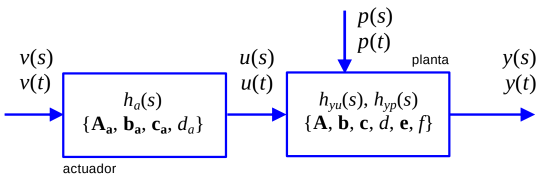
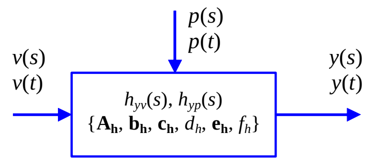
Representación Simplificada

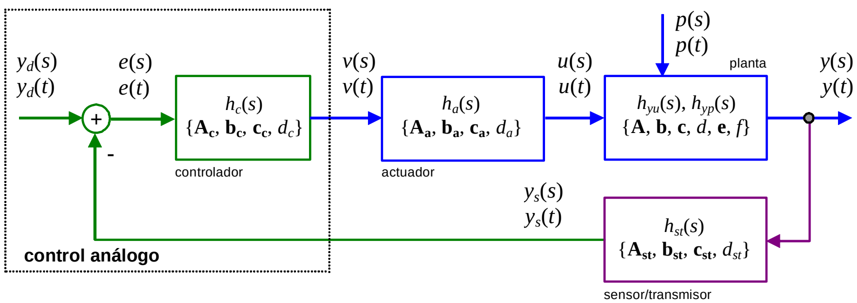
Problema Reducir la notación de sistemas intrconectados.

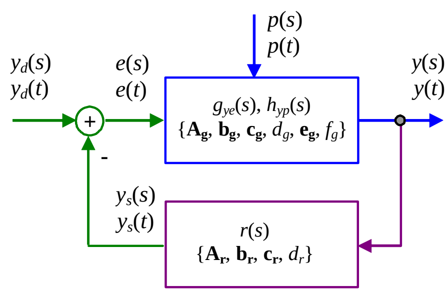
Sistemas Continuos en L.A.



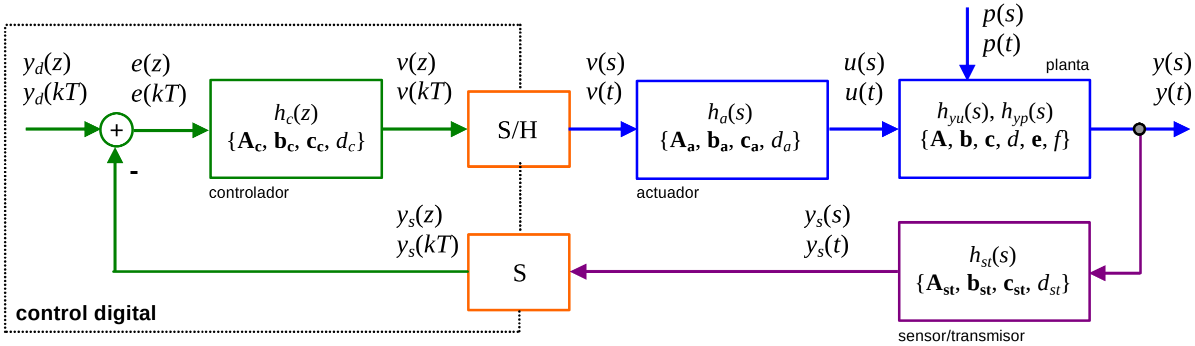


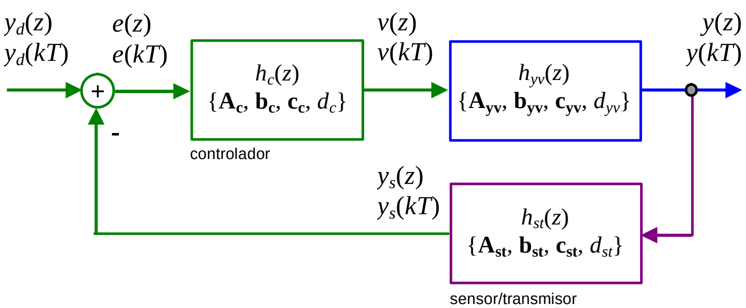
Sistemas Continuos en L.C.

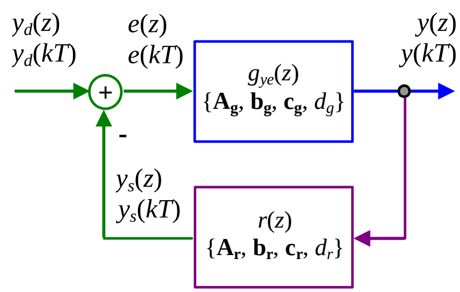




Sistemas Híbridos en L.C.

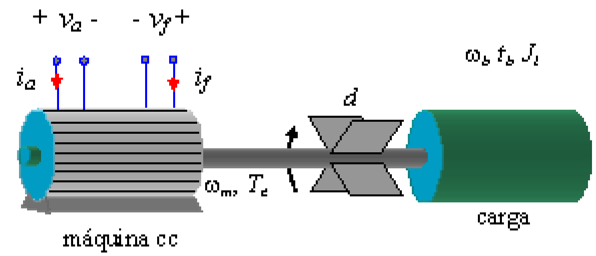






Sensibilidad en Sistemas en L.A. y en L.C.

Problema Comparar la sensibilidad en L.A. y L.C.



Parámetros

$d := 0.08$ $\qquad R := 1.2$ $\qquad t_l := 60$

$k_m := 0.6$ $\qquad L := 50 \cdot 10^{-3}$ $\qquad J_l := 0.135$

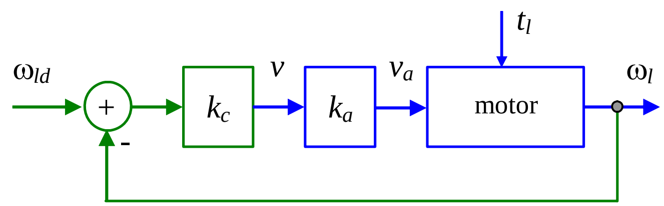
$$A := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \qquad b := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$

Modelo

$$v_a = L \cdot \frac{d}{dt} i_a + R \cdot i_a + k_m \cdot \omega \qquad J_l \cdot \frac{d}{dt} \omega = k_m \cdot i_a - d \cdot \omega - t_l \qquad e := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix} \qquad c := \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Variables de Estado

$x_1 = i_a \qquad x_2 = \omega$



Función de Transferencia en L.A.

$$h_{\omega l\_va}(s) := c \cdot (s \cdot \text{identity}(2) - A)^{-1} \cdot b$$

Se considera el caso en L.C. con un controlador k_c. La F. de T. en L.D.

$$k_a := 100 \qquad k_c := \frac{0.95}{h_{\omega l\_va}(0) \cdot (1 - 0.95) \cdot k_a} \qquad k_c = 0.144 \qquad g(s, k_c) := k_c \cdot k_a \cdot h_{\omega l\_va}(s)$$

La F. de T. en L.C.

$$h_{\omega l\_\omega ld\_k\omega}(s, k_\omega) := \frac{g(s, k_\omega)}{1 + g(s, k_\omega)}$$

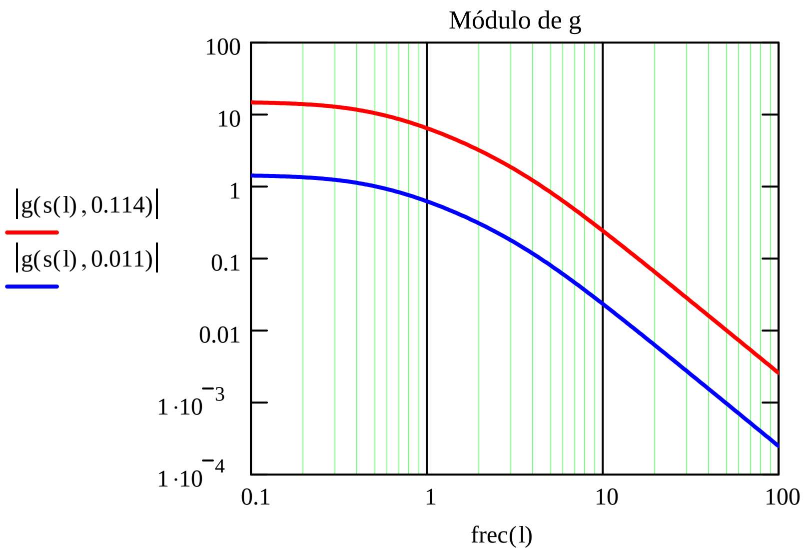
La sensibilidad de la F. de T. en L.C. respecto de la ganacia k_ω es,

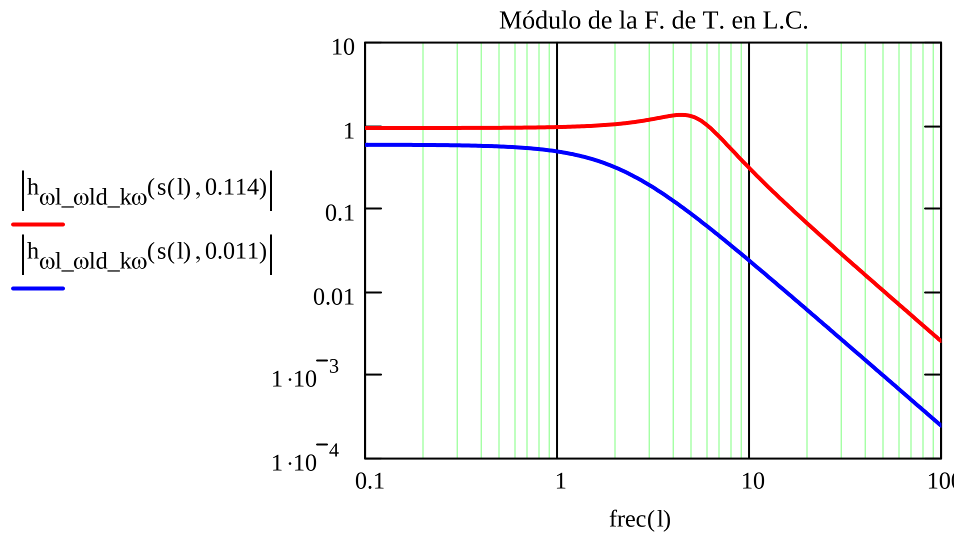
$$S_{\omega l\_\omega ld\_k\omega}(s, k_\omega) := \frac{1}{1 + g(s, k_\omega)}$$

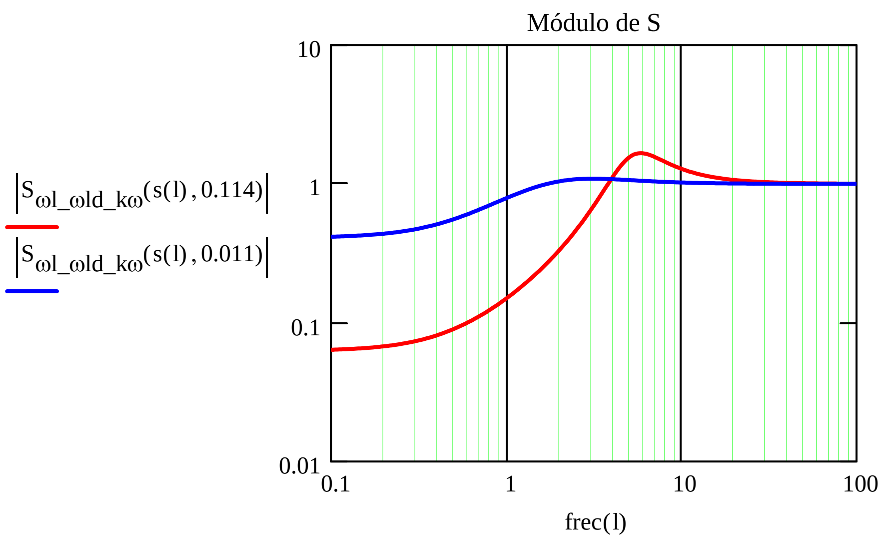
Los Diagramas de Bode son,

$$l_{max} := 250 \qquad l := 0 .. l_{max} \qquad rg := \frac{180}{\pi} \qquad rr := \frac{60}{2 \cdot \pi}$$

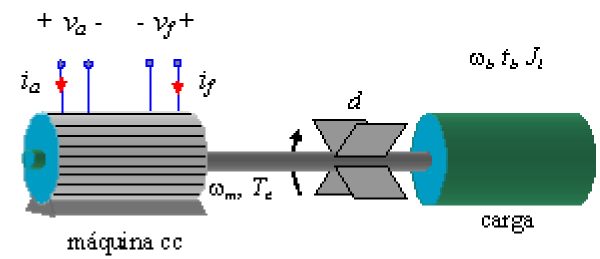
$$f_{min} := 10^{-1} \qquad f_{max} := 10^2 \qquad ratio := \log\left(\frac{f_{max}}{f_{min}}\right) \cdot \frac{1}{l_{max}} \qquad frec(l) := f_{min} \cdot 10^{l \cdot ratio} \qquad s(l) := j \cdot 2 \cdot \pi \cdot frec(l)$$

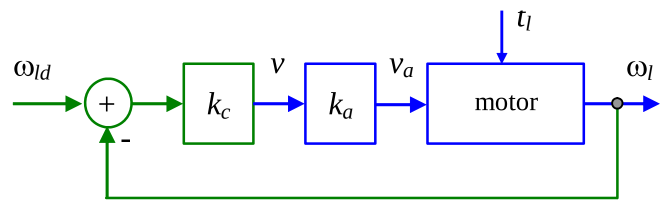

Módulo de g
|g(s(l), 0.114)|
|g(s(l), 0.011)|
100
10
1
0.1
0.01
1·10^−3
1·10^−4
0.1
1
10
100
frec(l)


Módulo de la F. de T. en L.C.
|h_ωl_ωld_kω(s(l), 0.114)|
|h_ωl_ωld_kω(s(l), 0.011)|
10
1
0.1
0.01
1·10^−3
1·10^−4
0.1
1
10
100
frec(l)


Módulo de S
|S_ωl_ωld_kω(s(l), 0.114)|
|S_ωl_ωld_kω(s(l), 0.011)|
10
1
0.1
0.01
0.1
1
10
100
frec(l)

Problema Comparar el efecto de las perturbaciones en L.A. y L.C.





