

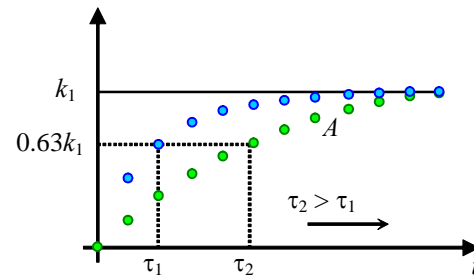
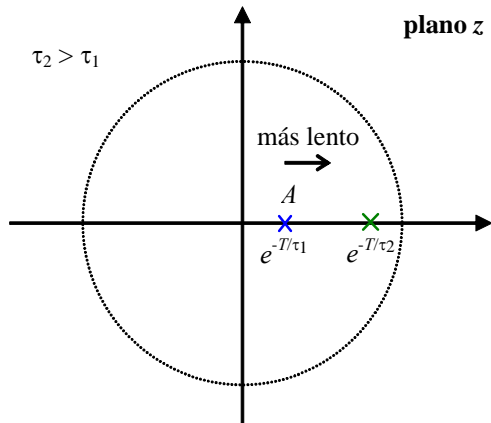
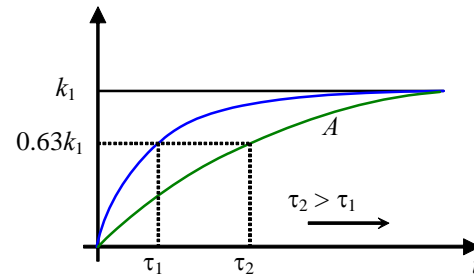
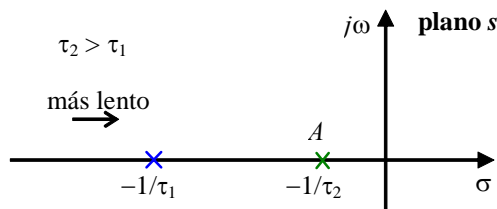
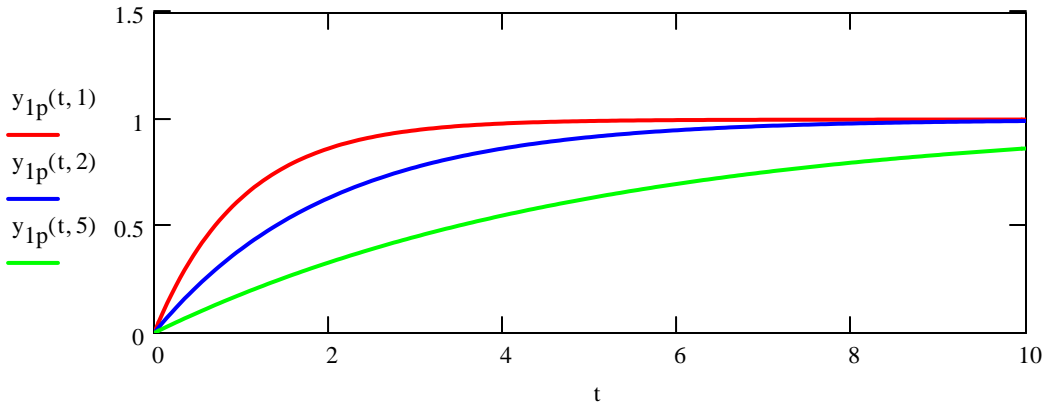
# Comportamiento Transitorio de Sistemas de Primer Orden

**Problema** Sistemas de Primer con un polo y/o cero extra.

**Parámetros**  $k_p := 1$        $n_f := 200$        $t_f := 10$        $t := 0, \frac{t_f}{n_f} .. t_f$

## Sistema de Primer Orden

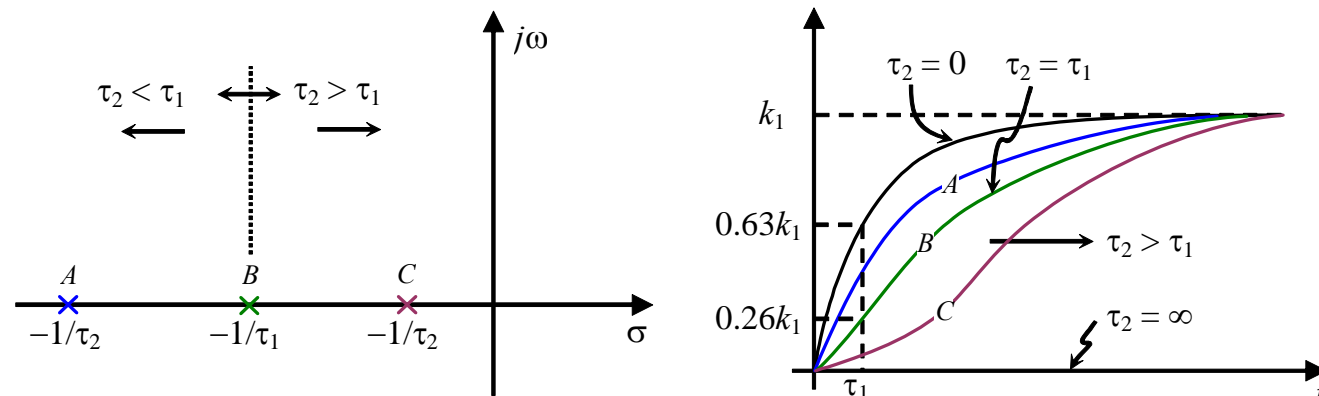
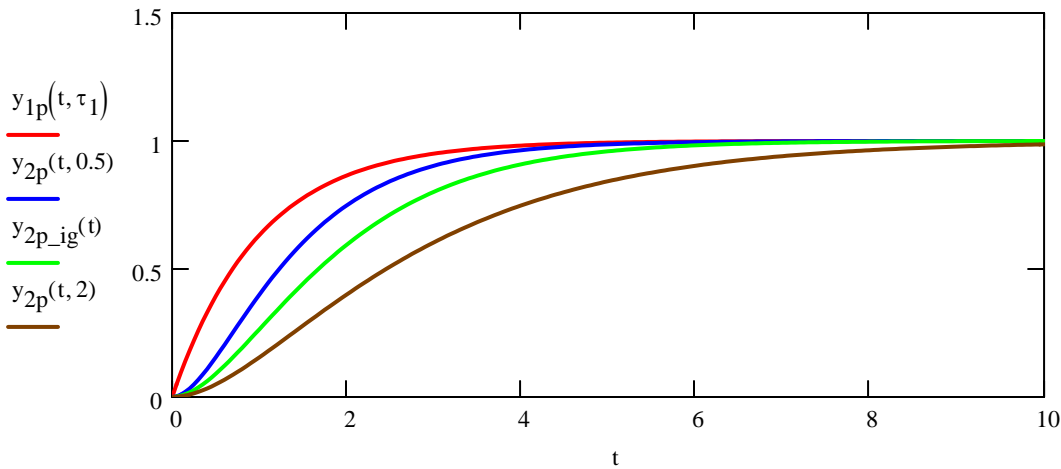
$$g(s) = k_p \cdot \frac{1}{\tau_1 \cdot s + 1} \quad y(s) = k_p \cdot \frac{1}{s} \cdot \frac{1}{\tau_1 \cdot s + 1} \quad y_{1p}(t, \tau_1) := k_p \cdot \left( 1 - e^{-\frac{t}{\tau_1}} \right) \cdot \Phi(t)$$



Sistema de Primer Orden y un Polo Adicional

$$\tau_1 := 1 \quad g(s) = k_p \cdot \frac{1}{\tau_1 \cdot s + 1} \cdot \frac{1}{\tau_2 \cdot s + 1} \quad y_{2p}(t, \tau_2) := k_p \cdot \left[ 1 - \frac{\tau_1}{\tau_1 - \tau_2} \cdot e^{\left(\frac{-t}{\tau_1}\right)} + \frac{\tau_2}{\tau_1 - \tau_2} \cdot e^{\left(\frac{-t}{\tau_2}\right)} \right] \cdot \Phi(t)$$

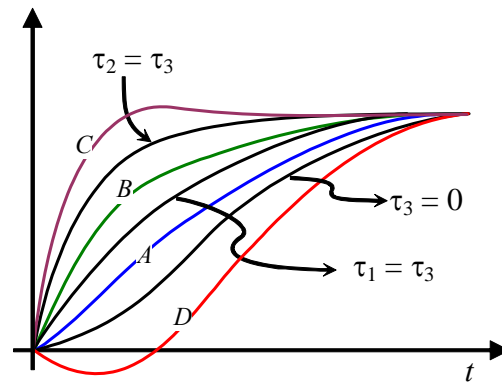
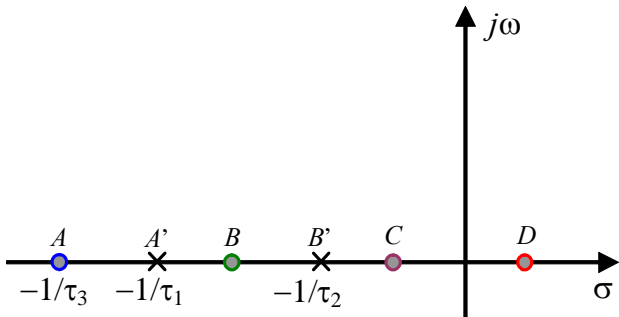
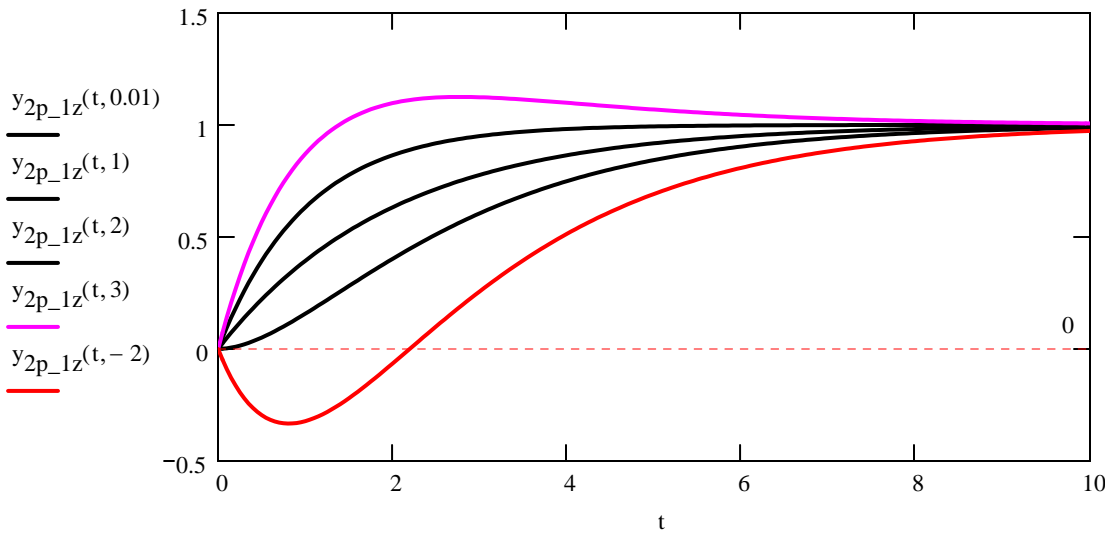
$$y(s) = k_p \cdot \frac{1}{s} \cdot \frac{1}{\tau_1 \cdot s + 1} \cdot \frac{1}{\tau_2 \cdot s + 1} \quad y_{2p\_ig}(t) := k_p \cdot \left[ 1 - e^{\left(\frac{-t}{\tau_1}\right)} - \frac{1}{\tau_1} \cdot t \cdot e^{\left(\frac{-t}{\tau_1}\right)} \right] \cdot \Phi(t)$$



Sistema de Primer Orden, un Polo Adicional y un Cero.

$$\tau_1 := 1 \quad \tau_2 := 2 \quad g(s) = k_p \cdot \frac{1}{\tau_1 \cdot s + 1} \cdot \frac{\tau_3 \cdot s + 1}{\tau_2 \cdot s + 1} \quad y_{2p\_1z}(t, \tau_3) := k_p \cdot \left[ 1 + \frac{\tau_2 - \tau_3}{\tau_1 - \tau_2} \cdot e^{\left(\frac{-t}{\tau_2}\right)} + \frac{\tau_3 - \tau_1}{\tau_1 - \tau_2} \cdot e^{\left(\frac{-t}{\tau_1}\right)} \right] \cdot \Phi(t)$$

$$y(s) = k_p \cdot \frac{1}{s} \cdot \frac{1}{\tau_1 \cdot s + 1} \cdot \frac{\tau_3 \cdot s + 1}{\tau_2 \cdot s + 1}$$



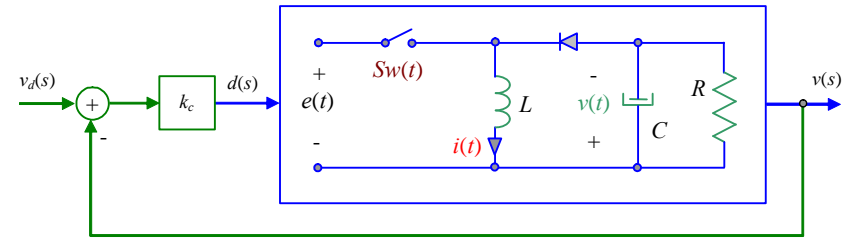
**Problema** Estudiar si el reductor/elevador de tensión oscilaría con un controlador de ganancia.

