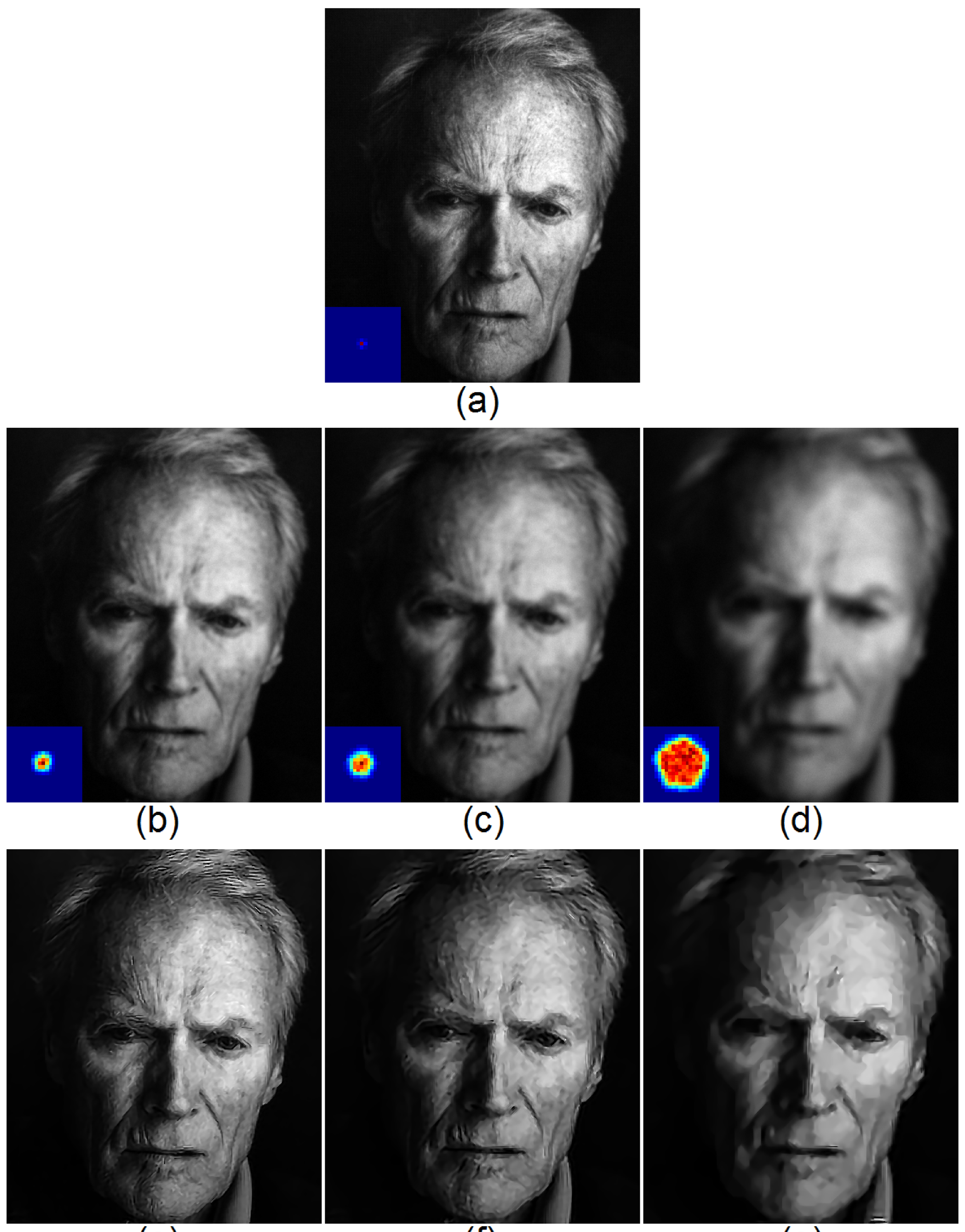
Digital Refocusing Experiment

We designed an experiment for systematically defocusing a target picture, while also retrieving the respective PSF from an illuminated optical fiber. We used a QImaging RETIGA EXi Monochrome 12-Bit Cooled CCD camera attached to a Computar H6Z0812 8-48 mm F1.2 zoom lens. First we retrieved pictures of several image thumbnails of natural images of 100×80 pixels each, while the second experiment used a larger target image of 350×500 pixels.



(a) (b) (c) (d)

Digital refocusing comparison: (a) Focused images; (b) Defocused images (Inset: measured PSF); (c) **BTV** restored images; (d) **NF4** restored images.



(a) (b) (c) (d) (e) (f) (g)

Digital refocusing experiment: (a) Focused image; (b) Moderate defocus; (c) Medium defocus; (d) Strong defocus (Insets: $4\times$ measured **PSF**); (e) **NF4** restored image (b); (f) **NF4** restored image (c); (g) **NF4** restored image (d).

Conclusion

We presented a novel image restoration algorithm that comfortably surpassed the state-of-the-art in terms of **ISNR** for compactly supported **PSF** blurring kernels. We also empirically demonstrated that the algorithm is successfully suitable for restoring defocused images, showing a clear improvement in the perceivable visual quality.

References

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*corresponding email: `estebanvera@u.arizona.edu`

IEEE International Conference on Image Processing (ICIP) 2011, Brussels, Belgium.