Representación del Sistema en L.A.

$$\omega_l(s) = h_{\omega l\_va}(s) \cdot v_a(s) + h_{\omega l\_tl}(s) \cdot t_l(s)$$

donde,

$$h_{\omega l\_va}(s) := c \cdot (s \cdot \mathrm{identity}(2) - A)^{-1} \cdot b$$

$$h_{\omega l\_tl}(s) := c \cdot (s \cdot \mathrm{identity}(2) - A)^{-1} \cdot e$$

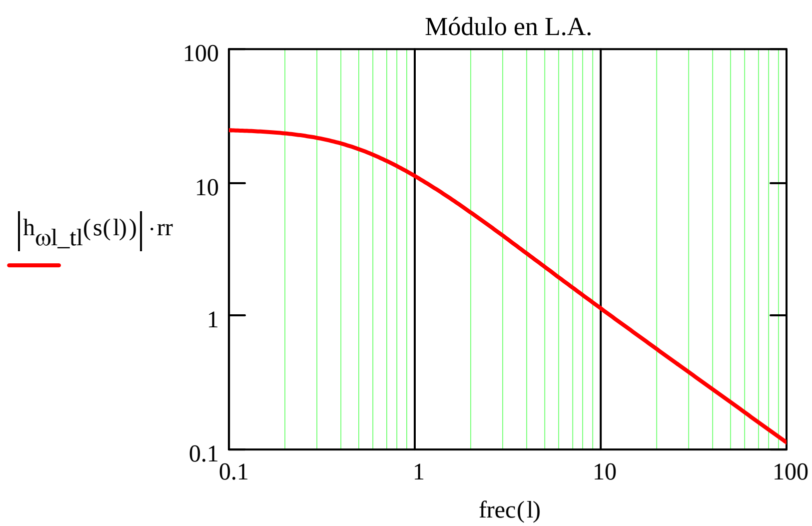
en L.C. se tiene

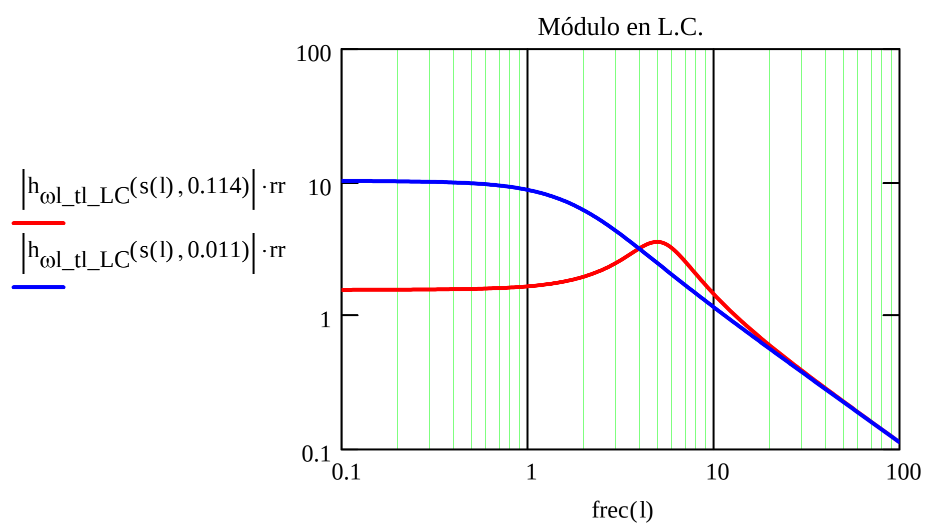
$$\omega_l(s) = h_{\omega l\_va}(s) \cdot \left[k_c \cdot k_a \cdot \left(\omega_{ld}(s) - \omega_l(s)\right)\right] + h_{\omega l\_tl}(s) \cdot t_l(s)$$

$$\omega_l(s) = \frac{h_{\omega l\_va}(s) \cdot k_c \cdot k_a}{1 + h_{\omega l\_va}(s) \cdot k_c \cdot k_a} \cdot \omega_{ld}(s) + \frac{h_{\omega l\_tl}(s)}{1 + h_{\omega l\_va}(s) \cdot k_c \cdot k_a} \cdot t_l(s) \qquad k_c = 0.144 \qquad g(s, k_c) := k_c \cdot k_a \cdot h_{\omega l\_va}(s)$$

En L.A. las **p** afectan en el factor, $h_{\omega l\_tl}(s)$, y en L.C. en el factor, $h_{\omega l\_tl\_LC}(s, k_c) := \dfrac{h_{\omega l\_tl}(s)}{1 + h_{\omega l\_va}(s) \cdot k_c \cdot k_a}$

Los Diagramas de Bode son,





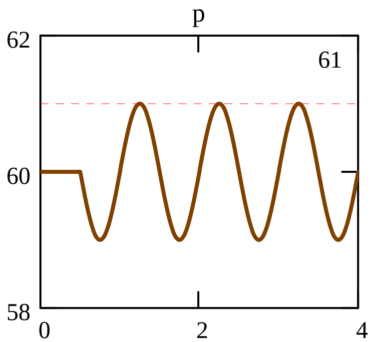
Simulación

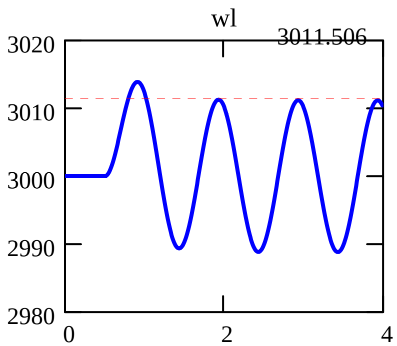
$V_a := \frac{3000}{rr}\cdot k_m + \left(t_l + d\cdot\frac{3000}{rr}\right)\cdot\frac{R}{k_m}$ $t_d := 0.5$ $v_a(t) := V_a$

$t_f := 4$ $n_f := 400$ $m := 0..n_f$ $u(t) := v_a(t)$ $p(t) := t_l + \sin(2\cdot\pi\cdot t)\cdot\Phi(t - t_d)$

$D(t,x) := A\cdot(x_1 \;\; x_2)^T + b\cdot u(t) + e\cdot p(t)$ $Z_a := \mathrm{rkfixed}\left[-\left[A^{-1}\cdot(b\cdot u(0) + e\cdot p(0))\right], 0, t_f, n_f, D\right]$

Simulación en L.A. con v_a para tener 3000 y luego aparece una perturbación unitaria con periodo de **1 Hz**.





La F. de T. en L.A. predice una amplitud de la oscilación en rpm de,

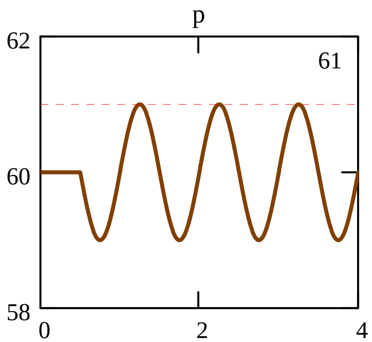
$\left|h_{\omega l\_tl}(j\cdot 2\cdot\pi)\right|\cdot rr = 11.174$

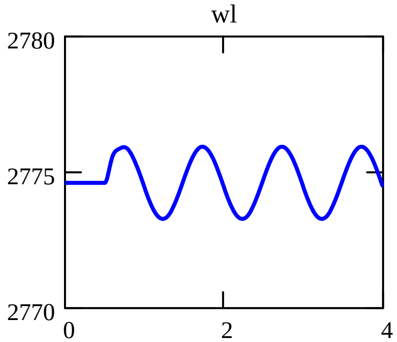
Simulación en L.C. con Ω_{ld} = 3000 y luego aparece una perturbación unitaria con periodo de **1 Hz**.

$\Omega_{ld} := 3000$ $\omega_{ld}(t) := \Omega_{ld}\cdot rr^{-1}$ $CI := -\left[(A - b\cdot k_c\cdot k_a\cdot c)^{-1}\cdot(b\cdot k_c\cdot k_a\cdot\omega_{ld}(0) + e\cdot p(0))\right]$

$D(t,x) := A\cdot(x_1 \;\; x_2)^T + b\cdot\left[k_c\cdot k_a\cdot(\omega_{ld}(t) - x_2)\right] + e\cdot p(t)$ $CI^T = (138.741 \;\; 290.557)$

$Z_a := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$





La F. de T. en L.A. predice una amplitud de la oscilación en rpm de,

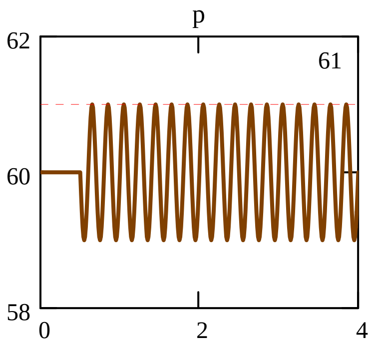
$\left|h_{\omega l\_tl\_LC}(j\cdot 2\cdot\pi, 0.114)\right|\cdot rr = 1.667$

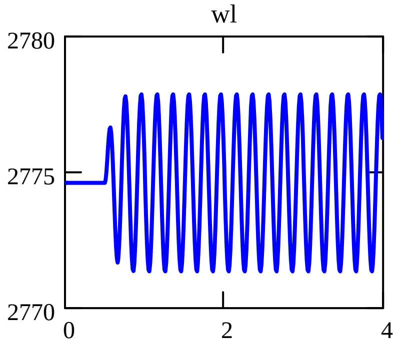
Simulación en L.C. con Ω_{ld} = 3000 y luego aparece una perturbación unitaria con periodo de **5 Hz**.

$p(t) := t_l + \sin(2\cdot\pi\cdot 5\cdot t)\cdot\Phi(t - t_d)$

$D(t,x) := A\cdot(x_1 \;\; x_2)^T + b\cdot\left[k_c\cdot k_a\cdot(\omega_{ld}(t) - x_2)\right] + e\cdot p(t)$

$Z_a := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$





La F. de T. en L.A. predice una amplitud de la oscilación en rpm de,

$\left|h_{\omega l\_tl\_LC}(j\cdot 2\cdot\pi\cdot 5, 0.114)\right|\cdot rr = 3.593$

Problema Estudiar el efecto de ruido en la realimentación.