**Parámetros**

$$L := 12 \cdot 10^{-3} \quad C := 250 \cdot 10^{-6} \quad R := 10 \quad d_o := 0.5 \quad e_o := 10 \quad \Delta e := 0.5$$

**Punto de operación**

$$v_o := \frac{d_o}{1 - d_o} \cdot e_o \quad i_o := \frac{v_o}{R \cdot (1 - d_o)} \quad v_o = 10 \quad i_o = 2 \quad u_o := d_o \quad p_o := e_o$$



**Modelo Lineal Normalizado.**

$$A_n := \begin{bmatrix} \frac{-1}{R \cdot C} & \frac{1}{R \cdot C} \\ \frac{-R}{L} \cdot (1 - d_o)^2 & 0 \end{bmatrix} \quad b_n := \begin{bmatrix} \frac{-d_o}{R \cdot C \cdot (1 - d_o)} \\ \frac{R}{L} \cdot (1 - d_o) \end{bmatrix} \quad e_n := \begin{bmatrix} 0 \\ \frac{R}{L} \cdot (1 - d_o)^2 \end{bmatrix} \quad c_n := (1 \ 0)$$

**Variables de Estado**

$$x_1 = v \quad x_2 = i$$

**Función de Transferencia en L.A.**

$$h_{vd}(s) := \frac{1}{1 - d_o} \cdot \frac{s \cdot \frac{-1}{R \cdot C} \cdot d_o + \frac{(1 - d_o)^2}{L \cdot C}}{s^2 + s \cdot \frac{1}{R \cdot C} + \frac{(1 - d_o)^2}{L \cdot C}}$$

**cero**

$$\text{polyroots} \left[ \begin{bmatrix} \frac{(1 - d_o)^2}{L \cdot C} \\ \frac{-1}{R \cdot C} \cdot d_o \end{bmatrix} \right] = 416.667$$

**polos**

$$\text{polyroots} \left[ \begin{bmatrix} \frac{(1 - d_o)^2}{L \cdot C} \\ \frac{1}{R \cdot C} \\ 1 \end{bmatrix} \right] = \begin{pmatrix} -200 - 208.167i \\ -200 + 208.167i \end{pmatrix}$$

**Función de Transferencia en L.C.**

$$k_c := 0.5$$

$$h_{vvd}(s) := \frac{\frac{k_c}{1 - d_o} \cdot \left[ s \cdot \frac{-1}{R \cdot C} \cdot d_o + \frac{(1 - d_o)^2}{L \cdot C} \right]}{s^2 + s \cdot \frac{1}{R \cdot C} \cdot \left( 1 - k_c \cdot \frac{d_o}{1 - d_o} \right) + \frac{(1 - d_o)^2}{L \cdot C} \cdot \left( 1 + k_c \cdot \frac{1}{1 - d_o} \right)}$$

**cero**

$$\text{polyroots} \left[ \begin{bmatrix} \frac{(1 - d_o)^2}{L \cdot C} \\ \frac{-1}{R \cdot C} \cdot d_o \end{bmatrix} \right] = 416.667$$

**polos**

$$\text{polyroots} \left[ \begin{bmatrix} \frac{(1 - d_o)^2}{L \cdot C} \cdot \left( 1 + k_c \cdot \frac{1}{1 - d_o} \right) \\ \frac{1}{R \cdot C} \cdot \left( 1 - k_c \cdot \frac{d_o}{1 - d_o} \right) \\ 1 \end{bmatrix} \right] = \begin{pmatrix} -100 + 395.811i \\ -100 - 395.811i \end{pmatrix}$$

### Simulación

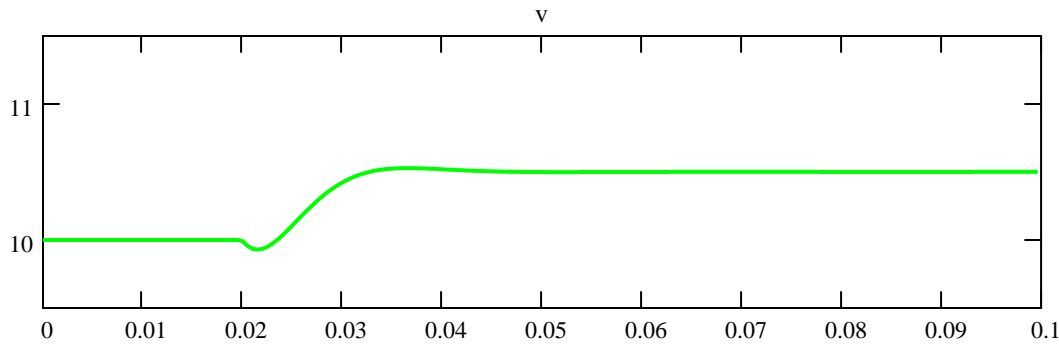
$$n_f := 200 \quad t_f := 0.1 \quad t := 0, \frac{t_f}{n_f} .. t_f \quad m := 0 .. n_f$$

L.A.

$$\Delta d(t) := 0.25 \cdot \Phi(t - 0.02) \quad \Delta e(t) := 0 \quad k_a := 1$$

$$D(t, \Delta x) := A_n \cdot (\Delta x_1 \ \Delta x_2)^T + b_n \cdot k_a \cdot \Delta d(t) + e_n \cdot \Delta e(t) \quad CI := (0 \ 0)^T \quad Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$$

Oscila establemente. Hay un error en est. est.

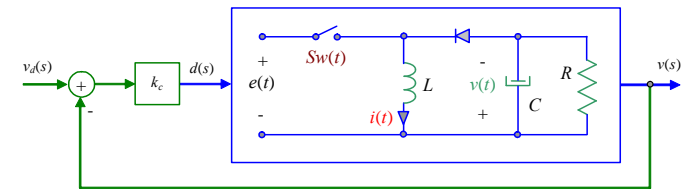
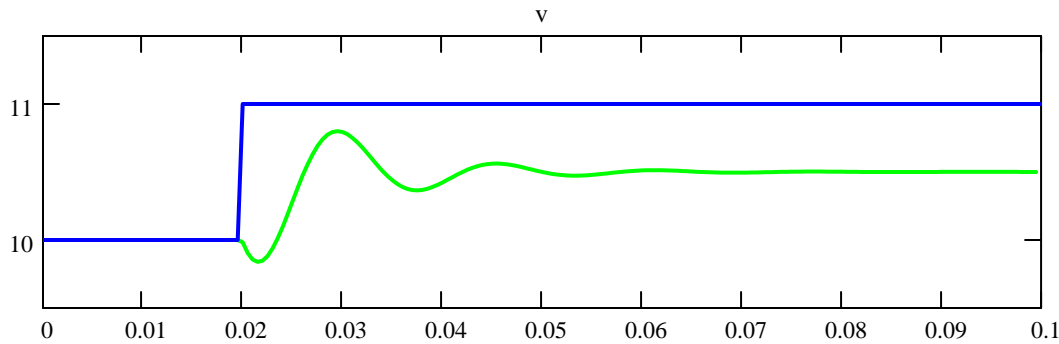


L.C.

$$\Delta v_d(t) := 1 \Phi(t - 0.02) \quad \Delta e(t) := 0 \quad k_c := 0.5$$

$$D(t, \Delta x) := A_n \cdot (\Delta x_1 \ \Delta x_2)^T + b_n \cdot (k_c \cdot k_a) \cdot (\Delta v_d(t) - \Delta x_1) + e_n \cdot \Delta e(t) \quad CI := (0 \ 0)^T \quad Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$$

Oscila eternamente.



# Comportamiento Transitorio de Sistemas de 2<sup>do</sup> Orden Continuos

**Problema** Sistemas de Primer con un polo y/o cero extra.

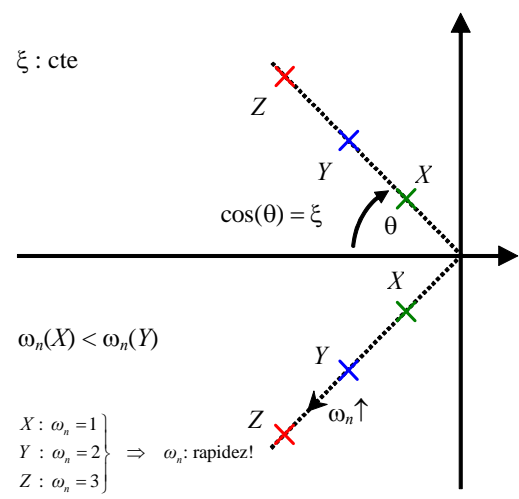
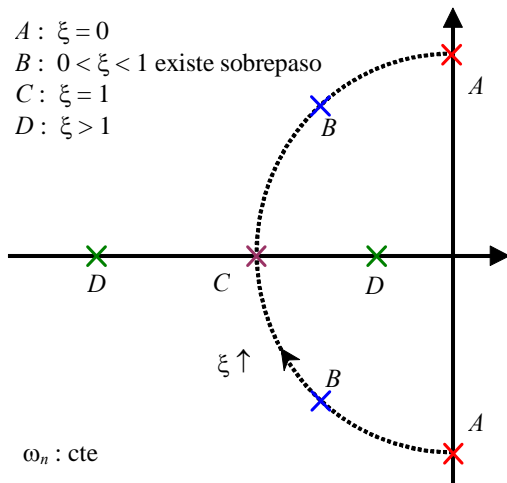
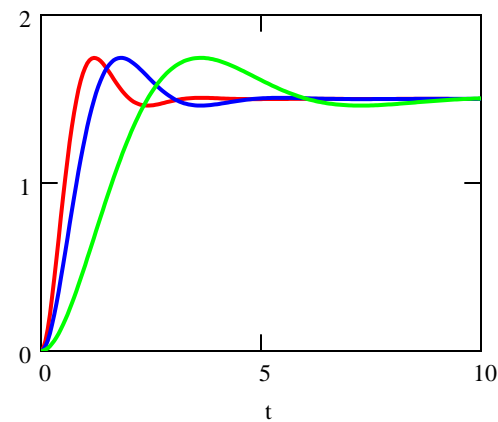
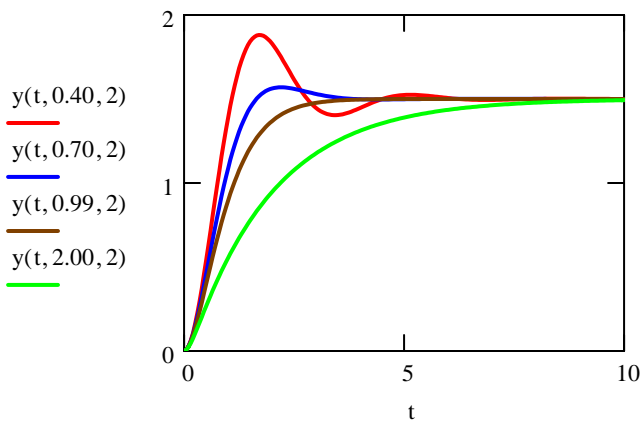
**Parámetros**  $k_p := 1.5$     $n_f := 200$     $t_f := 10$     $t := 0, \frac{t_f}{n_f} \dots t_f$

## Sistema de Segundo Orden

$$g(s) = k_p \cdot \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$$

$$y(s) = k_p \cdot \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2} \cdot \frac{1}{s}$$

$$y(t, \xi, \omega_n) := k_p \cdot \left( 1 - \frac{1}{\sqrt{1 - \xi^2}} \cdot e^{-\xi \cdot \omega_n \cdot t} \cdot \sin\left(\omega_n \cdot \sqrt{1 - \xi^2} \cdot t + \arccos(\xi)\right) \right) \cdot \Phi(t)$$

