

$$M=3$$

$$\begin{pmatrix} a_1 & b_1 & 0 \\ c_1 & a_2 & b_2 \\ 0 & c_2 & a_3 \end{pmatrix} = \begin{pmatrix} l_1 & 0 & 0 \\ c_1 & l_2 & 0 \\ 0 & c_2 & l_3 \end{pmatrix} \begin{pmatrix} 1 & m_1 & 0 \\ 0 & 1 & m_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a_1 = l_1$$

$$b_1 = l_1 m_1$$

$$a_2 = c_1 m_1 + l_2$$

$$b_2 = l_2 m_2$$

$$a_3 = c_2 m_2 + l_3$$

$$l_1 = a_1 \quad m_1 = \frac{b_1}{l_1}$$

$$l_2 = a_2 - c_1 m_1 \quad m_2 = \frac{b_2}{l_2}$$

$$l_3 = a_3 - c_2 m_2$$

$$M=4 \quad \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ c_1 & a_2 & b_2 & 0 \\ 0 & c_2 & a_3 & b_3 \\ 0 & 0 & c_3 & a_4 \end{pmatrix} = \begin{pmatrix} l_1 & 0 & 0 & 0 \\ c_1 & l_2 & 0 & 0 \\ 0 & c_2 & l_3 & 0 \\ 0 & 0 & c_3 & l_4 \end{pmatrix} \begin{pmatrix} 1 & m_1 & 0 & 0 \\ 0 & 1 & m_2 & 0 \\ 0 & 0 & 1 & m_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$a_1 = l_1$$

$$b_1 = l_1 m_1$$

$$l_1 = a_1$$

$$m_1 = \frac{b_1}{l_1}$$

$$a_2 = c_1 m_1 + l_2$$

$$b_2 = l_2 m_2$$

$$l_2 = a_2 - c_1 m_1$$

$$m_2 = \frac{b_2}{l_2}$$

$$a_3 = c_2 m_2 + l_3$$

$$b_3 = l_3 m_3$$

$$l_3 = a_3 - c_2 m_2$$

$$m_3 = \frac{b_3}{l_3}$$

$$a_4 = c_3 m_3 + l_4$$

$$l_4 = a_4 - c_3 m_3$$

$$m=3$$

Además:

$$LUx = d \Rightarrow \begin{cases} Ux = Y \\ LY = d \end{cases}$$

$$\begin{pmatrix} l_1 & 0 & 0 \\ c_1 & l_2 & 0 \\ 0 & c_2 & l_3 \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$Y_1 = d_1$$

$$Y_2 = (d_2 - c_1 Y_1) / l_2$$

$$Y_3 = (d_3 - c_2 Y_2) / l_3$$

$$\begin{pmatrix} 1 & m_1 & 0 \\ 0 & 1 & m_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$$

$$X_3 = Y_3$$

$$X_2 = Y_2 - m_2 X_3$$

$$X_1 = Y_1 - m_1 X_2$$

Observando, nos damos cuenta que:

$$l_1 = a_1$$

$$Y_1 = d_1 / l_1 \rightarrow 1 \text{ flop (división)}$$

Para $i = 2 \dots m$.

$$\text{Por iteración: } \begin{cases} 1 \text{ flop (división)} \leftarrow m_{i-1} = b_{i-1} / l_{i-1} \\ 2 \text{ flops (resta, división)} \leftarrow l_i = a_i - c_{i-1} m_{i-1} \\ 3 \text{ flops (resta, div, mult)} \leftarrow Y_i = (d_i - c_{i-1} Y_{i-1}) / l_i \end{cases}$$

$$X_m = Y_m$$

Por $k = m-1 \dots 1 \rightarrow 2 \text{ flops (resta, multiplicación)}$ por iteración

$$X_k = Y_k - m_k X_{k+1}$$

$$\text{flops} = 1 + \sum_{i=2}^m 8 = 1 - 8 + \sum_{i=1}^m 8$$

$$= 8 \sum_{i=1}^m 1 - 7 = 8m - 7 \text{ flops.}$$