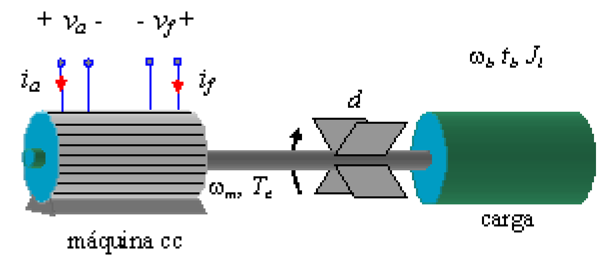
Representación del Sistema en L.A.

$$\omega_l(s) = h_{\omega l\_va}(s)\cdot v_a(s) + h_{\omega l\_tl}(s)\cdot t_l(s)$$

donde,

$$h_{\omega l\_va}(s) := c\cdot(s\cdot \text{identity}(2) - A)^{-1}\cdot b$$

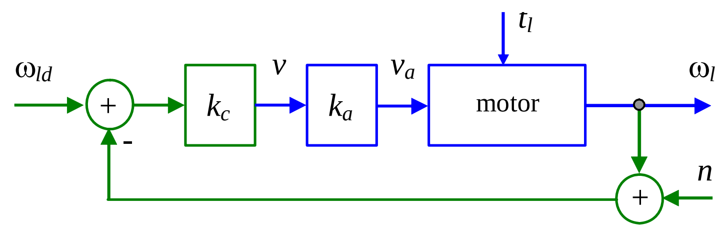
$$h_{\omega l\_tl}(s) := c\cdot(s\cdot \text{identity}(2) - A)^{-1}\cdot e$$



en L.C. y con ruido se tiene

$$\omega_l(s) = h_{\omega l\_va}(s)\cdot\left[k_c\cdot k_a\cdot\left(\omega_{ld}(s) - \omega_l(s) + n(s)\right)\right] + h_{\omega l\_tl}(s)\cdot t_l(s)$$

$$\omega_l(s) = \frac{h_{\omega l\_va}(s)\cdot k_c\cdot k_a}{1 + h_{\omega l\_va}(s)\cdot k_c\cdot k_a}\cdot\omega_{ld}(s) + \frac{-h_{\omega l\_va}(s)\cdot k_c\cdot k_a}{1 + h_{\omega l\_va}(s)\cdot k_c\cdot k_a}\cdot n(s) + \frac{h_{\omega l\_tl}(s)}{1 + h_{\omega l\_va}(s)\cdot k_c\cdot k_a}\cdot t_l(s)$$

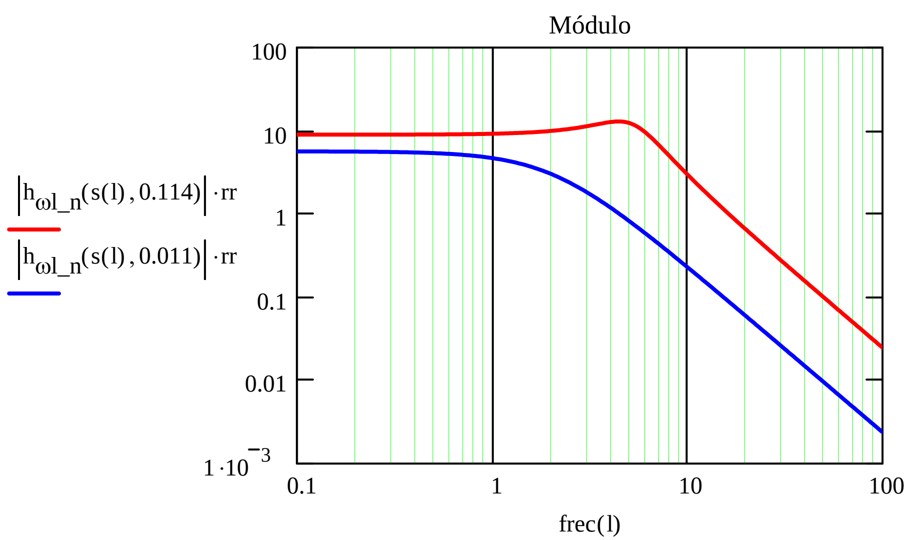


$k_c = 0.144$ $\qquad g(s, k_c) := k_a\cdot k_c\cdot h_{\omega l\_va}(s)$

En L.C. el ruido **n** afecta en el factor,

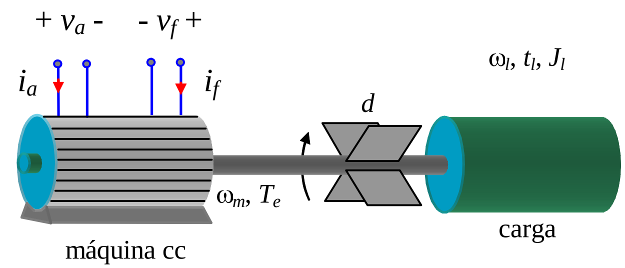
$$h_{\omega l\_n}(s, k_c) := \frac{-h_{\omega l\_va}(s)\cdot k_c\cdot k_a}{1 + h_{\omega l\_va}(s)\cdot k_c\cdot k_a}$$

El Diagrama de Bode es,



Ganancia DC y Constante de Tiempo

Problema Estudiar el efecto de la realimentación en la ganancia DC y constante de tiempo en el sistema resultante.

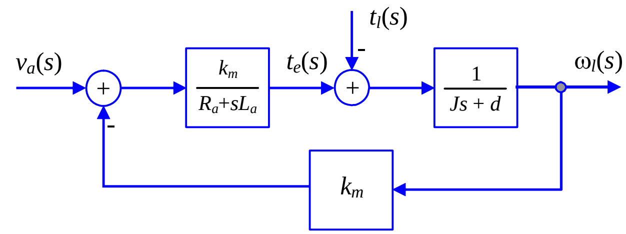


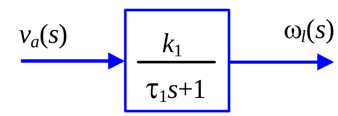
$$\frac{\omega_l(s)}{v_a(s)} = \frac{k_m}{(R_a + L_a s)(Js + d) + k_m k_m}$$

En L.A.

$$\frac{\omega_l(s)}{v_a(s)} = \frac{k_1}{\tau_1 s + 1} \qquad k_1 = \frac{k_m}{R_a d + k_m k_m} \qquad \tau_1 = \frac{R_a J}{R_a d + k_m k_m}$$

$$\omega_l(t) = \mathcal{L}^{-1}\left\{\frac{k_1}{\tau_1 s + 1}\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{k_1}{s} - \frac{k_1 \tau_1}{\tau_1 s + 1}\right\} = k_1 - k_1 e^{-t/\tau_1} = k_1(1 - e^{-t/\tau_1})$$

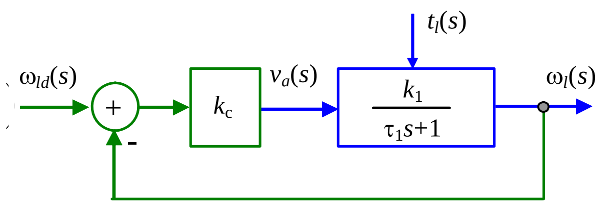




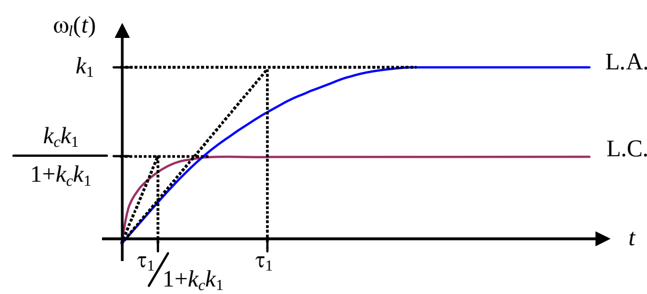
En L.C.

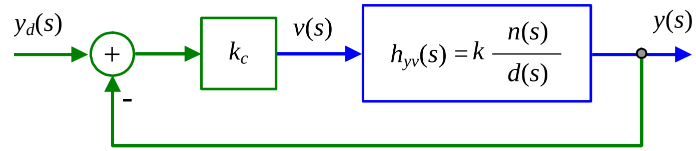
$$\frac{\omega_l(s)}{\omega_{ld}(s)} = \frac{k_c k_1}{\tau_1 s + 1 + k_c k_1}$$

$$\omega_l(t) = \mathcal{L}^{-1}\left\{\frac{k_c k_1}{\tau_1 s + 1 + k_c k_1}\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{k_c k_1}{1 + k_c k_1}\left(\frac{1}{s} - \frac{1}{s + (1 + k_c k_1)/\tau_1}\right)\right\} = \frac{k_c k_1}{1 + k_c k_1}\left(1 - e^{-t/[\tau_1/(1 + k_c k_1)]}\right)$$



Característica	L.A.		L.C.	L.C. con $k_c \to 0$	L.C. con $k_c \to \infty$
constante de tiempo	τ_1	>	$\frac{\tau_1}{1 + k_c k_1}$	τ_1	0
ganancia	k_1		$\frac{k_c k_1}{1 + k_c k_1}$	0	1





Problema Estudiar el efecto de la realimentación en las oscilaciones del sistema.

$$\frac{y(s)}{y_d(s)} = \frac{k_c \cdot k}{k_c \cdot k + (s+a)\cdot(s+bb)}$$

$$\frac{y(s)}{y_d(s)} = \frac{k_c \cdot k}{s^2 + (a+bb)\cdot s + a\cdot bb + k_c \cdot k} \qquad k := 5 \qquad g(s) = k\frac{1}{(s+a)(s+b)}$$

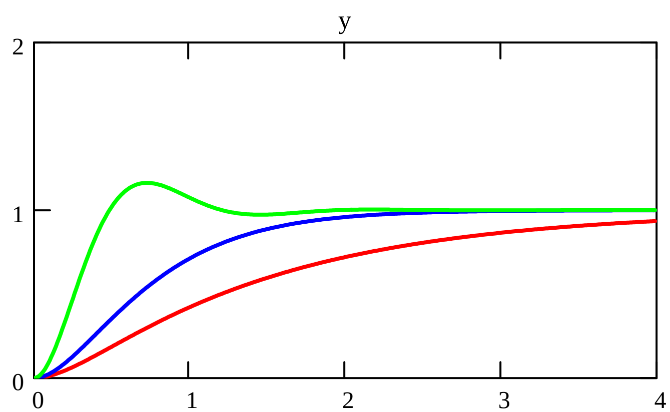
$$A(a,\beta,k_c) := \begin{pmatrix} 0 & 1 \\ -a\cdot\beta - k_c\cdot k & -a-\beta \end{pmatrix} \qquad b(k_c) := \begin{pmatrix} 0 \\ k_c \cdot k \end{pmatrix} \qquad c := (1 \;\; 0) \qquad CI := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Caso i): a = 0, b > 0

$a := 0 \quad \beta := 5 \quad k_c := 0.5\cdot\beta^2\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \;\; x_2)^T + b(k_c)\cdot 1 \quad Z_a := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 0 \quad \beta := 5 \quad k_c := \beta^2\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \;\; x_2)^T + b(k_c)\cdot 1 \quad Z_b := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 0 \quad \beta := 5 \quad k_c := 4\cdot\beta^2\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \;\; x_2)^T + b(k_c)\cdot 1 \quad Z_c := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

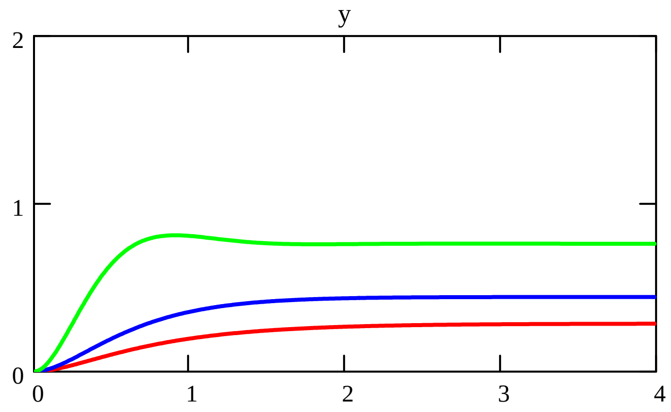


Caso ii): a > 0, b >0

$a := 1 \quad \beta := 5 \quad k_c := 0.5\cdot\left[(a+\beta)^2 - 4\cdot a\cdot\beta\right]\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_d := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 1 \quad \beta := 5 \quad k_c := 1.0\cdot\left[(a+\beta)^2 - 4\cdot a\cdot\beta\right]\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_e := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 1 \quad \beta := 5 \quad k_c := 4.0\cdot\left[(a+\beta)^2 - 4\cdot a\cdot\beta\right]\cdot(4\cdot k)^{-1} \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_f := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

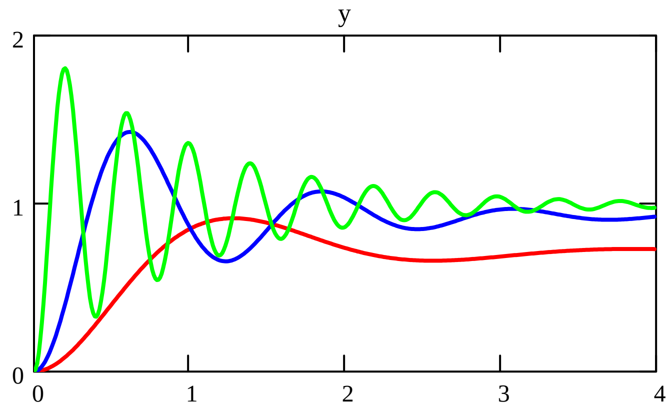


Caso iii): a = re + j im, b = re - j im

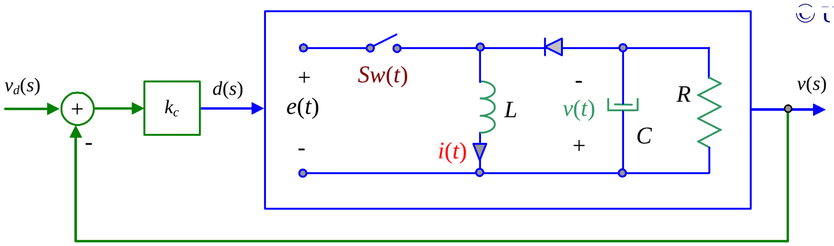
$a := 1 + j \quad \beta := 1 - j \quad k_c := 1 \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_g := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 1 + j \quad \beta := 1 - j \quad k_c := 5 \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_h := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$

$a := 1 + j \quad \beta := 1 - j \quad k_c := 50 \quad D(t,x) := A(a,\beta,k_c)\cdot(x_1 \ \ x_2)^T + b(k_c)\cdot 1 \quad Z_i := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$



Problema Estudiar si el reductor/elevador de tensión oscilaría con un controlador de ganancia.