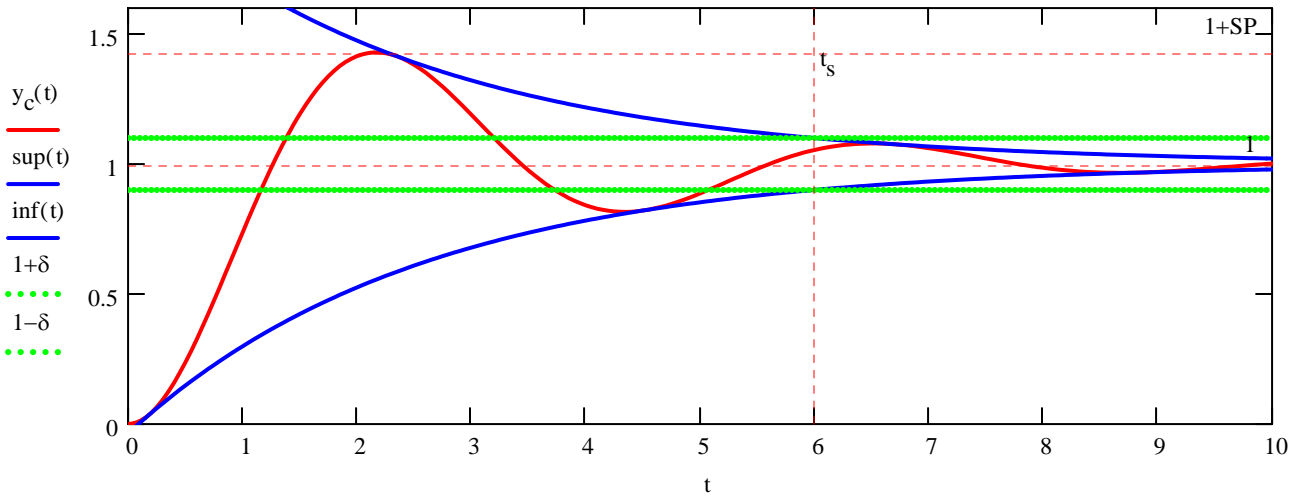


# Magnitudes Características en Sistemas de Segundo Orden

**Problema** Relacionar las características en el tiempo con los parámetros del sistema.

$$\xi := 0.26 \quad \omega_n := 1.5 \quad \delta := 0.10 \quad k_p := 1 \quad y_c(t) := k_p \cdot \left( 1 - \frac{1}{\sqrt{1-\xi^2}} \cdot e^{-\xi \cdot \omega_n \cdot t} \cdot \sin\left(\omega_n \cdot \sqrt{1-\xi^2} \cdot t + \arccos(\xi)\right) \right) \cdot \Phi(t)$$

$$\text{sup}(t) := 1 + \frac{e^{-\xi \cdot \omega_n \cdot t}}{\sqrt{1-\xi^2}} \quad \text{inf}(t) := 1 - \frac{e^{-\xi \cdot \omega_n \cdot t}}{\sqrt{1-\xi^2}} \quad t_s := 6 \quad \text{SP} := 0.43$$



**El sobrepaso, SP.**

$$\text{SP}(\xi) := e^{\frac{-\pi \cdot \xi}{\sqrt{1-\xi^2}}}$$

$$\xi := 0, 0.0999 \dots 1$$

**El tiempo de asentamiento,  $t_s$ .**

Aproximado :

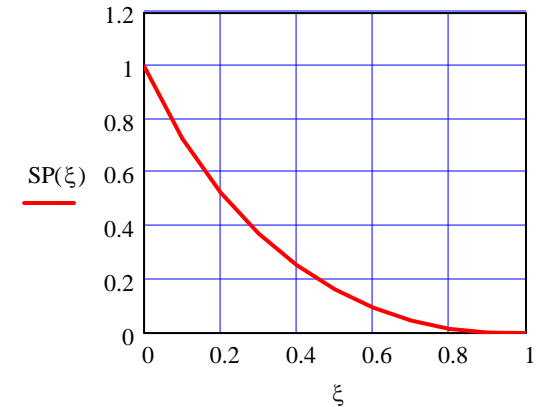
$$\delta = e^{-\xi \cdot \omega_n \cdot t_s} \quad \omega_n = \ln\left(\frac{1}{\delta}\right) \cdot \frac{1}{\xi \cdot t_s} \quad \xi \cdot \omega_n = \ln\left(\frac{1}{\delta}\right) \cdot \frac{1}{t_s}$$

$$\xi_0 := \frac{1}{\omega_n \cdot t_s} \cdot \ln\left(\frac{1}{\delta}\right) \quad \xi_0 = 0.256$$

Exacto :

$$\delta = \frac{e^{-\xi \cdot \omega_n \cdot t_s}}{\sqrt{1-\xi^2}} \quad \omega_n = \ln\left(\frac{1}{\delta \cdot \sqrt{1-\xi^2}}\right) \cdot \frac{1}{\xi \cdot t_s} \quad \xi \cdot \omega_n = \ln\left(\frac{1}{\delta \cdot \sqrt{1-\xi^2}}\right) \cdot \frac{1}{t_s} \quad \xi := 0.26$$

$$\xi_0 := \frac{1}{\omega_n \cdot t_s} \cdot \ln\left(\frac{1}{\delta \cdot \sqrt{1-\xi^2}}\right) \quad \xi_0 = 0.26$$



# Comportamiento Transitorio de Sistemas de 2<sup>do</sup> Orden Discretos

**Problema** Revisar las respuestas de sistemas de segundo orden discreto en función de sus parámetros.

**Parámetros**  $k_p := 1.5$     $n_f := 200$     $t_f := 12$     $t := 0, \frac{t_f}{n_f} .. t_f$     $T := 0.5$     $\xi := 0.26$     $\omega_n := 2$

Función de Transferencia Equivalente de Segundo Orden Estándar

$$h(s) = k_p \cdot \left( 1 - \frac{\exp(-\xi \cdot \omega_n \cdot T)}{\sqrt{1 - \xi^2}} \cdot \sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T + \text{acos}(\xi)) \right) \cdot \frac{z - \frac{\sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T - \text{acos}(\xi)) + \sqrt{1 - \xi^2} \cdot \exp(-\xi \cdot \omega_n \cdot T)}{\sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T + \text{acos}(\xi)) - \sqrt{1 - \xi^2} \cdot \exp(\xi \cdot \omega_n \cdot T)}}{z^2 - 2 \cdot z \cdot \exp(-\xi \cdot \omega_n \cdot T) \cdot \cos(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T) + \exp(-2 \cdot \xi \cdot \omega_n \cdot T)}$$

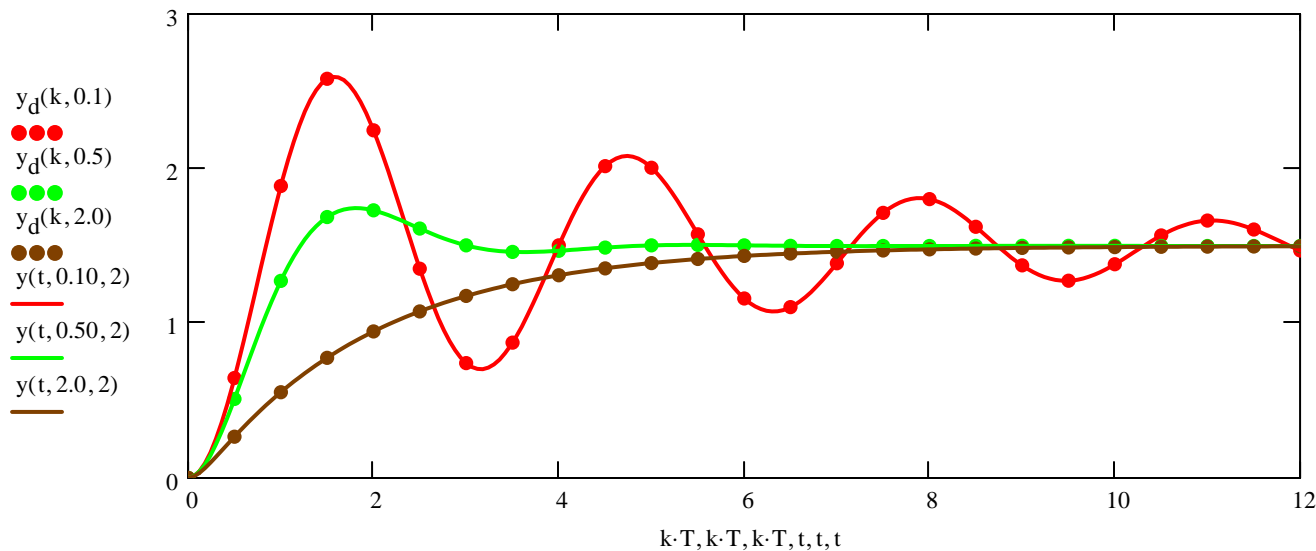
Parámetros

$$A_d(\xi) := \begin{pmatrix} 0 & -\exp(-2 \cdot \xi \cdot \omega_n \cdot T) \\ 1 & 2 \cdot \exp(-\xi \cdot \omega_n \cdot T) \cdot \cos(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T) \end{pmatrix} \quad b_d(\xi) := \begin{pmatrix} \frac{\sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T - \text{acos}(\xi)) + \sqrt{1 - \xi^2} \cdot \exp(-\xi \cdot \omega_n \cdot T)}{\sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T + \text{acos}(\xi)) - \sqrt{1 - \xi^2} \cdot \exp(\xi \cdot \omega_n \cdot T)} \\ 1 \end{pmatrix}$$

Simulación

$$k := 0, 1 .. \frac{t_f}{T}$$

$$x_o := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad x_d(k, \xi) := \text{if} \left( k = 0, x_o, A_d(\xi)^k \cdot x_o + \sum_{j=0}^{k-1} A_d(\xi)^{k-j-1} \cdot b_d(\xi) \right) \quad y_d(k, \xi) := x_d(k, \xi)_2 \cdot \left[ k_p \cdot \left( 1 - \frac{\exp(-\xi \cdot \omega_n \cdot T)}{\sqrt{1 - \xi^2}} \cdot \sin(\omega_n \cdot \sqrt{1 - \xi^2} \cdot T + \text{acos}(\xi)) \right) \right]$$



# Valores Característicos en Función de la Frecuencia

**Problema** Graficar el valor máximo  $M_p$ , la frecuencia a la cual se produce el máximo  $\omega_p$  y el ancho de banda BW como función del factor de amortiguamiento.

**Parámetros** 
$$h(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$$

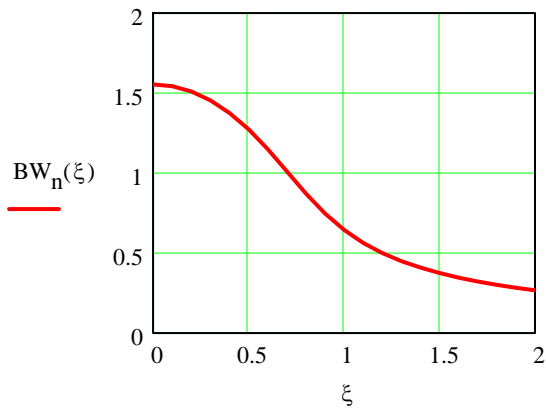
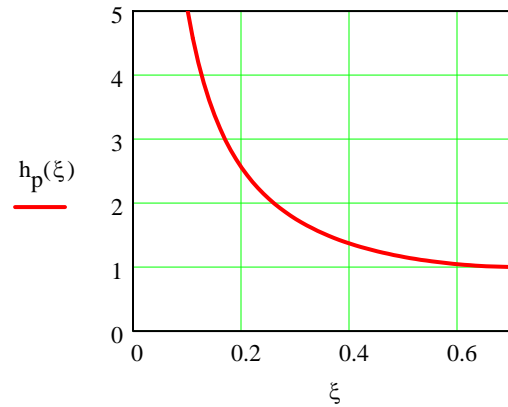
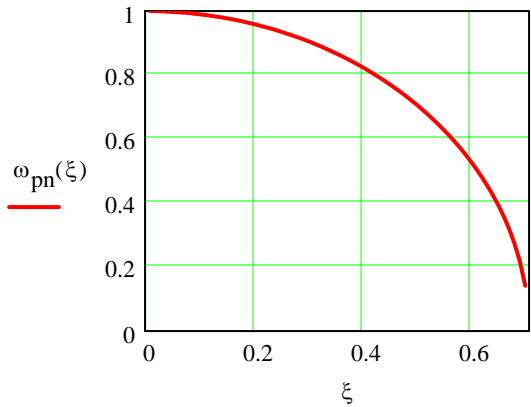
Para la función de segundo orden, los gráficos son:

$\omega_{pn}(\xi) := \sqrt{1 - 2 \cdot \xi^2}$  peak frequency

$h_p(\xi) := \frac{1}{2 \cdot \xi \cdot \sqrt{1 - \xi^2}}$  peak magnitude

$BW_n(\xi) := \sqrt{1 - 2 \cdot \xi^2 + \sqrt{4 \cdot \xi^4 - 4 \cdot \xi^2 + 2}}$  band width

$\xi := 0.01, 0.02 \dots \frac{\sqrt{2}}{2}$



# Reducción de Orden de la F. de T.

**Problema** Ilustrar los casos y métodos bajo los cuales el orden de una F. de T. puede ser reducido para simplificar su análisis.

**Parametros**

$$g(s) := \frac{1000}{(s + 6) \cdot (s + 1 + 2j) \cdot (s + 1 - 2j)} \qquad g(s) := \frac{1000}{(s + 6) \cdot (s^2 + 2s + 5)}$$

**El Bode** El Bode de la Función de Transferencia es:

$$f_{\min} := 10^{-2} \quad f_{\max} := 10^1 \quad n_{\max} := 250 \quad \text{ratio} := \log\left(\frac{f_{\max}}{f_{\min}}\right) \cdot \frac{1}{n_{\max}}$$

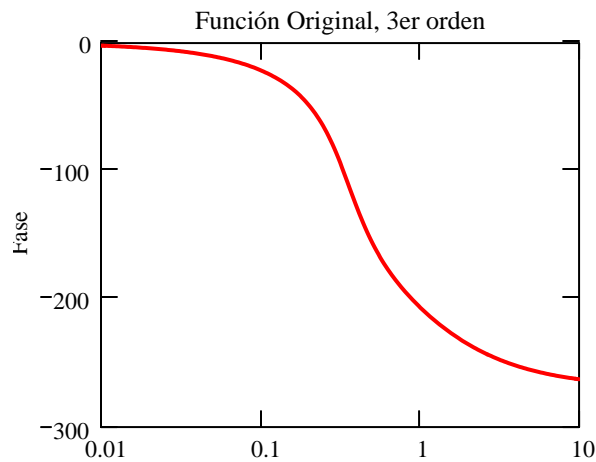
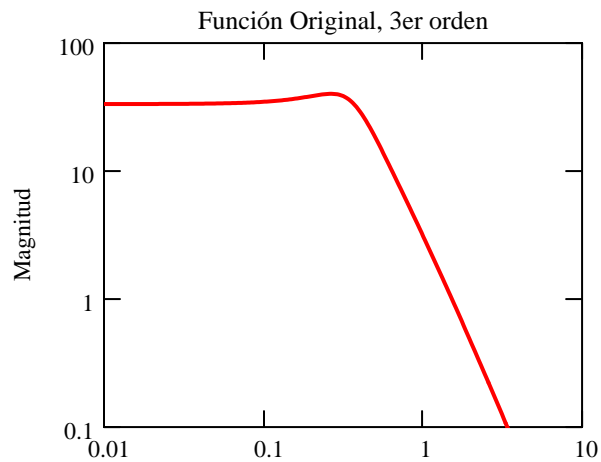
$$n := 0 .. n_{\max}$$

$$f_{\text{rec}}(n) := f_{\min} \cdot 10^{n \cdot \text{ratio}} \qquad w(n) := 2 \cdot \pi \cdot f_{\text{rec}}(n) \qquad s(n) := w(n) \cdot j$$

$$M_O(n) := |g(s(n))|$$

$$P_O(n) := \frac{180}{\pi} \cdot \arg(g(s(n)))$$

$$F_O(n) := \text{if}(P_O(n) > 0, P_O(n) - 360, P_O(n))$$



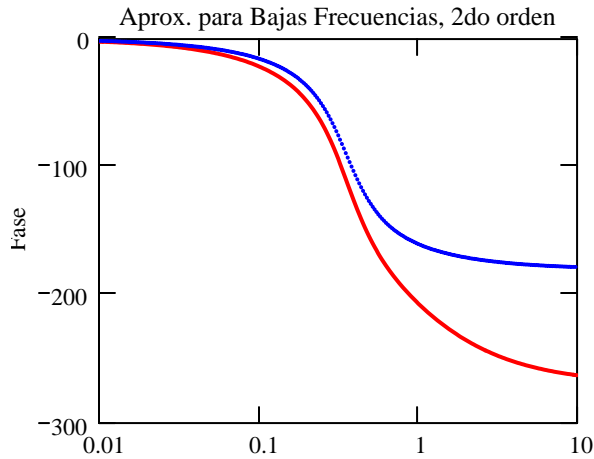
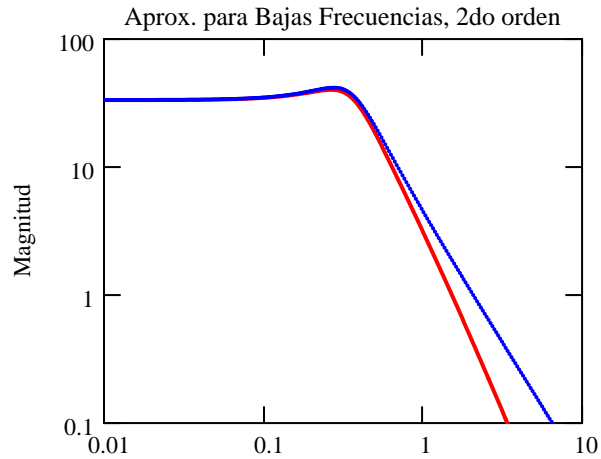
Aproximación a bajas frecuencias.

$$g(s) := \frac{1000}{6 \cdot (s^2 + 2 \cdot s + 5)}$$

$$M_b(n) := |g(s(n))|$$

$$P_b(n) := \frac{180}{\pi} \cdot \arg(g(s(n)))$$

$$F_b(n) := \text{if}(P_b(n) > 0, P_b(n) - 360, P_b(n))$$



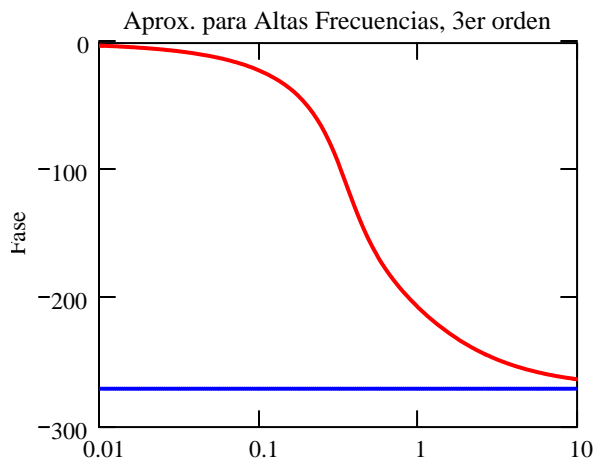
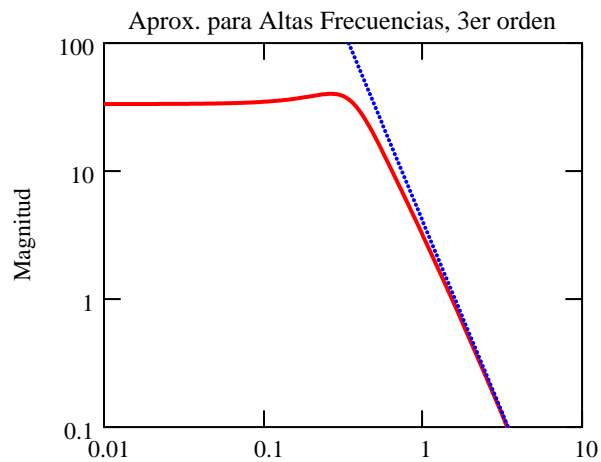
Aproximación a altas frecuencias.

$$g(s) := \frac{1000}{s^3}$$

$$M_a(n) := |g(s(n))|$$

$$P_a(n) := \frac{180}{\pi} \cdot \arg(g(s(n)))$$

$$F_a(n) := \text{if}(P_a(n) > 0, P_a(n) - 360, P_a(n))$$



La función original:

$$g(s) := \frac{60}{(s + 1) \cdot (s + 2) \cdot (s + 3)}$$

$$g(s) := \frac{60}{s^3 + 6 \cdot s^2 + 11 \cdot s + 6}$$

$$M_O(n) := |g(s(n))|$$

$$P_O(n) := \frac{180}{\pi} \cdot \arg(g(s(n)))$$

$$F_O(n) := \text{if}(P_O(n) > 0, P_O(n) - 360, P_O(n))$$

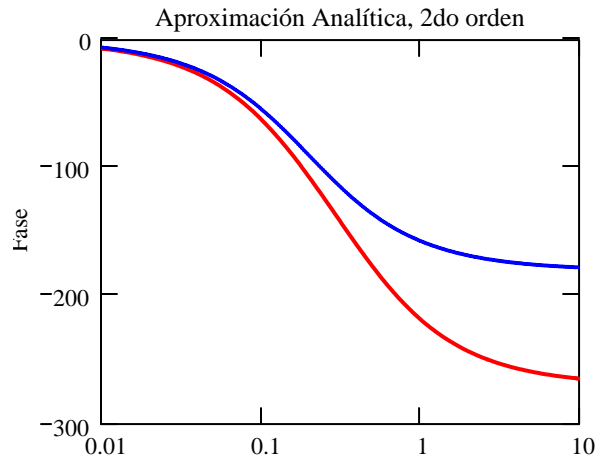
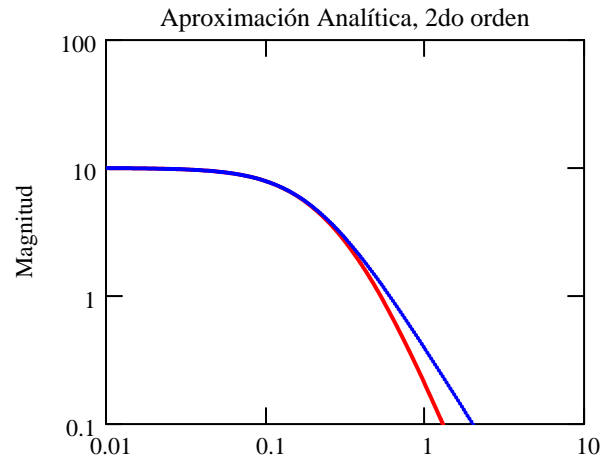
La función resultante:

$$g(s) := \frac{16}{s^2 + 2.584 \cdot s + 1.60}$$

$$M_a(n) := |g(s(n))|$$

$$P_a(n) := \frac{180}{\pi} \cdot \arg(g(s(n)))$$

$$F_a(n) := \text{if}(P_a(n) > 0, P_a(n) - 360, P_a(n))$$



**Problema Reducir el Orden del Sistema de Levitación magnética.**

**Parámetros**

$$\begin{aligned}
 R &:= 1 & L &:= 50 \cdot 10^{-3} & g &:= 9.8 \\
 m &:= 0.250 & k_i &:= 3 \cdot 10^{-3} & a &:= 0.02 \\
 K &:= 24.5 & l_1 &:= 0.5 & d &:= 1.5 & l_0 &:= 0.3
 \end{aligned}$$

**Modelo Total.**

$$\frac{d}{dt}i = \frac{e}{L} - \frac{R}{L} \cdot i \quad \frac{d}{dt}x = v \quad \frac{d}{dt}v = -g + \frac{k_i}{m} \cdot \frac{i^2}{l_1 - x + a} + \frac{K}{m} \cdot (l_0 - x) - \frac{d}{m} \cdot v$$

**Punto de Operación y Entradas.**

la corriente  $i_f$  para tener la bola a 30 cm desde el piso en  $t = 0$  es,

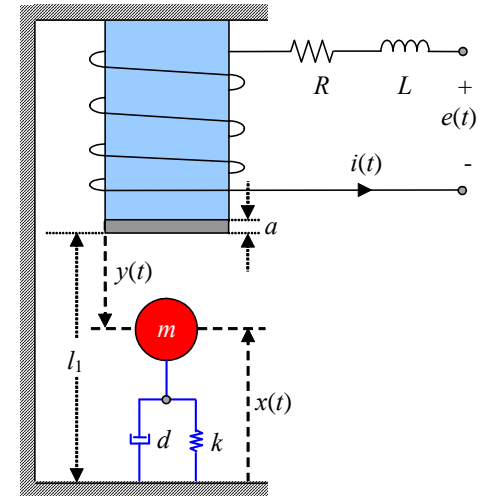
$$x_f := \frac{30}{100} \quad i_f := \frac{1}{k_i} \cdot \sqrt{k_i \cdot \left( g \cdot m \cdot l_1 - g \cdot m \cdot x_f + g \cdot m \cdot a + K \cdot x_f \cdot l_1 - K \cdot l_0 \cdot l_1 + K \cdot l_0 \cdot x_f - K \cdot l_0 \cdot a - K \cdot x_f^2 + K \cdot x_f \cdot a \right)}$$

por lo que la tensión  $e_f$  a aplicar es,  $e_f := i_f R$   $e_f = 13.404$  punto de operación es,  $i_0 := i_f$   $x_0 := x_f$   $v_0 := 0$

$$A := \begin{bmatrix} \frac{-R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ 2 \cdot \frac{k_i}{m} \cdot \frac{i_0}{l_1 - x_0 + a} & \frac{k_i}{m} \cdot \frac{i_0^2}{(l_1 - x_0 + a)^2} & -\frac{K}{m} & -\frac{d}{m} \end{bmatrix} \quad b := \begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