Parámetros

$L := 12\cdot10^{-3}$ $C := 250\cdot10^{-6}$ $R := 10$

$d_o := 0.5$ $e_o := 10$ $\Delta e := 0.5$

Punto de operación

$v_o := \frac{d_o}{1-d_o}\cdot e_o$ $v_o = 10$ $i_o := \frac{v_o}{R\cdot(1-d_o)}$ $u_o := d_o$ $p_o := e_o$ $i_o = 2$

Variables de Estado

$x_1 = v$ $x_2 = i$

Modelo Lineal Normalizado.

$$A_n := \begin{bmatrix} \frac{-1}{R\cdot C} & \frac{1}{R\cdot C} \\ \frac{-R}{L}\cdot(1-d_o)^2 & 0 \end{bmatrix} \quad b_n := \begin{bmatrix} \frac{-d_o}{R\cdot C\cdot(1-d_o)} \\ \frac{R}{L}\cdot(1-d_o) \end{bmatrix} \quad e_n := \begin{bmatrix} 0 \\ \frac{R}{L}\cdot(1-d_o)^2 \end{bmatrix} \quad c_n := (1 \quad 0)$$

Función de Transferencia en L.A.

$$h_{vndn}(s) := \frac{1}{1-d_o}\cdot\frac{s\cdot\frac{-1}{R\cdot C}\cdot d_o + \frac{(1-d_o)^2}{L\cdot C}}{s^2 + s\cdot\frac{1}{R\cdot C} + \frac{(1-d_o)^2}{L\cdot C}}$$

cero $s\cdot\frac{-1}{R\cdot C}\cdot d_o + \frac{(1-d_o)^2}{L\cdot C} = 0$

polos $s^2 + s\cdot\frac{1}{R\cdot C} + \frac{(1-d_o)^2}{L\cdot C} = 0$

$$zz := \lambda^2 + \lambda\cdot\frac{1}{R\cdot C} + \frac{(1-d_o)^2}{L\cdot C} \text{ coeffs}, \lambda \rightarrow \begin{pmatrix} 83333.3333333333333333 \\ 400 \\ 1 \end{pmatrix}$$

$$\text{polyroots}(zz) = \begin{pmatrix} -200 - 208.167i \\ -200 + 208.167i \end{pmatrix}$$

$$pp := \lambda\cdot\frac{-1}{R\cdot C}\cdot d_o + \frac{(1-d_o)^2}{L\cdot C} \text{ coeffs}, \lambda \rightarrow \begin{pmatrix} 83333.3333333333333333 \\ -200.0 \end{pmatrix}$$

$$\text{polyroots}(pp) = 416.667$$

Función de Transferencia en L.C.

$k_c := 0.5$

$$h_{vnvnd}(s) := \frac{\frac{k_c}{1-d_o}\cdot\left[s\cdot\frac{-1}{R\cdot C}\cdot d_o + \frac{(1-d_o)^2}{L\cdot C}\right]}{s^2 + s\cdot\frac{1}{R\cdot C}\cdot\left(1 - k_c\cdot\frac{d_o}{1-d_o}\right) + \frac{(1-d_o)^2}{L\cdot C}\cdot\left(1 + k_c\cdot\frac{1}{1-d_o}\right)}$$

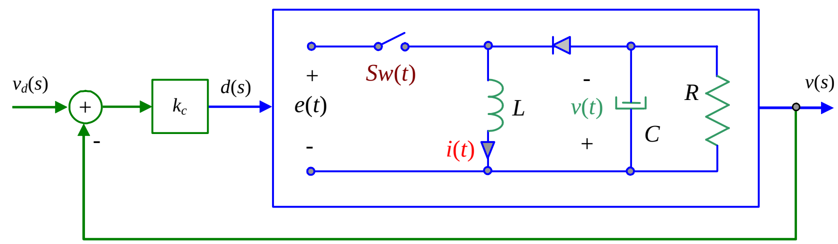
$h_{vnvnd}(0) = 0.5$

$$pp := \lambda^2 + \lambda\cdot\frac{1}{R\cdot C}\cdot\left(1 - k_c\cdot\frac{d_o}{1-d_o}\right) + \frac{(1-d_o)^2}{L\cdot C}\cdot\left(1 + k_c\cdot\frac{1}{1-d_o}\right) \text{ coeffs}, \lambda \rightarrow \begin{pmatrix} 166666.66666666666667 \\ 200.00000000000000000 \\ 1 \end{pmatrix}$$

$$\text{polyroots}(pp) = \begin{pmatrix} -100 + 395.811i \\ -100 - 395.811i \end{pmatrix}$$

Simulación en L.C.

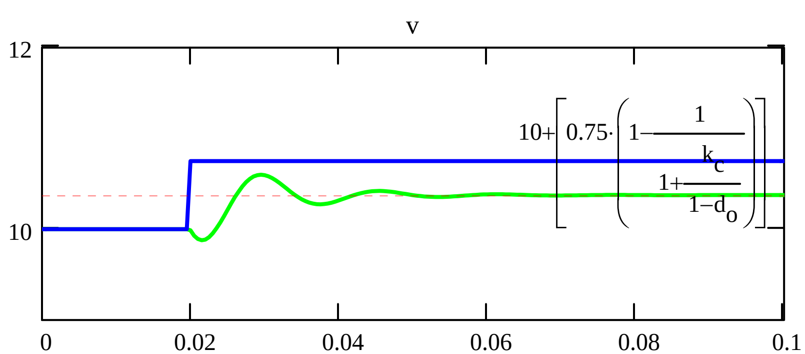
$k_a := 1$



Caso 1

$\Delta v_d(t) := 0.75\cdot\Phi(t - 0.02)$ $\quad \Delta e(t) := 0$ $\quad k_c := 0.5$ $\quad n_f := 200$ $\quad t_f := 0.1$ $\quad t := 0, \frac{t_f}{n_f} .. t_f$

$D(t, \Delta x) := A_n\cdot(\Delta x_1 \ \ \Delta x_2)^T + b_n\cdot(k_c\cdot k_a)\cdot(\Delta v_d(t) - \Delta x_1) + e_n\cdot\Delta e(t)$ $\quad CI := (0 \ \ 0)^T$ $\quad Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$



Oscila establemte. Hay un error en est. est.

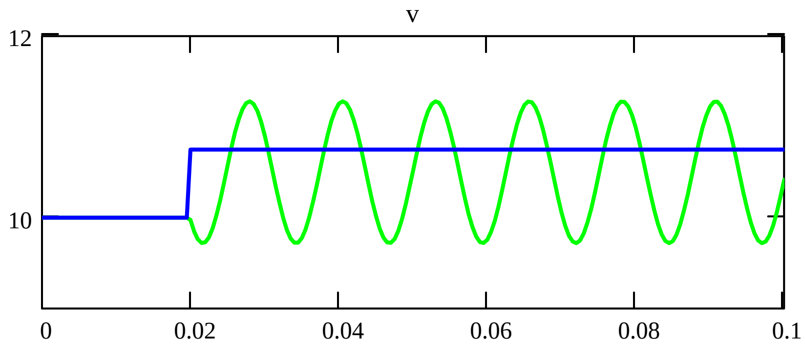
$$\text{polyroots}\left[\begin{bmatrix} \frac{(1-d_o)^2}{L\cdot C}\cdot\left(1 + k_c\cdot\frac{1}{1-d_o}\right) \\ \frac{1}{R\cdot C}\cdot\left(1 - k_c\cdot\frac{d_o}{1-d_o}\right) \\ 1 \end{bmatrix}\right] = \begin{pmatrix} -100 + 395.811i \\ -100 - 395.811i \end{pmatrix}$$

Caso 2

$\Delta v_d(t) := 0.75\,\Phi(t - 0.02) \quad \Delta e(t) := 0 \quad k_c := 1.0$

$D(t,\Delta x) := A_n \cdot (\Delta x_1 \ \ \Delta x_2)^T + b_n \cdot (k_c \cdot k_a) \cdot (\Delta v_d(t) - \Delta x_1) + e_n \cdot \Delta e(t)$

$Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$



Oscila eternamente.

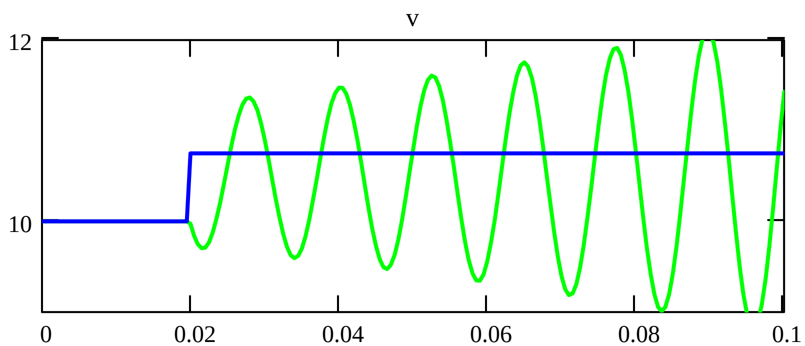
$$\text{polyroots}\left[\begin{bmatrix} \frac{(1-d_o)^2}{L \cdot C} \cdot \left(1 + k_c \cdot \frac{1}{1-d_o}\right) \\ \frac{1}{R \cdot C} \cdot \left(1 - k_c \cdot \frac{d_o}{1-d_o}\right) \\ 1 \end{bmatrix}\right] = \begin{pmatrix} -500i \\ 500i \end{pmatrix}$$

Caso 3

$\Delta v_d(t) := 0.75\,\Phi(t - 0.02) \quad \Delta e(t) := 0 \quad k_c := 1.05$

$D(t,\Delta x) := A_n \cdot (\Delta x_1 \ \ \Delta x_2)^T + b_n \cdot (k_c \cdot k_a) \cdot (\Delta v_d(t) - \Delta x_1) + e_n \cdot \Delta e(t)$

$Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$

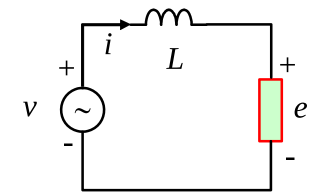


Oscila inestablemente.

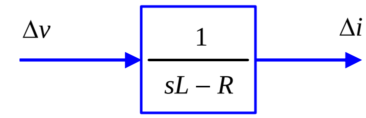
$$\text{polyroots}\left[\begin{bmatrix} \frac{(1-d_o)^2}{L \cdot C} \cdot \left(1 + k_c \cdot \frac{1}{1-d_o}\right) \\ \frac{1}{R \cdot C} \cdot \left(1 - k_c \cdot \frac{d_o}{1-d_o}\right) \\ 1 \end{bmatrix}\right] = \begin{pmatrix} 10 - 508.167i \\ 10 + 508.167i \end{pmatrix}$$

Problema Estudiar el efecto en la estabilidad de sistemas.

$$\frac{\Delta i(s)}{\Delta v(s)} = \frac{1}{L\cdot s + R} \qquad R := 1 \qquad L := 0.5 \qquad \frac{\Delta i(s)}{\Delta i_d(s)} = \frac{k_c}{k_c + L\cdot s - R} = \frac{\frac{k_c}{L}}{s + \frac{(k_c - R)}{L}}$$



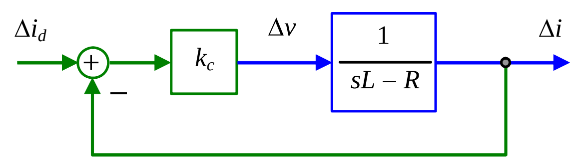
$$a(k_c) := \frac{-(k_c - R)}{L} \qquad b(k_c) := \frac{k_c}{L} \qquad c := 1 \qquad CI := 0 \qquad t_f := 4$$