**Variables de Estado**

$$x_1 = i_a \quad x_2 = x \quad x_3 = \frac{d}{dt}x = v$$



**Modelo Lineal**

$$\begin{aligned}
 c &:= (0 \quad 1 \quad 0) \\
 \text{eigenvals}(A) &= \begin{pmatrix} -3 + 6.667i \\ -3 - 6.667i \\ -20 \end{pmatrix} \\
 e_0 &:= e_f
 \end{aligned}$$

**F. de T. Total**

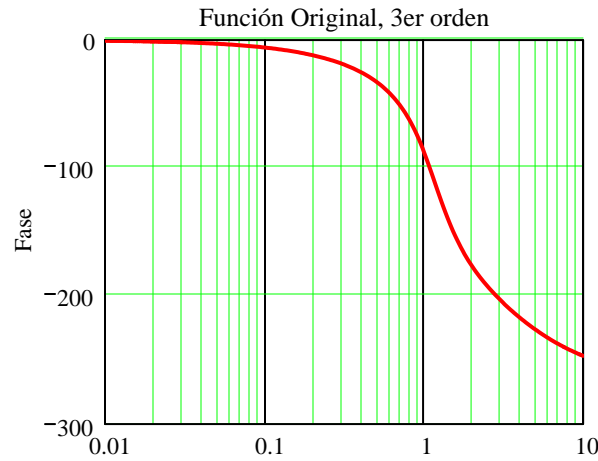
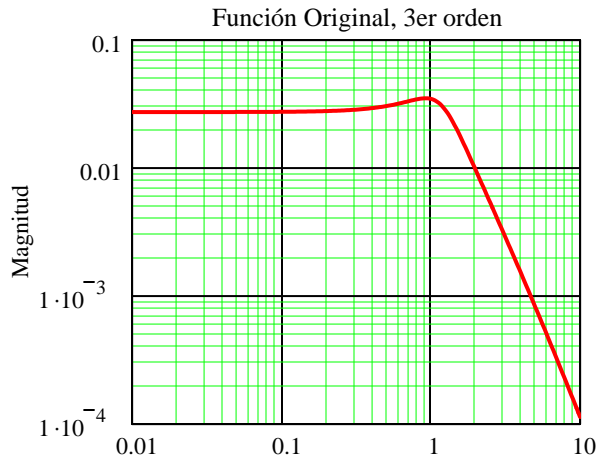
$$g_0(s) := c \cdot (s \cdot \text{identity}(3) - A)^{-1} \cdot b$$

**El Bode**

El Bode de la F. de T. es:  $f_{\min} := 10^{-2}$   $f_{\max} := 10^1$   $n_{\max} := 250$   $\text{ratio} := \log\left(\frac{f_{\max}}{f_{\min}}\right) \cdot \frac{1}{n_{\max}}$   $n := 0 \dots n_{\max}$

$$\text{frec}(n) := f_{\min} \cdot 10^{n \cdot \text{ratio}} \quad w(n) := 2 \cdot \pi \cdot \text{frec}(n) \quad s(n) := w(n) \cdot j$$

$$M_O(n) := |g_0(s(n))| \quad P_O(n) := \frac{180}{\pi} \cdot \arg(g_0(s(n))) \quad F_O(n) := \text{if}(P_O(n) > 0, P_O(n) - 360, P_O(n))$$



**Modelo Reducido.**

$$e = R \cdot i$$

$$\frac{d}{dt}x = v \quad \frac{d}{dt}v = -g + \frac{k_i}{m} \cdot \frac{\left(\frac{e}{R}\right)^2}{l_1 - x + a} + \frac{K}{m} \cdot (l_0 - x) - \frac{d}{m} \cdot v$$

**Variables de Estado**

$$x_1 = x \quad x_2 = \frac{d}{dt}x = v$$

**Punto de Operación y Entradas.**

la corriente if1 para tener la bola a 30 cm desde el piso en  $t = 0$  es,

$$x_f := \frac{30}{100} \quad e_f := \frac{R}{k_i} \cdot \sqrt{\left[ k_i \cdot \left( g \cdot m \cdot l_1 - g \cdot m \cdot x_f + g \cdot m \cdot a + K \cdot x_f \cdot l_1 - K \cdot l_0 \cdot l_1 + K \cdot l_0 \cdot x_f - K \cdot l_0 \cdot a - K \cdot x_f^2 + K \cdot x_f \cdot a \right) \right]}$$

por lo que la tensión ef a aplicar es,  $e_f = 13.404$  punto de operación es,  $x_0 := x_f \quad v_0 := 0 \quad e_0 := e_f$

**Modelo Lineal**

$$A_r := \begin{bmatrix} 0 & 1 \\ \frac{k_i}{m} \cdot \frac{\left(\frac{e_0}{R}\right)^2}{(l_1 - x_0 + a)^2} - \frac{K}{m} & -\frac{d}{m} \end{bmatrix}$$

$$b_r := \begin{bmatrix} 0 \\ \frac{k_i}{m} \cdot \frac{2}{R} \cdot \left(\frac{e_0}{R}\right) \end{bmatrix}$$

$$c_r := (1 \ 0)$$

**F. de T. Reducida**

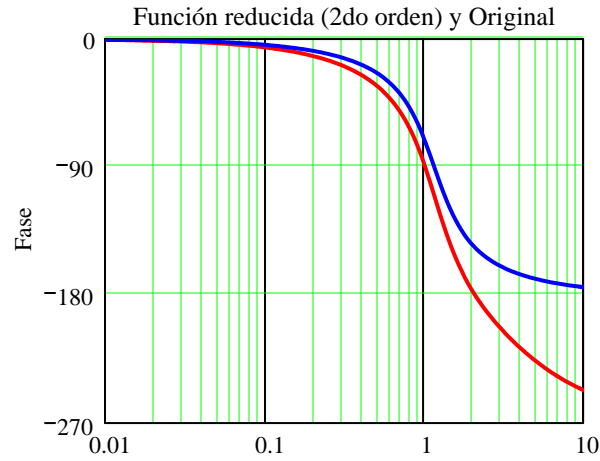
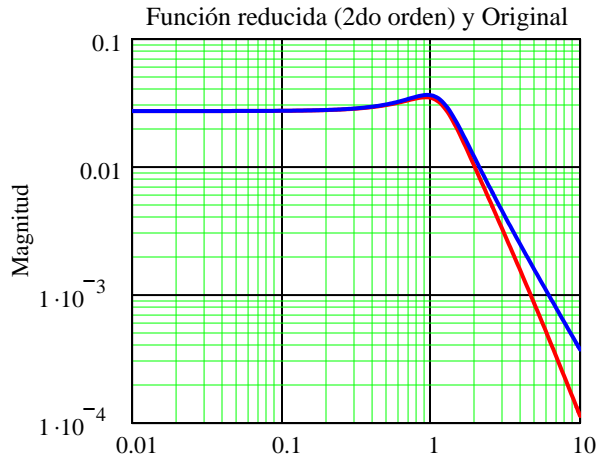
$$g_r(s) := c_r \cdot (s \cdot \text{identity}(2) - A_r)^{-1} \cdot b_r$$

$$\text{eigenvals}(A_r) = \begin{pmatrix} -3 + 6.667i \\ -3 - 6.667i \end{pmatrix}$$

$$M_r(n) := |g_r(s(n))|$$

$$P_r(n) := \frac{180}{\pi} \cdot \arg(g_r(s(n)))$$

$$F_r(n) := \text{if}(P_r(n) > 0, P_r(n) - 360, P_r(n))$$



**Simulación en L.A. Caso Total**

$$\Delta e_o := \frac{0.04}{g_o(0)}$$

$$\Delta e_o = 1.462$$

$$\Delta e(t) := \Delta e_o \cdot \Phi(t - 0.5)$$

$$-A^{-1} \cdot b \cdot \Delta e_o = \begin{pmatrix} 1.462 \\ 0.04 \\ 0 \end{pmatrix}$$

$$t_f := 3$$

$$l_f := 500$$

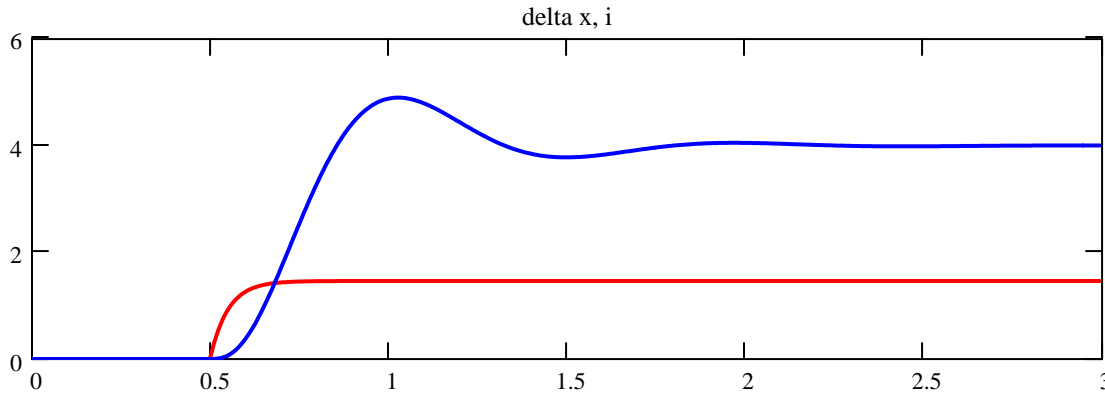
$$l := 0 .. l_f$$

$$t := 0, \frac{t_f}{l_f} .. t_f$$

$$D(t, x) := A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^T + b \cdot \Delta e(t)$$

$$CI := (0 \ 0 \ 0)^T$$

$$Z_c := \text{rkfixed}(CI, 0, t_f, l_f, D)$$



**Simulación  
en L.A.  
Caso Orden  
Reducido**

$$\Delta e_0 := \frac{0.04}{g_r(0)}$$

$$\Delta e_0 = 1.462$$

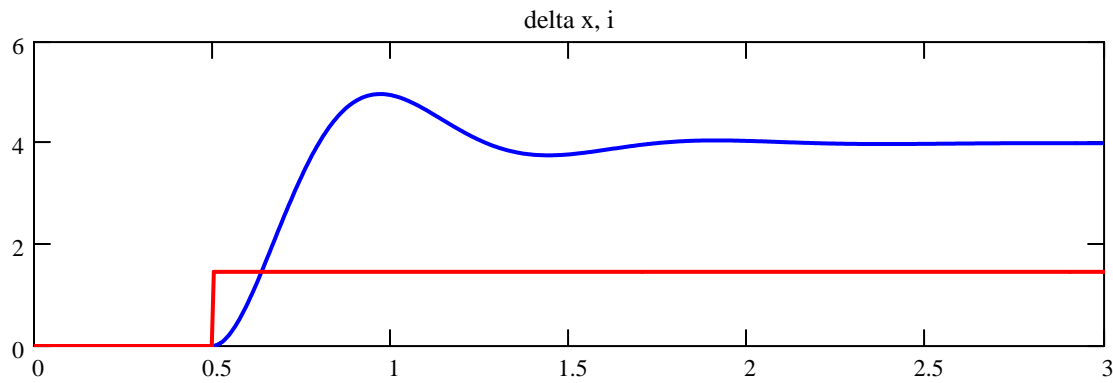
$$\Delta e(t) := \Delta e_0 \cdot \Phi(t - 0.5)$$

$$-A_r^{-1} \cdot b_r \cdot \Delta e_0 = \begin{pmatrix} 0.04 \\ 0 \end{pmatrix}$$