Caso i): $k_c < R$

$$k_c := \frac{R}{2} \qquad D(t,x) := a(k_c)\cdot x_1 + b(k_c)\cdot 1$$

$$Z_a := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$$



Caso ii): $k_c = R$

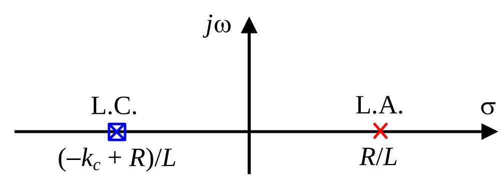
$$k_c := R \qquad D(t,x) := a(k_c)\cdot x_1 + b(k_c)\cdot 1$$

$$Z_b := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$$

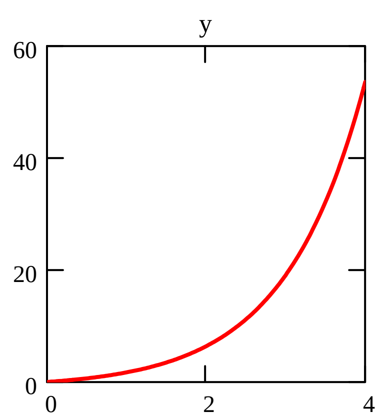
Caso iii): $k_c > R$

$$k_c := 2\cdot R \qquad D(t,x) := a(k_c)\cdot x_1 + b(k_c)\cdot 1$$

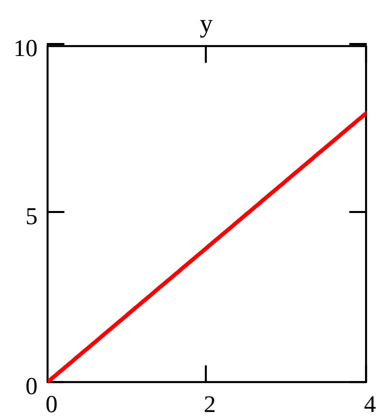
$$Z_c := \mathrm{rkfixed}(CI, 0, t_f, n_f, D)$$



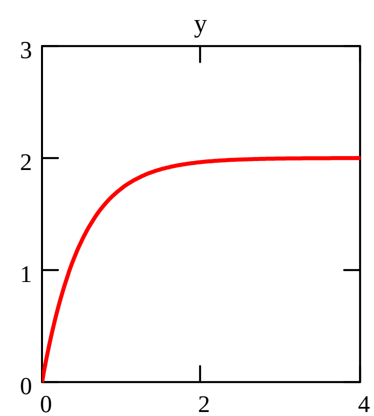
Caso i



Caso ii

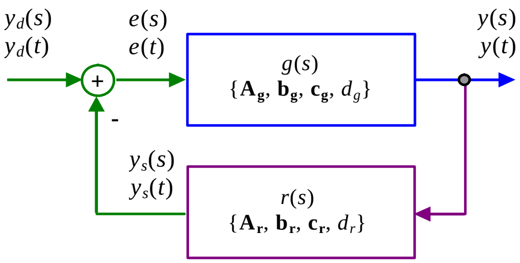


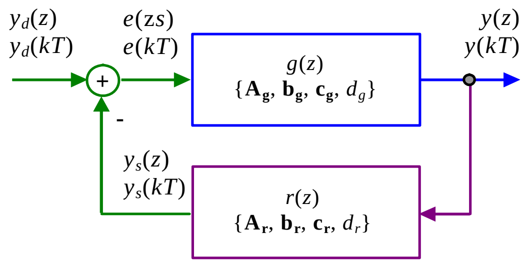
Caso iii



Error en Estado Estacionario

Problema Establecer expresiones y definiciones relacionadas con el error en estado estacionario.





$$e(s) = \frac{y_d(s)}{1+g(s)r(s)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} se(s) = \lim_{s\to 0} \frac{sy_d(s)}{1+g(s)r(s)}$$

$$e(z) = \frac{y_d(z)}{1+g(z)r(z)}$$

$$e_{ss} = \lim_{k\to\infty} e(kT) = \lim_{z\to 1} \frac{z-1}{z} e(z) = \lim_{z\to 1} \frac{z-1}{z} \frac{y_d(z)}{1+g(z)r(z)}$$

Def.: La F. de T. dada por el producto $g(s)r(s)$ en el caso continuo y por $g(z)r(z)$ en el caso discreto se conoce como F. de T. en **Lazo Directo** (L.D.) y se representa por $l(s)$ o $l(z)$ según corresponda. Así, $l(s) = g(s)r(s)$ y $l(z) = g(z)r(z)$. La expresión general de $l(s)$ se asumirá,

$$l(s) = g(s)r(s) = k\frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + 1}{s^N(a_{n-N}s^{n-N} + a_{n-N-1}s^{n-N-1} + \cdots + 1)},$$

y la expresión general de $l(z)$ se asumirá,

$$l(z) = g(z)r(z) = k\frac{b_m z^m + b_{m-1}z^{m-1} + \cdots + 1}{(z-1)^N(a_{n-N}z^{n-N} + a_{n-N-1}z^{n-N-1} + \cdots + 1)}.$$

Entradas Normalizadas

$$u(t) = \frac{t^m}{m!}u(t) \qquad u(s) = \frac{1}{s^{m+1}}$$

Entradas continuas, m = 0: escalón; m = 1: rampa; m = 2: parábola.

$$u(kT) = \frac{(kT)^m}{m!}u(kT)$$

Entradas discretas

m = 0: escalón

$$u(z) = \frac{z}{z-1}$$

m = 1: rampa

$$u(z) = T\frac{z}{(z-1)^2}$$

m = 2: parábola

$$u(z) = \frac{T^2}{2}\frac{z(z+1)}{(z-1)^3}$$

Coeficientes de Error Estático para Sistemas Continuos

- De **posición** k_p. Se define para **entrada escalón**.

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+g(s)r(s)} \frac{1}{s} = \lim_{s\to 0} \frac{1}{1+g(s)r(s)} = \frac{1}{1+\lim_{s\to 0} g(s)r(s)} = \frac{1}{1+k_p}.$$

Por lo tanto, $k_p = \lim_{s\to 0} g(s)r(s)$.

i) Para sistemas Tipo 0: $k_p = \lim_{s\to 0} g(s)r(s) = k$ $\Rightarrow$ $e_{ss} = 1/(1+k_p)$.

ii) Para sistemas Tipo 1: $k_p = \infty$ $\Rightarrow$ $e_{ss} = 0$.

iii) Para sistemas Tipo 2: $k_p = \infty$ $\Rightarrow$ $e_{ss} = 0$.

- De **velocidad** k_v. Se define para **entrada rampa**.

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+g(s)r(s)} \frac{1}{s^2} = \lim_{s\to 0} \frac{1}{1+g(s)r(s)} \frac{1}{s} = \frac{1}{\lim_{s\to 0} sg(s)r(s)} = \frac{1}{k_v}.$$

Por lo tanto, $k_v = \lim_{s\to 0} sg(s)r(s)$.

i) Para sistemas Tipo 0: $k_v = 0$ $\Rightarrow$ $e_{ss} = \infty$.

ii) Para sistemas Tipo 1: $k_v = \lim_{s\to 0} sg(s)r(s) = k$ $\Rightarrow$ $e_{ss} = 1/k_v$.

iii) Para sistemas Tipo 2: $k_v = \infty$ $\Rightarrow$ $e_{ss} = 0$.

- De **aceleración** k_a. Se define para **entrada parábola**.

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+g(s)r(s)} \frac{1}{s^3} = \lim_{s\to 0} \frac{1}{1+g(s)r(s)} \frac{1}{s^2} = \frac{1}{\lim_{s\to 0} s^2 g(s)r(s)} = \frac{1}{k_a}.$$

Por lo tanto, $k_a = \lim_{s\to 0} s^2 g(s)r(s)$.

i) Para sistemas Tipo 0: $k_a = 0$ $\Rightarrow$ $e_{ss} = \infty$.

ii) Para sistemas Tipo 1: $k_a = 0$ $\Rightarrow$ $e_{ss} = \infty$.

iii) Para sistemas Tipo 2: $k_a = \lim_{s\to 0} s^2 g(s)r(s) = k$ $\Rightarrow$ $e_{ss} = 1/k_a$.

Coeficientes de Error Estático para Sistemas Discretos

- De **posición** k_p. Se define para **entrada escalón**.

$$e_{ss} = \lim_{z\to 1} \frac{z-1}{z} \frac{1}{1+g(z)r(z)} \frac{z}{z-1} = \lim_{z\to 1} \frac{1}{1+g(z)r(z)} = \frac{1}{1+\lim_{z\to 1} g(z)r(z)} = \frac{1}{1+k_p}$$

Por lo tanto, $k_p = \lim_{z\to 1} g(z)r(z)$.

i) Para sistemas Tipo 0: $k_p = \lim_{z\to 1} g(z)r(z)$ $\Rightarrow$ $e_{ss} = 1/(1+k_p)$.

ii) Para sistemas Tipo 1: $k_p = \infty$ $\Rightarrow$ $e_{ss} = 0$.

iii) Para sistemas Tipo 2: $k_p = \infty$ $\Rightarrow$ $e_{ss} = 0$.

- De **velocidad** k_v. Se define para **entrada rampa**.

$$e_{ss} = \lim_{z\to 1} \frac{z-1}{z} \frac{1}{1+g(z)r(z)} T \frac{z}{(z-1)^2} = \lim_{z\to 1} \frac{T}{1+g(z)r(z)} \frac{1}{z-1} = \frac{T}{\lim_{z\to 1}(z-1)g(z)r(z)} = \frac{1}{k_v}.$$

Por lo tanto, $k_v = \lim_{z\to 1}(z-1)g(z)r(z)/T$.

i) Para sistemas Tipo 0: $k_v = 0$ $\Rightarrow$ $e_{ss} = \infty$.

ii) Para sistemas Tipo 1: $k_v = \lim_{z\to 1}(z-1)g(z)r(z)/T$ $\Rightarrow$ $e_{ss} = 1/k_v$.

iii) Para sistemas Tipo 2: $k_v = \infty$ $\Rightarrow$ $e_{ss} = 0$.

- De **aceleración** k_a. Se define para **entrada parábola**.

$$e_{ss} = \lim_{z\to 1} \frac{z-1}{z} \frac{1}{1+g(z)r(z)} \frac{T^2}{2} \frac{z(z+1)}{(z-1)^3} = \lim_{z\to 1} \frac{T^2}{1+g(z)r(z)} \frac{1}{(z-1)^2} = \frac{T^2}{\lim_{z\to 1}(z-1)^2 g(z)r(z)} = \frac{1}{k_a}$$

.

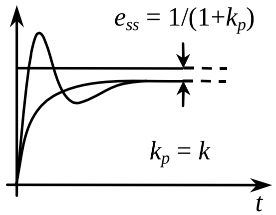
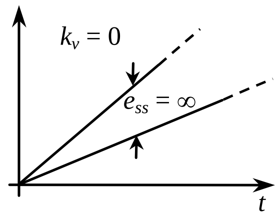
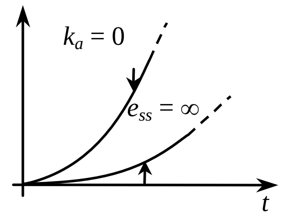
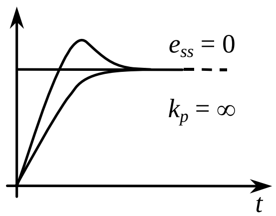
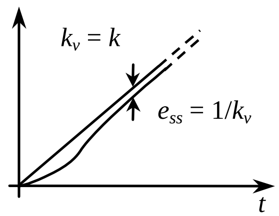
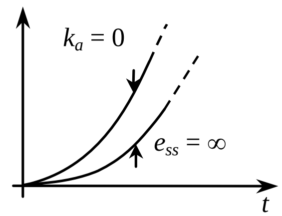
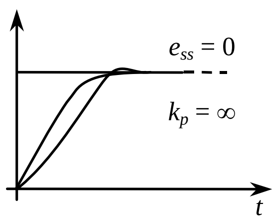
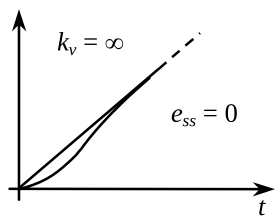
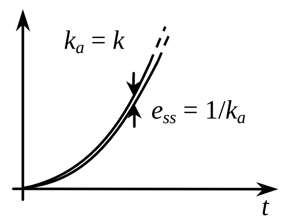
Por lo tanto, $k_a = \lim_{z\to 1}(z-1)^2 g(z)r(z)/T^2$.

i) Para sistemas Tipo 0: $k_a = 0$ $\Rightarrow$ $e_{ss} = \infty$.

ii) Para sistemas Tipo 1: $k_a = 0$ $\Rightarrow$ $e_{ss} = \infty$.

iii) Para sistemas Tipo 2: $k_a = \lim_{z\to 1}(z-1)^2 g(z)r(z)/T^2$ $\Rightarrow$ $e_{ss} = 1/k_a$.