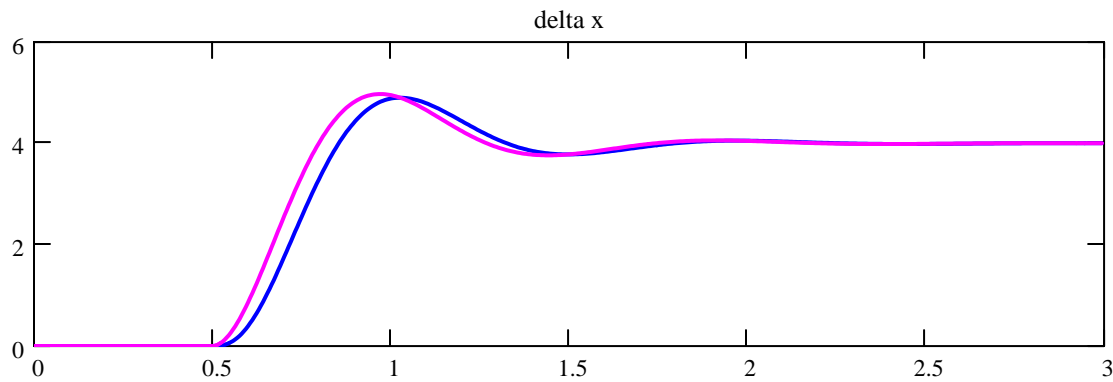
$$D_r(t, x) := A_r \cdot \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T + b_r \cdot \Delta e(t)$$

$$CI := (0 \ 0)^T$$

$$Z_r := \text{rkfixed}(CI, 0, t_f, l_f, D_r)$$



**Compara-  
ción**



# Retardos en Sistemas Realimentados.

**Problema** Ilustrar el efecto del retardo en sistemas realimentados.

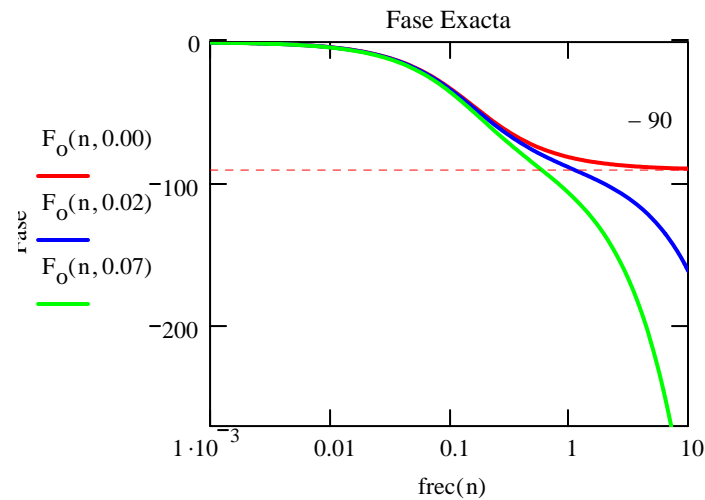
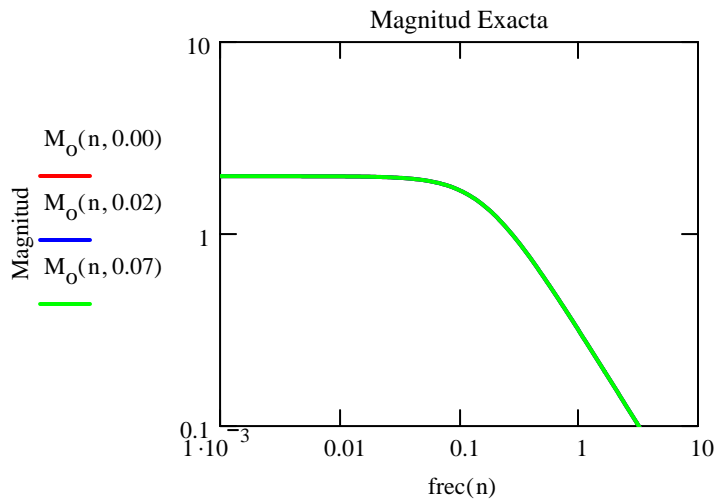
$$f_{\min} := 10^{-3} \quad f_{\max} := 10^1 \quad n_{\max} := 250 \quad \text{ratio} := \log\left(\frac{f_{\max}}{f_{\min}}\right) \cdot \frac{1}{n_{\max}}$$

**Parametros**  $k_p := 2 \quad \tau := 1$

$$n := 0 .. n_{\max} \quad \text{frec}(n) := f_{\min} \cdot 10^{n \cdot \text{ratio}} \quad s(n) := 2 \cdot \pi \cdot \text{frec}(n) \cdot j$$

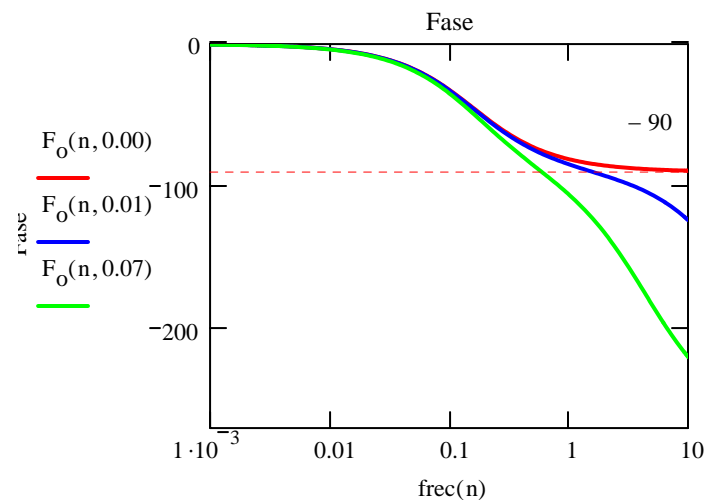
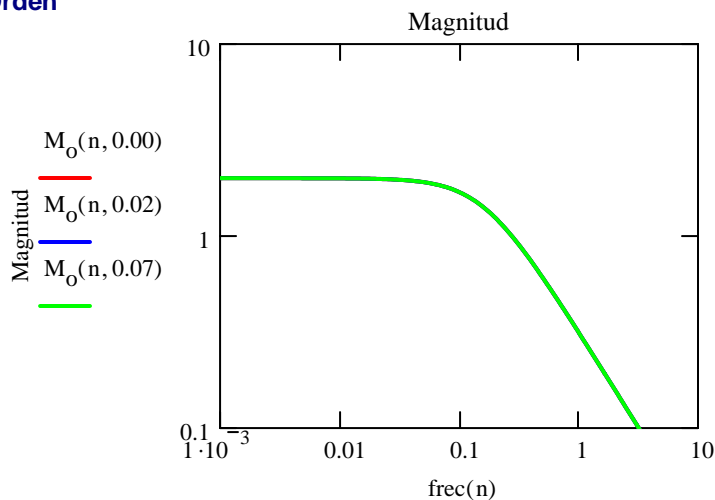
**El Bode Exacto**

$$g(s, t_r) := k_p \cdot \frac{1}{\tau \cdot s + 1} \cdot e^{-t_r \cdot s} \quad M_o(n, t_r) := |g(s(n), t_r)| \quad P_o(n, t_r) := \frac{180}{\pi} \cdot \arg(g(s(n), t_r)) \quad F_o(n, t_r) := \text{if}(P_o(n, t_r) > 0, P_o(n, t_r) - 360, P_o(n, t_r))$$



**El Bode Aproximación de Primer Orden**

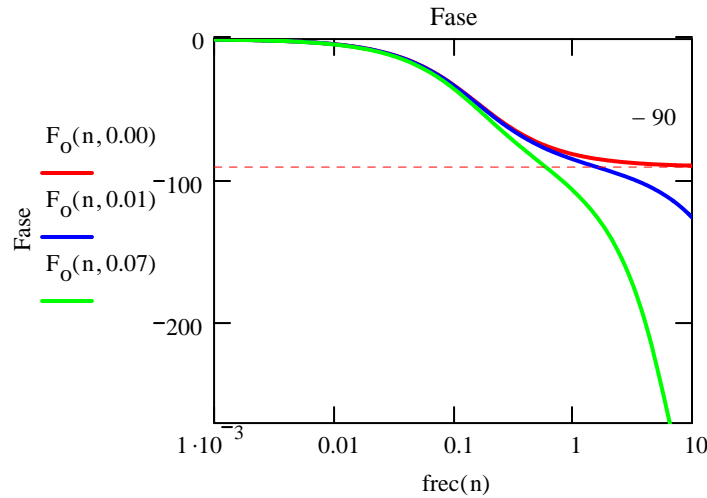
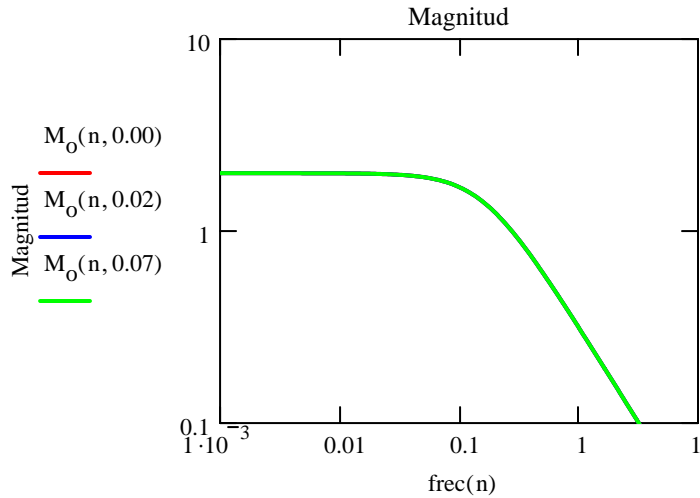
$$g(s, t_r) := k_p \cdot \frac{1}{\tau \cdot s + 1} \cdot \frac{1 - \frac{t_r}{2} \cdot s}{1 + \frac{t_r}{2} \cdot s} \quad M_o(n, t_r) := |g(s(n), t_r)| \quad P_o(n, t_r) := \frac{180}{\pi} \cdot \arg(g(s(n), t_r)) \quad F_o(n, t_r) := \text{if}(P_o(n, t_r) > 0, P_o(n, t_r) - 360, P_o(n, t_r))$$



**El Bode**  
**Aproximación**  
**de Segundo**  
**Orden**

$$g(s, t_r) := k_p \cdot \frac{1}{\tau \cdot s + 1} \cdot \frac{1 - \frac{t_r}{2} \cdot s + \frac{1}{2} \cdot \left(\frac{t_r}{2}\right)^2 \cdot s^2}{1 + \frac{t_r}{2} \cdot s + \frac{1}{2} \cdot \left(\frac{t_r}{2}\right)^2 \cdot s^2}$$

$$M_O(n, t_r) := |g(s(n), t_r)| \quad P_O(n, t_r) := \frac{180}{\pi} \cdot \arg(g(s(n), t_r)) \quad F_O(n, t_r) := \text{if}(P_O(n, t_r) > 0, P_O(n, t_r) - 360, P_O(n, t_r))$$



**Respuestas**  
**en el Tiempo**

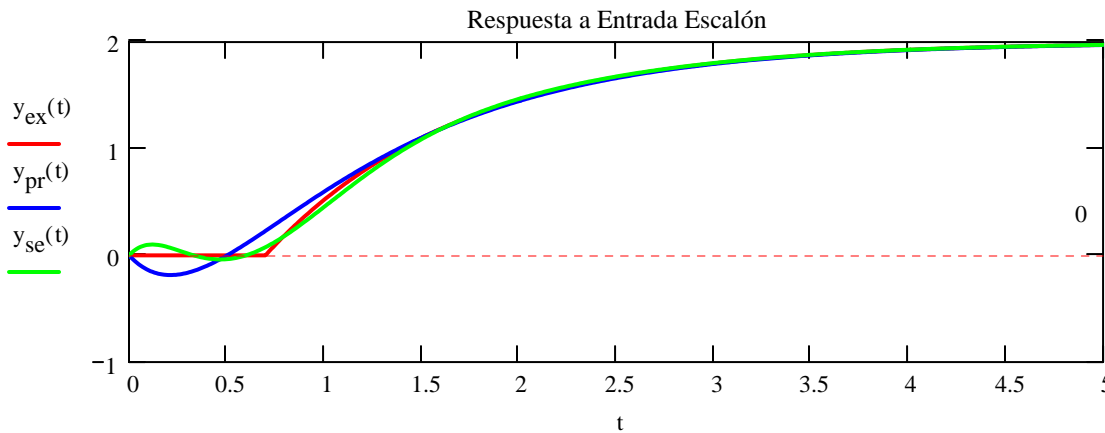
$$y_{ex}(t) := k_p \cdot \left[ -e^{\left[ \frac{-(t-t_r)}{\tau} \right]} + 1 \right] \cdot \Phi(t - t_r)$$

$$y_{pr}(t) := k_p \cdot \left[ \frac{t_r + 2 \cdot \tau}{t_r - 2 \cdot \tau} \cdot e^{\left( \frac{-t}{\tau} \right)} - 2 \cdot \frac{t_r}{t_r - 2 \cdot \tau} \cdot e^{\left( -2 \cdot \frac{t}{t_r} \right)} + 1 \right]$$

$$t_r := 0.7$$

$$t := 0, 0.01 \dots 5$$

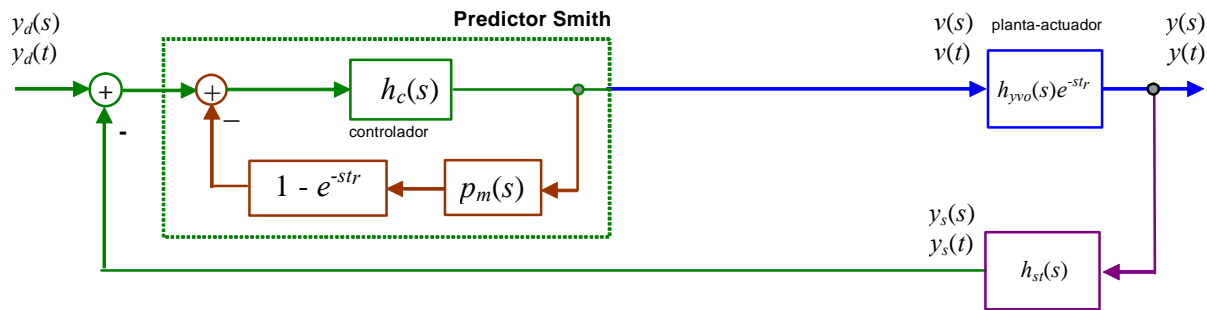
$$y_{se}(t) := \left[ \frac{8 \cdot \tau \cdot t_r \cdot \cos\left(2 \cdot \frac{t}{t_r}\right) - (4 \cdot t_r^2 - 8 \cdot \tau \cdot t_r) \cdot \sin\left(2 \cdot \frac{t}{t_r}\right)}{-4 \cdot \tau \cdot t_r + t_r^2 + 8 \cdot \tau^2} \cdot e^{\left(-2 \cdot \frac{t}{t_r}\right)} - \frac{4 \cdot \tau \cdot t_r + t_r^2 + 8 \cdot \tau^2}{-4 \cdot \tau \cdot t_r + t_r^2 + 8 \cdot \tau^2} \cdot e^{\left(\frac{-t}{\tau}\right)} - \frac{-8 \cdot \tau^2 - t_r^2 + 4 \cdot \tau \cdot t_r}{-4 \cdot \tau \cdot t_r + t_r^2 + 8 \cdot \tau^2} \right] \cdot k_p$$



# Predictor Smith

**Problema** Deducir las expresiones del Predictor Smith.

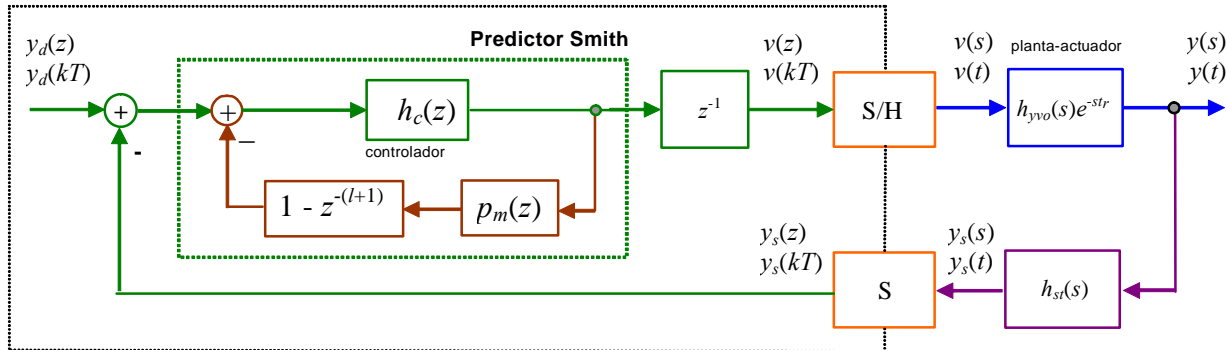
**Continuo**



$$\frac{v(s)}{e(s)} = \frac{h_c(s)}{1 + h_c(s)(1 - e^{-st_r})p_m(s)}$$

$$\begin{aligned} \frac{y(s)}{y_d(s)} &= \frac{\frac{h_c(s)}{1 + h_c(s)(1 - e^{-st_r})p_m(s)} h_{yvo}(s)e^{-st_r}}{1 + \frac{h_c(s)}{1 + h_c(s)(1 - e^{-st_r})p_m(s)} h_{yvo}(s)e^{-st_r} h_{st}(s)} = \frac{h_c(s)h_{yvo}(s)e^{-st_r}}{1 + h_c(s)(1 - e^{-st_r})p_m(s) + h_c(s)h_{yvo}(s)h_{st}(s)e^{-st_r}} \\ &= \frac{h_c(s)h_{yvo}(s)e^{-st_r}}{1 + h_c(s)p_m(s) - h_c(s)p_m(s)e^{-st_r} + h_c(s)h_{yvo}(s)h_{st}(s)e^{-st_r}} \end{aligned} \quad \frac{y(s)}{y_d(s)} = \frac{h_c(s)h_{yvo}(s)}{1 + h_c(s)h_{yvo}(s)h_{st}(s)} e^{-st_r}$$

Discreto



$$\frac{v(z)}{e(z)} = z^{-1} \frac{h_c(z)}{1 + h_c(z)(1 - z^{-(l+1)})p_m(z)}$$

$$\begin{aligned} \frac{y(z)}{y_d(z)} &= \frac{z^{-1} \frac{h_c(z)}{1 + h_c(z)(1 - z^{-(l+1)})p_m(z)} h_{yvo}(z) z^{-l}}{1 + z^{-1} \frac{h_c(z)}{1 + h_c(z)(1 - z^{-(l+1)})p_m(z)} h_{yvo}(z) z^{-l} h_{st}(z)} \\ &= \frac{z^{-1} h_c(z) h_{yvo}(z) z^{-l}}{1 + h_c(z)(1 - z^{-(l+1)})p_m(z) + z^{-1} h_c(z) h_{yvo}(z) h_{st}(z) z^{-l}} \\ &= \frac{h_c(z) h_{yvo}(z) z^{-(l+1)}}{1 + h_c(z)p_m(z) - h_c(z)p_m(z)z^{-(l+1)} + h_c(z)h_{yvo}(z)h_{st}(z)z^{-(l+1)}} \end{aligned}$$

$$\frac{y(z)}{y_d(z)} = \frac{h_c(z)h_{yvo}(z)}{1 + h_c(z)h_{yvo}(z)h_{st}(z)} z^{-(l+1)}$$

# Predictor Smith Continuo

**Problema** Ilustrar el Predictor Smith en un sistema con retardo.

**Parámetros**  $t_r := 1$        $t_{sd} := 4.5$        $SP := \frac{30}{100}$        $\delta := \frac{15}{100}$

## Sistema de Segundo Orden

$$h_{yv}(s) = \frac{k}{s \cdot (s + p)} \quad \text{en L.C.:} \quad \frac{k}{s \cdot (s + p) + k} = \frac{k}{s^2 + p \cdot s + k} = k_p \cdot \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$$

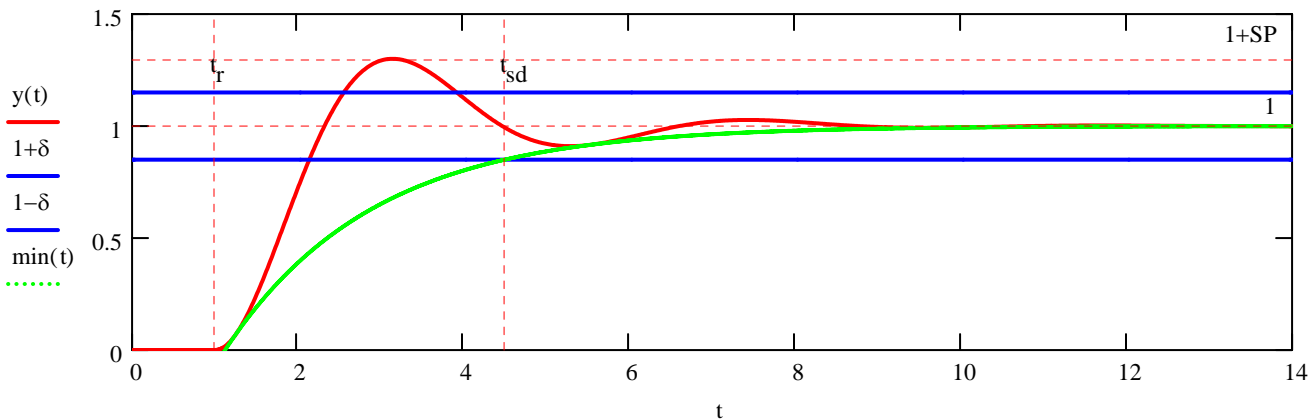
$$SP = e^{\frac{-\pi \cdot \xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi(SP) := \frac{-\ln(SP)}{\sqrt{\ln(SP)^2 + \pi^2}} \quad \xi := \xi(SP) \quad \xi = 0.358 \quad t_s := t_{sd} - t_r \quad t_s = 3.5$$

$$\delta = \frac{e^{-\xi \cdot \omega_n \cdot t_s}}{\sqrt{1-\xi^2}} \Rightarrow \omega_n(\xi) := \frac{-\ln(\delta \cdot \sqrt{1-\xi^2})}{\xi \cdot t_s} \quad \omega_n := \omega_n(\xi) \quad \omega_n = 1.569$$

$$k_p := 1 \quad k := \omega_n^2 \quad p := 2 \cdot \xi \cdot \omega_n \quad k = 2.463 \quad p = 1.123$$

$$n_f := 200 \quad t_f := 14 \quad t := 0, 0.01 \dots t_f \quad \min(t) := 1 - \frac{1}{\sqrt{1-\xi^2}} \cdot e^{-\xi \cdot \omega_n \cdot (t-t_r)}$$

$$y(t) := k_p \cdot \left[ 1 - \frac{1}{\sqrt{1-\xi^2}} \cdot e^{-\xi \cdot \omega_n \cdot (t-t_r)} \cdot \sin\left[\omega_n \cdot \sqrt{1-\xi^2} \cdot (t-t_r) + \arccos(\xi)\right] \right] \cdot \Phi(t-t_r)$$



# Predictor Smith Discreto (Estanque)

**Problema** Ilustrar la eficacia del control realimentado.

**Estanque** Parámetros.

$$f_{s0} := 0 \quad h_a(s) = e^{-t_r \cdot s} \quad A_e := 2.5 \quad h_d(t) := 2 \cdot \Phi(t) \quad t_r := 0.5$$

**Modelo Continuo**

$$\frac{d}{dt}h(t) = \frac{1}{A_e} \cdot (v(t - t_r) - f_s(t)) \quad A_t := 0 \quad b_t := \frac{1}{A_e} \quad e_t := -\frac{1}{A_e} \quad c_t := 1$$

**Modelo Discreto**

$$T := 0.25$$

$$A_k := \begin{pmatrix} 1 & \frac{T}{A_e} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad b_k := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad e_k := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad c_k := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^T$$

**Caso I** Entrada.

$$v(k) := \Phi(k \cdot T - 4 \cdot T) - \Phi(k \cdot T - 8 \cdot T)$$

**Simulación Discreto**

$$\xi_o := (2 \ 0 \ 0)^T \quad \xi(k) := \text{if} \left( k = 0, \xi_o, A_k^k \cdot \xi_o + \sum_{j=0}^{k-1} A_k^{k-j-1} \cdot b_k \cdot v(j) \right)$$

**Entrada Continua**

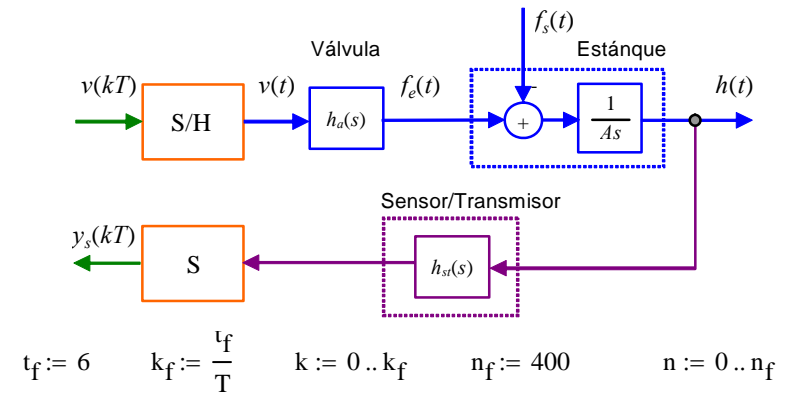
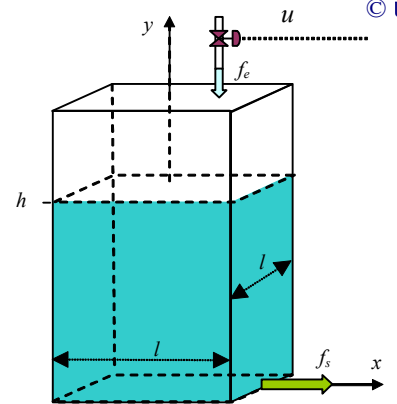
$$f_e(t) := v \left( \text{if} \left( \text{trunc} \left( \frac{t}{T} \right) < 0, 0, \text{trunc} \left( \frac{t}{T} \right) \right) \right)$$