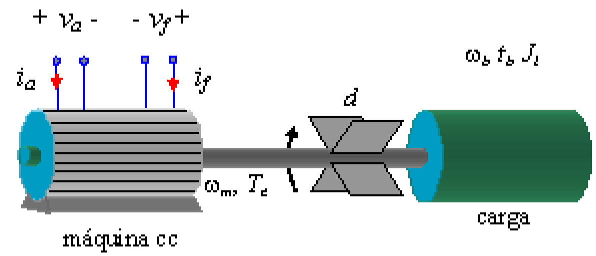
Tipo versus Entrada

Entrada	Escalón	Rampa	Parábola
Tipo 0			
Tipo 1			
Tipo 2			

Cuadro Resumen

Entrada	Escalón	Rampa	Parábola
Cte. de error continuo	$k_p = \lim_{s\to 0} g(s)r(s)$	$k_v = \lim_{s\to 0} sg(s)r(s)$	$k_a = \lim_{s\to 0} s^2 g(s)r(s)$
Cte. de error discreto	$k_p = \lim_{z\to 1} g(z)r(z)$	$k_v = \lim_{z\to 1}(z-1)g(z)r(z)/T$	$k_a = \lim_{z\to 1}(z-1)^2 g(z)r(z)/T^2$
Tipo de Sistema	Error en estado estacionario		
0	$1/(1+k_p)$	∞	∞
1	0	$1/k_v$	∞
2	0	0	$1/k_a$

Error en Estado Estacionario en Sistemas Continuos



Problema Definir el Controlador para lograr error en S.S. dados.

Parámetros $d := 0.08$ $R := 1.2$ $t_l := 60$ $k_m := 0.6$ $L := 50 \cdot 10^{-3}$ $J_l := 0.135$

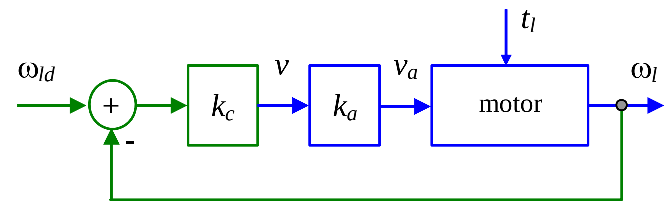
Modelo $v_a = L \cdot \frac{d}{dt} i_a + R \cdot i_a + k_m \cdot \omega$ $J_l \cdot \frac{d}{dt} \omega = k_m \cdot i_a - d \cdot \omega - t_l$

$$A := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \quad b := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \quad e := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix} \quad c := (0 \ \ 1)$$

Variables de Estado

$x_1 = i_a$ $x_2 = \omega$

Función de Transferencia en L.A. $h_{\omega l\_va}(s) := c \cdot (s \cdot \text{identity}(2) - A)^{-1} \cdot b$



Para saber si el sistema es Tipo 0 se puede obtener la ganancia dc. Si es el resultado es una constante, entonces el sistema es Tipo 0. Este es el caso del motor pues,

$h_{\omega l\_va}(0) = 1.316$ $k_a := 100$

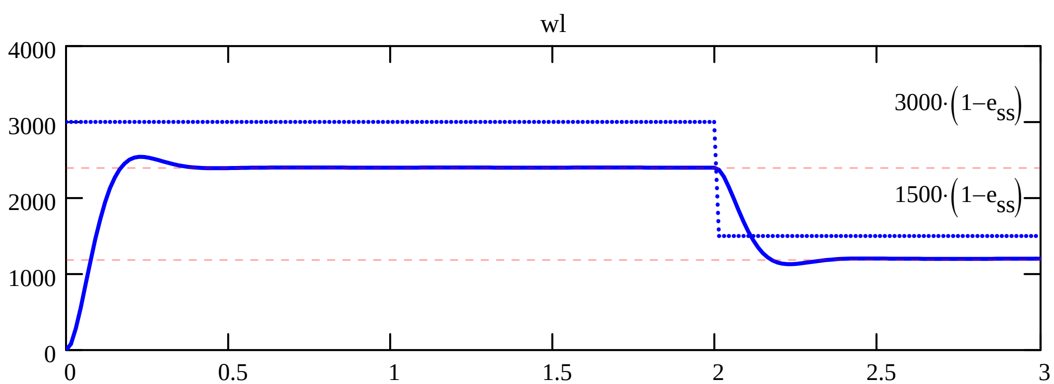
Controlador de Ganancia k_ω Para tener un error en S.S. de 20 % para **entrada escalón**, entonces,

$$e_{ss} := \frac{20}{100} \quad k_c := \frac{1 - e_{ss}}{h_{\omega l\_va}(0) \cdot e_{ss} \cdot k_a} \quad k_c \cdot 10^3 = 30.4$$

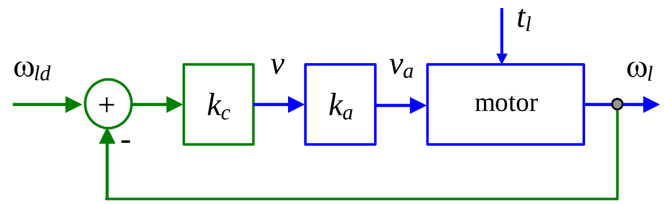
Simulación en L.C. con $\Omega_{ld} = 3000$ y **perturbación nula**. $t_f := 3$ $n_f := 200$

$\Omega_{ld} := 3000$ $\omega_{ld}(t) := \Omega_{ld} \cdot (1 - 0.5 \cdot \Phi(t-2)) \cdot rr^{-1}$ $p(t) := 0$ $t := 0, \frac{t_f}{n_f} .. t_f$

$D(t,x) := A \cdot (x_1 \ \ x_2)^T + b \cdot \left[k_c \cdot k_a \cdot (\omega_{ld}(t) - x_2)\right] + e \cdot p(t)$ $CI := (0 \ \ 0)^T$ $Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$



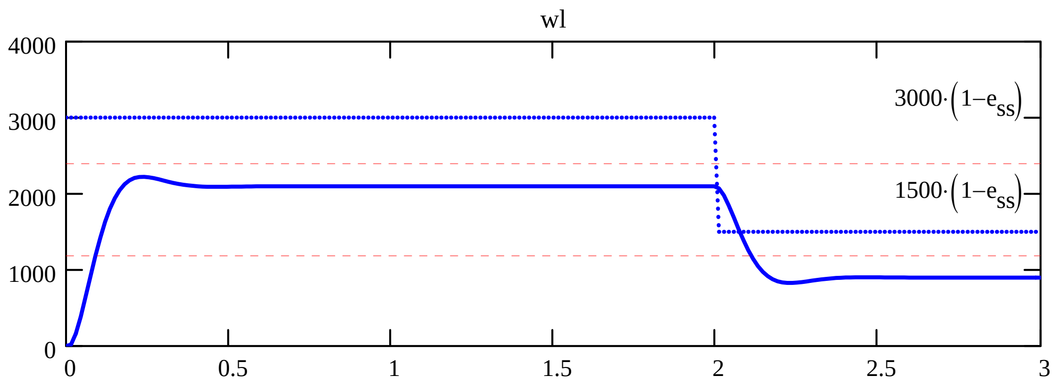
Se logra e_{ss} = 20% pues la perturbación es igual a cero.



Simulación en L.C. con Ω_{ld} = 3000 y **perturbación no nula**.

$\Omega_{ld} := 3000$ $\quad \omega_{ld}(t) := \Omega_{ld}\cdot(1 - 0.5\cdot\Phi(t-2))\cdot rr^{-1}$ $\quad p(t) := t_l$ $\quad t := 0, \frac{t_f}{n_f} .. t_f$

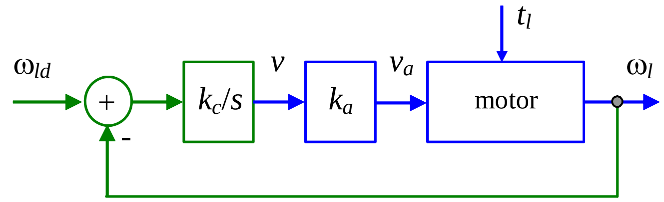
$D(t,x) := A\cdot(x_1 \;\; x_2)^T + b\cdot[k_c\cdot k_a\cdot(\omega_{ld}(t) - x_2)] + e\cdot p(t)$ $\quad CI := (0 \;\; 0)^T$ $\quad Z_a := rkfixed(CI, 0, t_f, n_f, D)$



No se logra pues la perturbación es distinta de cero.

Controlador con Integrador k_ω

Para tener un error en S.S. de 0.524 rad/s = 5 rpm para **entrada rampa** se utiliza un integrador con ganancia k_ω. El valor de la ganancia es,



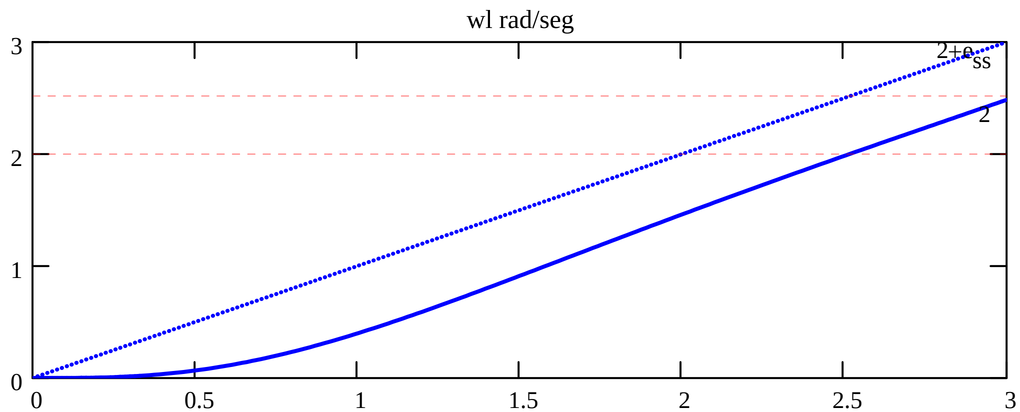
$e_{ss} := 0.524$ $\quad k_c := \dfrac{1}{h_{\omega l\_va}(0)\cdot e_{ss}\cdot k_a}$ $\quad k_c\cdot 1000 = 14.504$

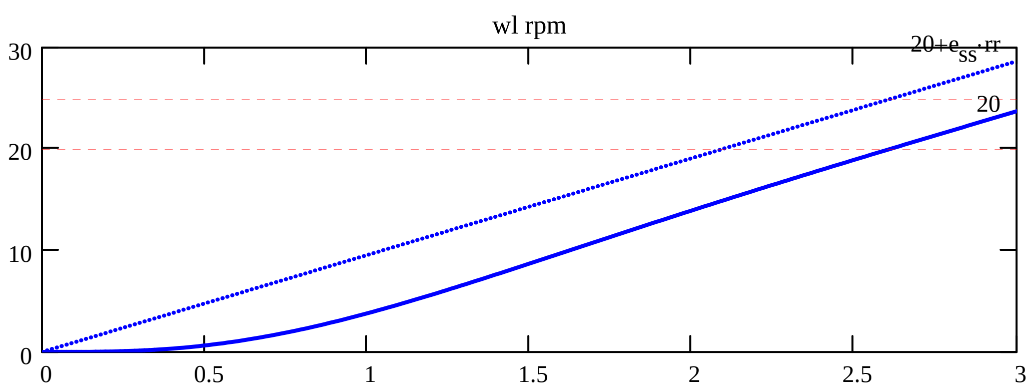
Simulación en L.C. con Ω_{ld} = 1 rad/seg y **perturbación nula**.

$\Omega_{ld} := 1$ $\quad \omega_{ld}(t) := \Omega_{ld}\cdot t$ $\quad p(t) := 0$

$A_p := \text{augment}\left(\text{augment}\left(A, b\cdot k_a\right)^T, \text{augment}\left(-c\cdot k_c, 0\right)^T\right)^T$ $\quad b_p := \text{augment}\left(b^T\cdot 0, k_c\right)^T$ $\quad e_p := \text{augment}\left(e^T, 0\right)^T$

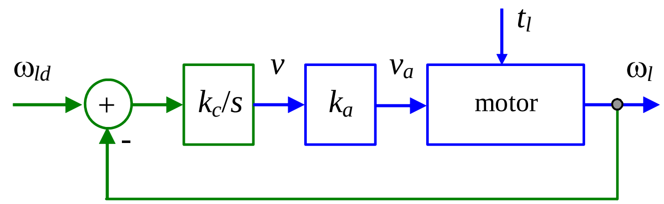
$D(t,x) := A_p\cdot\left(x_1 \;\; x_2 \;\; x_3\right)^T + b_p\cdot\omega_{ld}(t) + e_p\cdot p(t)$ $\quad CI := (0 \;\; 0 \;\; 0)^T$ $\quad Z_a := \text{rkfixed}\left(CI, 0, t_f, n_f, D\right)$





Controlador con Integrador k_ω

Para tener un error en S.S. de 0.524 rad/s = 5 rpm para **entrada rampa** se utiliza un integrador con ganancia k_ω. El valor de la ganancia es,



$e_{ss} := 0.524 \quad k_c := \dfrac{1}{h_{\omega l\_va}(0)\cdot e_{ss}\cdot k_a} \qquad k_c\cdot 1000 = 14.504$

Simulación en L.C. con Ω_{ld} = 1 rad/seg y **perturbación no nula**.

$\Omega_{ld} := 1 \qquad \omega_{ld}(t) := \Omega_{ld}\cdot t \qquad p(t) := 60$

$A_p := \text{augment}\left(\text{augment}(A, b\cdot k_a)^T, \text{augment}(-c\cdot k_c, 0)^T\right)^T$

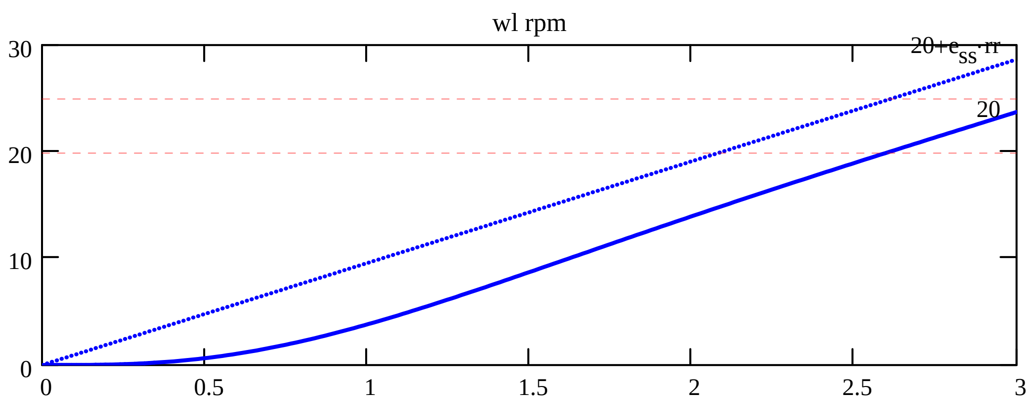
$b_p := \text{augment}\left(b^T\cdot 0, k_c\right)^T \qquad e_p := \text{augment}\left(e^T, 0\right)^T$

$D(t,x) := A_p\cdot\left(x_1 \;\; x_2 \;\; x_3\right)^T + b_p\cdot\omega_{ld}(t) + e_p\cdot p(t)$

$CI := -\left[A_p^{-1}\cdot\left(b_p\cdot\omega_{ld}(0) + e_p\cdot p(0)\right)\right]$

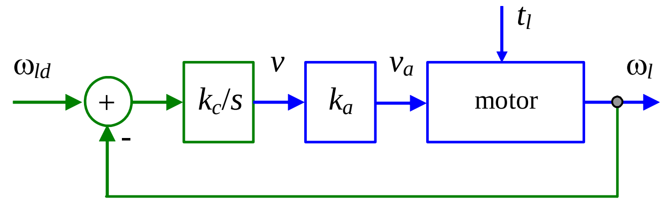
$Z_a := \text{rkfixed}\left(CI, 0, t_f, n_f, D\right)$

$CI^T = (100 \;\; 0 \;\; 1.2)$



Controlador con Integrador k_ω

Simulación para entrada escalón en la referencia de velocidad. Se conseva el controlador anterior y la perturbación no nula.



$\Omega_{ld} := 3000$

$\omega_{ld}(t) := \Omega_{ld}\cdot(1 - 0.\cdot\Phi(t-1))\cdot\frac{1}{rr}$

$p(t) := 60$

$A_p := \text{augment}\left(\text{augment}(A, b\cdot k_a)^T, \text{augment}(-c\cdot k_c, 0)^T\right)^T$

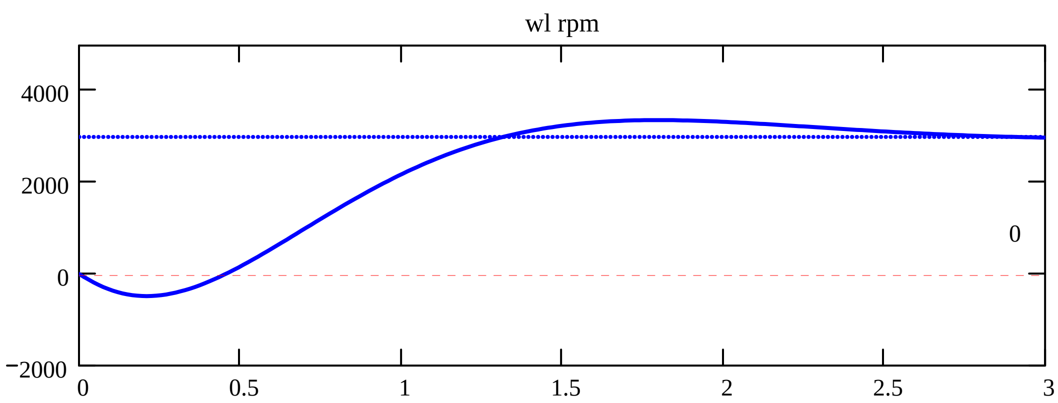
$b_p := \text{augment}\left(b^T\cdot 0, k_c\right)^T \qquad e_p := \text{augment}\left(e^T, 0\right)^T$

$D(t,x) := A_p\cdot\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T + b_p\cdot\omega_{ld}(t) + e_p\cdot p(t)$

$CI := -\left[A_p^{-1}\cdot\left(b_p\cdot\omega_{ld}(0) + e_p\cdot p(0)\right)\right]\cdot 0$

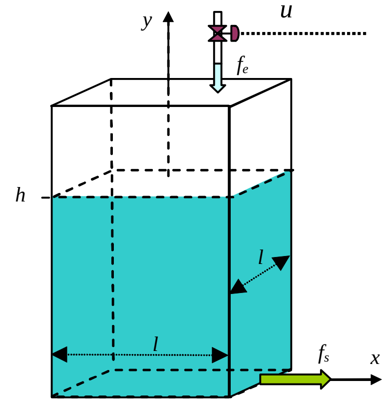
$Z_a := \text{rkfixed}(CI, 0, t_f, n_f, D)$

$CI^T = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$



Error en Estado Estacionario en Sistemas Discretos

Problema Ilustrar la eficacia del control realimentado.



Estanque Parámetros.

$f_{s0} := 0$ $\quad h_a(s) = 1$ $\quad A_e := 2.5$ $\quad h_d(t) := 2\cdot\Phi(t)$

Variable de Estado

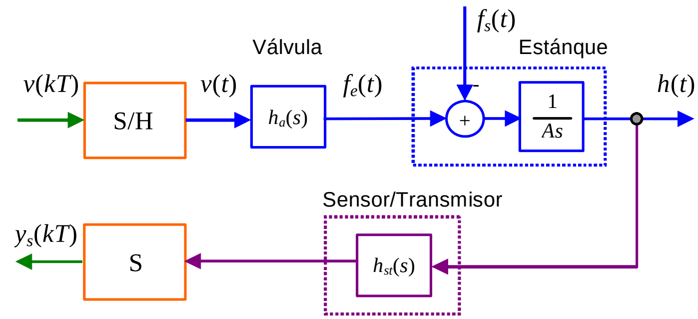
$x_1 = h$

Modelo Continuo

$$\frac{d}{dt}h = \frac{1}{A_e}\cdot(f_e - f_s) \qquad A_t := 0 \qquad b_t := \frac{1}{A_e} \qquad e_t := -\frac{1}{A_e} \qquad c_t := 1$$

Modelo Discreto $\quad T := 0.25$

$$A_k := 1 \qquad b_k := \frac{T}{A_e} \qquad e_k := -\frac{T}{A_e} \qquad c_k := 1$$



Caso I Entrada y perturbación.

$v(k) := 0 \qquad f_s(t) := f_{s0} + \Phi(t-1) \qquad p_d(k) := f_s(k\cdot T)$

Simulación Discreto

$$\xi_o := 2 \qquad \xi(k) := \text{if}\left(k = 0, \xi_o, A_k^{\,k}\cdot\xi_o + \sum_{j=0}^{k-1} A_k^{\,k-j-1}\cdot b_k\cdot v(j) + \sum_{j=0}^{k-1} A_k^{\,k-j-1}\cdot e_k\cdot p_d(j)\right)$$

$$t_f := 6 \qquad k_f := \frac{t_f}{T} \qquad k := 0..k_f \qquad n_f := 400 \qquad n := 0..n_f$$

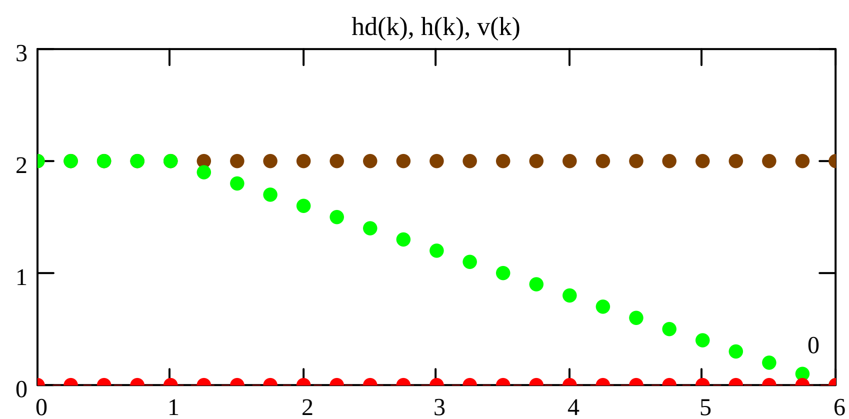
Entrada Continua

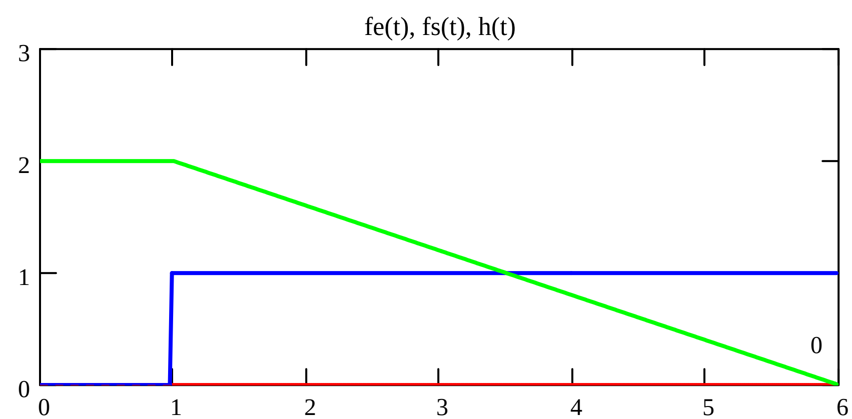
$$f_e(t) := v\left(\text{trunc}\left(\frac{t}{T}\right)\right)$$

Simulación Continuo

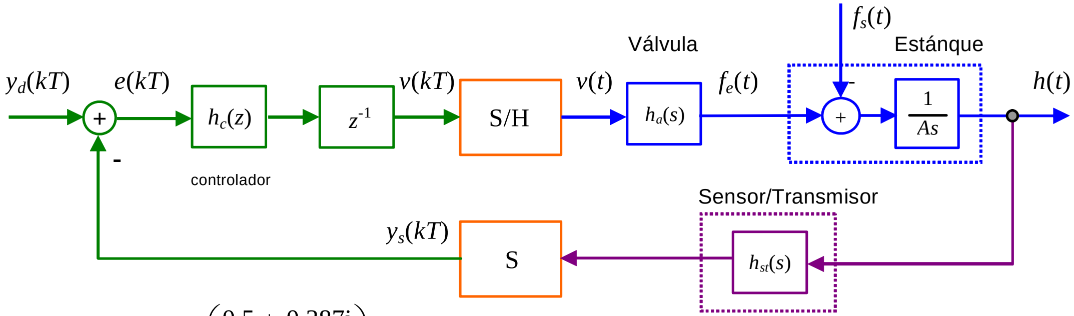
$$D(t,x) := A_t\cdot x_1 + b_t\cdot f_e(t) + e_t\cdot f_s(t) \qquad CI := 2$$

$$Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$$





Caso II — Controlador Discreto de Ganacia k_c. **Cambio en la referencia.**



Controlador Discreto

$k_c := 4 \quad A_{ck} := 0 \quad b_{ck} := 1 \quad c_{ck} := k_c \quad d_{ck} := 0$

Sistema Resultante

$A_{Tk} := \text{stack}(\text{augment}(A_k, b_k \cdot c_{ck}), \text{augment}(-b_{ck} \cdot c_k, A_{ck}))$

$c_{Tk} := \text{augment}(c_k, c_{ck} \cdot 0) \quad e_{Tk} := \text{stack}(e_k, c_{ck} \cdot 0)$

$b_{Tk} := \text{stack}(b_k \cdot 0, b_{ck})$

$h_{Tk}(z) := c_{Tk} \cdot (z \cdot \text{identity}(2) - A_{Tk})^{-1} \cdot b_{Tk}$

$\text{eigenvals}(A_{Tk}) = \begin{pmatrix} 0.5 + 0.387i \\ 0.5 - 0.387i \end{pmatrix} \quad h_{Tk}(1) = 1$