**Simulación Continuo**

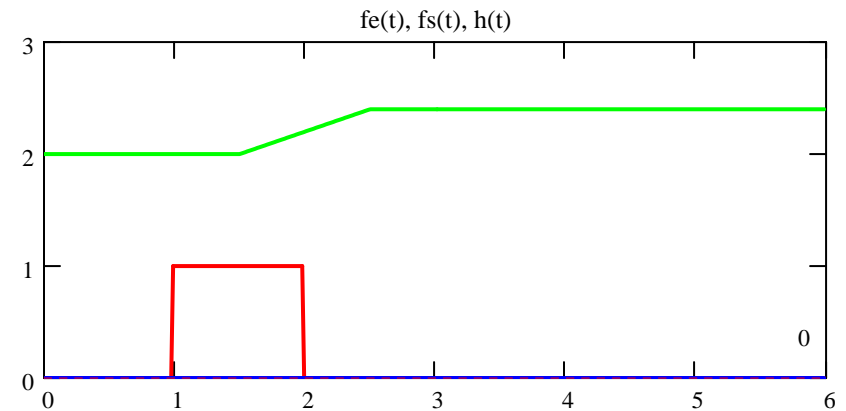
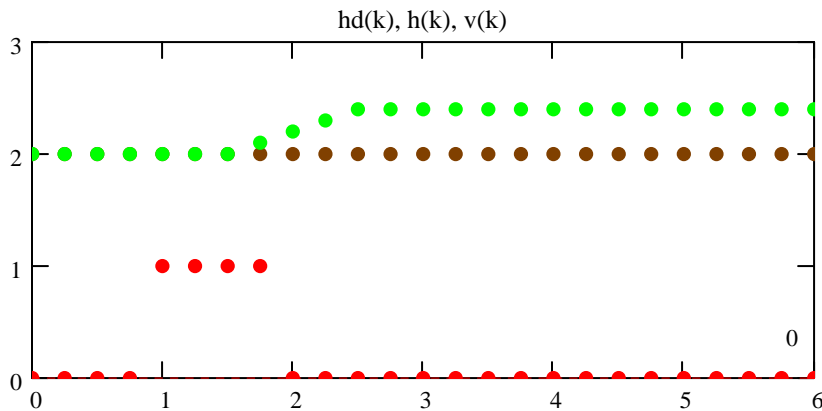
$$D(t, x) := A_t \cdot (x_1 \ x_1)^T + b_t \cdot f_e(t - t_r) \quad CI := (2 \ 0)^T$$

**Variable de Estado**

$$x_1 = h$$



$$Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$$



**Caso II** Controlador Discreto de Ganacia  $k_c$ . **Cambio en la referencia.**

Controlador Discreto

$$k_c := 4$$

$$A_{ck} := 0 \quad b_{ck} := 1 \quad c_{ck} := k_c \quad d_{ck} := 0$$

Sistema Resultante

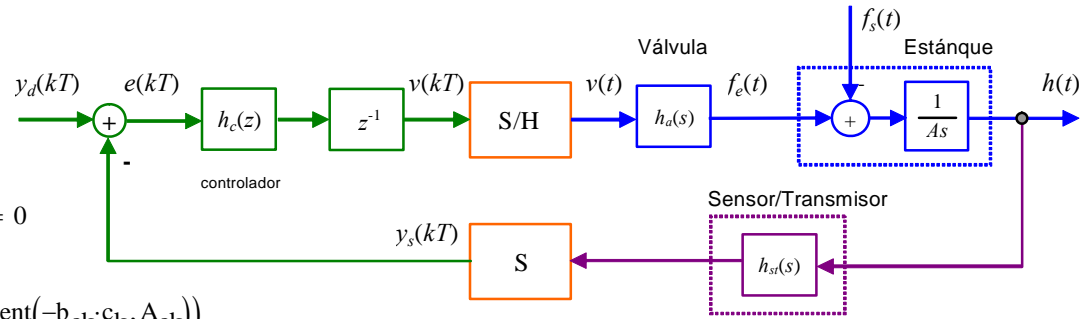
$$A_{Tk} := \text{stack}(\text{augment}(A_k, b_k \cdot c_{ck}), \text{augment}(-b_{ck} \cdot c_k, A_{ck}))$$

$$c_{Tk} := \text{augment}(c_k, c_{ck} \cdot 0) \quad e_{Tk} := \text{stack}(e_k, c_{ck} \cdot 0)$$

$$b_{Tk} := \text{stack}(b_k \cdot 0, b_{ck})$$

$$h_{Tk}(z) := c_{Tk} \cdot (z \cdot \text{identity}(4) - A_{Tk})^{-1} \cdot b_{Tk} \quad h_{Tk}(1) = 1$$

$$\text{eigenvals}(A_{Tk}) = \begin{pmatrix} 0.887 + 0.412i \\ 0.887 - 0.412i \\ -0.387 + 0.517i \\ -0.387 - 0.517i \end{pmatrix}$$



Entradas

$$h_d(t) := 2 \cdot \Phi(t) + \Phi(t - 1) \quad f_s(t) := f_{s0} \quad p_d(k) := f_s(k \cdot T)$$

Simulación Discreta

$$y_d(k) := h_d(k \cdot T)$$

$$\xi(k) := \text{if} \left( k = 0, \xi_0, A_{Tk}^k \cdot \xi_0 + \sum_{j=0}^{k-1} A_{Tk}^{k-j-1} \cdot b_{Tk} \cdot y_d(j) + \sum_{j=0}^{k-1} A_{Tk}^{k-j-1} \cdot e_{Tk} \cdot p_d(j) \right)$$

$$\xi_0 := (2 \ 0 \ 0 \ 0)^T$$

$$t_f := 6 \quad k_f := \frac{t_f}{T}$$

$$k := 0 \dots k_f \quad n_f := 400$$

$$n := 0 \dots n_f \quad t := 0, \frac{t_f}{n_f} \dots t_f$$

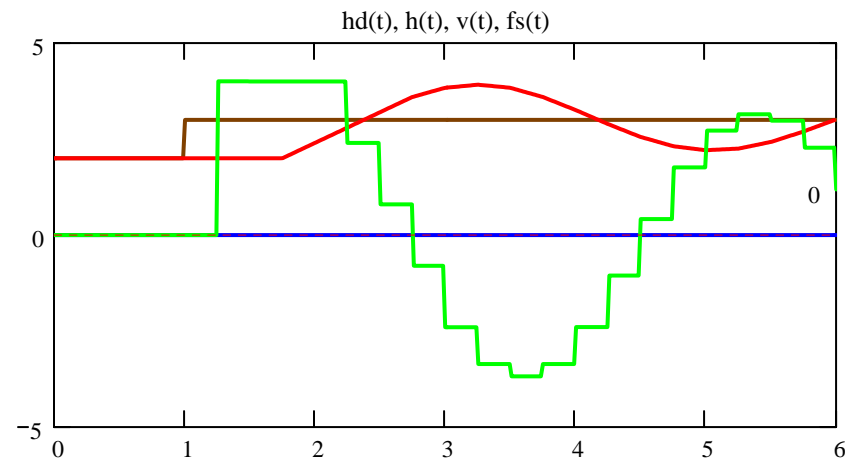
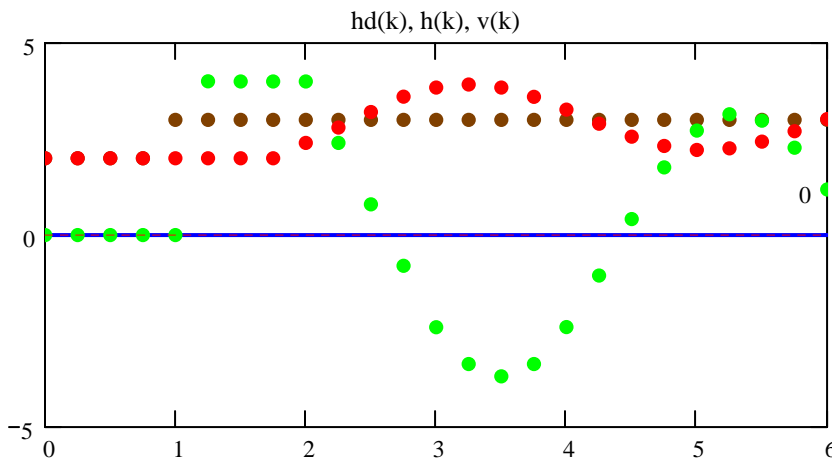
Entrada Continuo

$$v(k) := c_{ck} \cdot \xi(k) \cdot 4 \quad f_e(t) := c_{ck} \cdot \xi \left( \text{if} \left( \text{trunc} \left( \frac{t}{T} \right) < 0, 0, \text{trunc} \left( \frac{t}{T} \right) \right) \right) \cdot 4$$

Simulación Continua

$$D(t, x) := A_t \cdot x_1 + b_t \cdot f_e(t - t_f) + e_t \cdot f_s(t) \quad CI := 2$$

$$Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$$



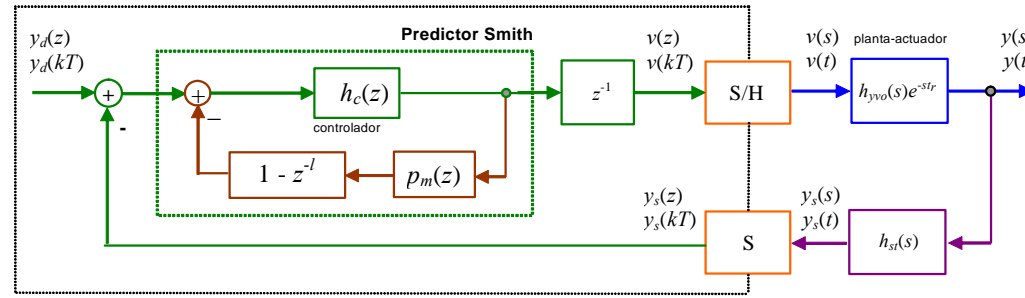
**Caso III**

**Controlador Discreto de Ganacia  $k_c$  y Predictor Smith para la planta.**

**Controlador Discreto**

$$k_c := 4$$

$$A_{ck} := \begin{pmatrix} 0 & 0 & 0 & k_c \cdot \frac{T}{A_e} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -k_c \cdot \frac{T}{A_e} \\ 0 & 0 & 1 & 1 \end{pmatrix}$$



$$b_{ck} := (0 \ 0 \ -k_c \ k_c)^T \quad c_{ck} := (0 \ 0 \ 0 \ 1) \quad d_{ck} := 0$$

**Sistema Resultante**

$$A_{Tk} := \text{stack}(\text{augment}(A_k, b_k \cdot c_{ck}), \text{augment}(-b_{ck} \cdot c_k, A_{ck}))$$

$$b_{Tk} := \text{stack}(b_k \cdot 0, b_{ck})$$

$$c_{Tk} := \text{augment}(c_k, c_{ck} \cdot 0) \quad e_{Tk} := \text{stack}(e_k, c_{ck}^T \cdot 0)$$

$$h_{Tk}(z) := c_{Tk} \cdot (z \cdot \text{identity}(4) - A_{Tk})^{-1} \cdot b_{Tk}$$

**Entradas**

$$h_d(t) := 2 \cdot \Phi(t) + \Phi(t - 1) \quad f_s(t) := f_{s0} \quad p_d(k) := f_s(k \cdot T)$$

**Simulación Discreta**

$$y_d(k) := h_d(k \cdot T)$$

$$\xi(k) := \text{if} \left( k = 0, \xi_0, A_{Tk}^k \cdot \xi_0 + \sum_{j=0}^{k-1} A_{Tk}^{k-j-1} \cdot b_{Tk} \cdot y_d(j) + \sum_{j=0}^{k-1} A_{Tk}^{k-j-1} \cdot e_{Tk} \cdot p_d(j) \right)$$

$$\xi_0 := (2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T$$

**Entrada Continuo**

$$v(k) := \xi(k)_7$$

$$f_e(t) := \xi \left( \text{if} \left( \text{trunc} \left( \frac{t}{T} \right) < 0, 0, \text{trunc} \left( \frac{t}{T} \right) \right) \right)_7$$

$$t_f := 6 \quad k_f := \frac{t_f}{T}$$

$$k := 0 \dots k_f \quad n_f := 400$$

$$n := 0 \dots n_f \quad t := 0, \frac{t_f}{n_f} \dots t_f$$

**Simulación Continua**

$$D(t, x) := A_t \cdot x_1 + b_t \cdot f_e(t - t_f) \quad CI := 2$$

$$Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$$