Entradas

$h_d(t) := 2 \cdot \Phi(t) + \Phi(t-1) \quad f_s(t) := f_{s0} \quad p_d(k) := f_s(k \cdot T)$

Simulación Discreta

$y_d(k) := h_d(k \cdot T) \quad \xi_o := (2 \;\; 0)^T$

$\xi(k) := \text{if}\left(k = 0, \xi_o, A_{Tk}{}^k \cdot \xi_o + \sum_{j=0}^{k-1} A_{Tk}{}^{k-j-1} \cdot b_{Tk} \cdot y_d(j) + \sum_{j=0}^{k-1} A_{Tk}{}^{k-j-1} \cdot e_{Tk} \cdot p_d(j)\right)$

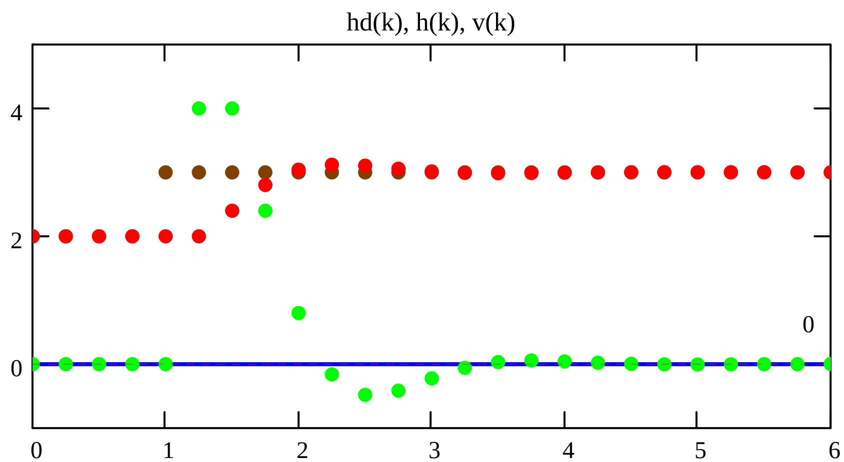
$t_f := 6 \quad k_f := \frac{t_f}{T} \quad k := 0..k_f \quad n_f := 400 \quad n := 0..n_f \quad t := 0, \frac{t_f}{n_f}..t_f$

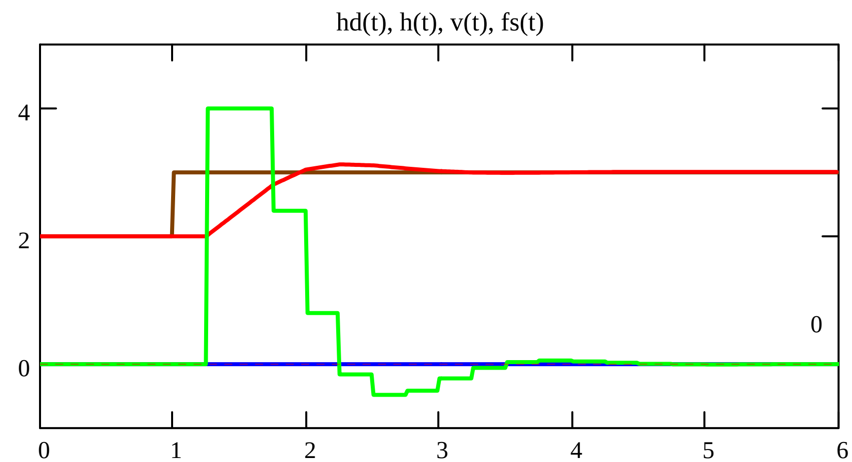
Entrada Continuo

$v(k) := c_{ck} \cdot \xi(k)_2 \quad f_e(t) := c_{ck} \cdot \xi\left(\text{trunc}\left(\frac{t}{T}\right)\right)_2$

Simulación Continua

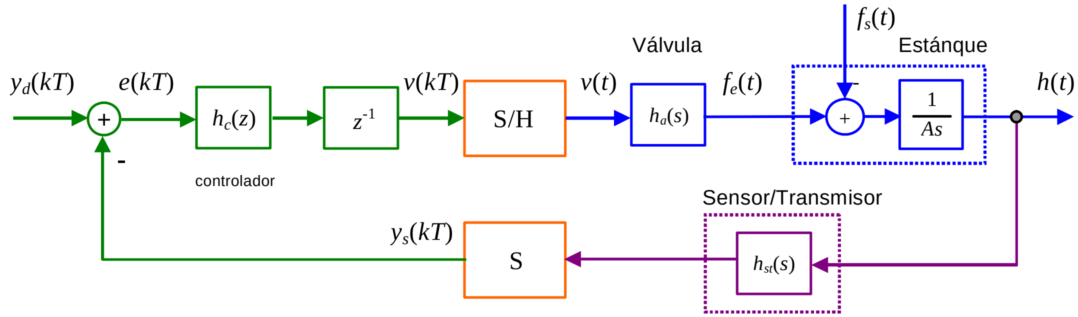
$D(t,x) := A_t \cdot x_1 + b_t \cdot f_e(t) + e_t \cdot f_s(t) \quad CI := 2 \quad Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$





Caso III Controlador Discreto de Ganacia k_c, **cambio en la perturbación**.

Controlador Discreto: $k_c := 4 \quad A_{ck} := 0 \quad b_{ck} := 1 \quad c_{ck} := k_c \quad d_{ck} := 0$



Sistema Resultante:

$A_{Tk} := \text{stack}\left(\text{augment}\left(A_k, b_k \cdot c_{ck}\right), \text{augment}\left(-b_{ck} \cdot c_k, A_{ck}\right)\right)$

$c_{Tk} := \text{augment}\left(c_k, c_{ck} \cdot 0\right) \qquad e_{Tk} := \text{stack}\left(e_k, c_{ck} \cdot 0\right)$

$b_{Tk} := \text{stack}\left(b_k \cdot 0, b_{ck}\right)$

$h_{Tk}(z) := c_{Tk} \cdot \left(z \cdot \text{identity}(2) - A_{Tk}\right)^{-1} \cdot e_{Tk}$

$\text{eigenvals}\left(A_{Tk}\right) = \begin{pmatrix} 0.5 + 0.387i \\ 0.5 - 0.387i \end{pmatrix} \qquad h_{Tk}(1) = -0.25$

Entradas: $h_d(t) := 2 \cdot \Phi(t) \qquad f_s(t) := f_{s0} + \Phi(t-1) \qquad p_d(k) := f_s(k \cdot T)$

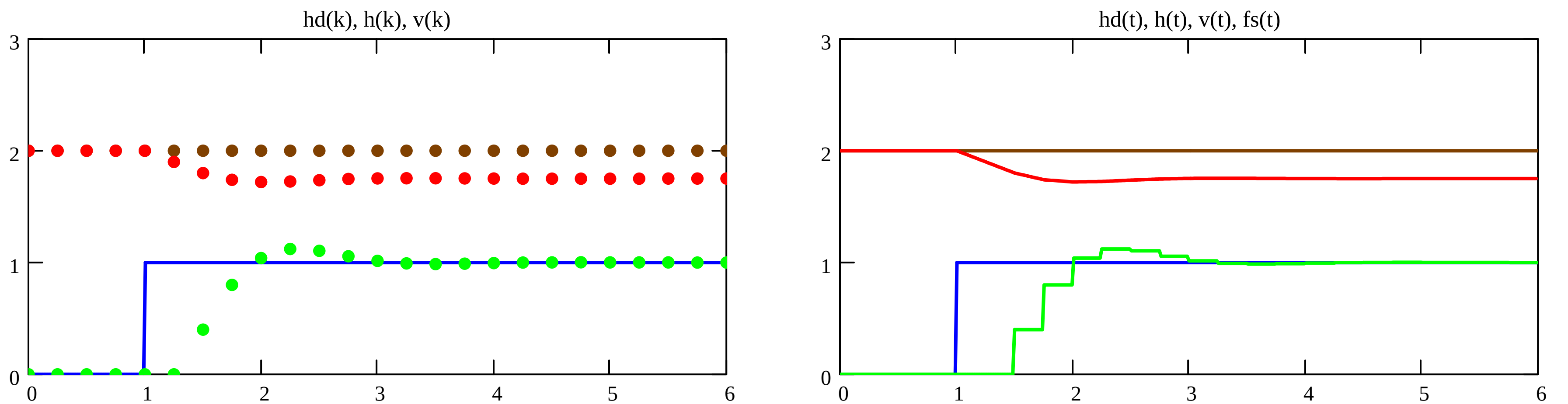
Simulación Discreta: $y_d(k) := h_d(k \cdot T) \qquad \xi_o := (2 \quad 0)^T$

$\xi(k) := \text{if}\left(k = 0, \xi_o, A_{Tk}^{\ k} \cdot \xi_o + \sum_{j=0}^{k-1} A_{Tk}^{\ k-j-1} \cdot b_{Tk} \cdot y_d(j) + \sum_{j=0}^{k-1} A_{Tk}^{\ k-j-1} \cdot e_{Tk} \cdot p_d(j)\right)$

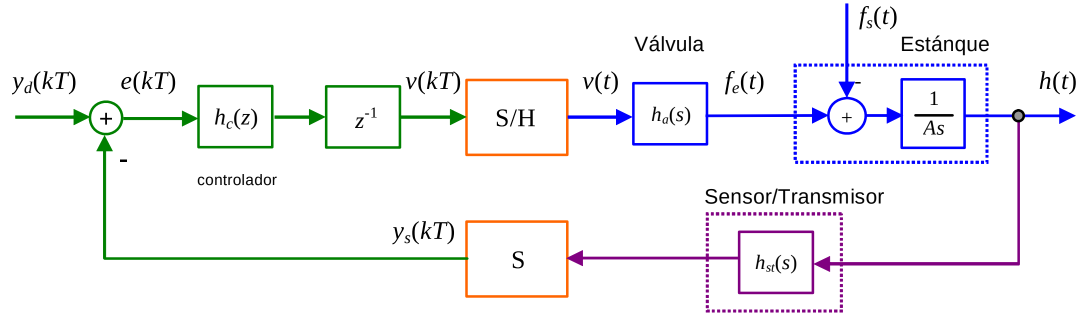
$t_f := 6 \quad k_f := \frac{t_f}{T} \quad k := 0 .. k_f \quad n_f := 400 \quad n := 0 .. n_f \quad t := 0, \frac{t_f}{n_f} .. t_f$

Entrada Continuo: $v(k) := c_{ck} \cdot \xi(k)_2 \qquad f_e(t) := c_{ck} \cdot \xi\left(\text{trunc}\left(\frac{t}{T}\right)\right)_2$

Simulación Continua: $D(t, x) := A_t \cdot x_1 + b_t \cdot f_e(t) + e_t \cdot f_s(t) \qquad CI := 2 \qquad Z_{al} := \text{rkfixed}\left(CI, 0, t_f, n_f, D\right)$



Caso IV Controlador Discreto de Ganacia k_c, **cambio rampa en la referencia**.



Controlador Discreto para e_{ss} dado

$e_{ss} := 2 \qquad k_c := \dfrac{T}{e_{ss} \cdot c_k \cdot b_k}$

$A_{ck} := 0 \qquad b_{ck} := 1 \qquad c_{ck} := k_c \qquad d_{ck} := 0$

Sistema Resultante

$A_{Tk} := \text{stack}\left(\text{augment}\left(A_k, b_k \cdot c_{ck}\right), \text{augment}\left(-b_{ck} \cdot c_k, A_{ck}\right)\right)$

$c_{Tk} := \text{augment}\left(c_k, c_{ck} \cdot 0\right) \qquad e_{Tk} := \text{stack}\left(e_k, c_{ck} \cdot 0\right)$

$b_{Tk} := \text{stack}\left(b_k \cdot 0, b_{ck}\right)$

$h_{Tk}(z) := c_{Tk} \cdot \left(z \cdot \text{identity}(2) - A_{Tk}\right)^{-1} \cdot b_{Tk}$

$\text{eigenvals}\left(A_{Tk}\right) = \begin{pmatrix} 0.854 \\ 0.146 \end{pmatrix}$

$h_{Tk}(1) = 1$

$k_v = \lim_{z \to 1} \dfrac{(z-1)}{T} \cdot \dfrac{k_c}{z} \cdot c_k \cdot \left(z \cdot \text{identity}(1) - A_k\right)^{-1} \cdot b_k$

$k_v = \dfrac{1}{T} \cdot k_c \cdot c_k \cdot b_k = \dfrac{1}{e_{ss}}$

Entradas

$h_d(t) := t \cdot \Phi(t) \qquad f_s(t) := f_{s0} \qquad p_d(k) := f_s(k \cdot T)$

Simulación Discreta

$y_d(k) := h_d(k \cdot T)$

$\xi_o := (2 \quad 0)^T$

$\xi(k) := \text{if}\left(k = 0, \xi_o, A_{Tk}{}^{k} \cdot \xi_o + \sum_{j=0}^{k-1} A_{Tk}{}^{k-j-1} \cdot b_{Tk} \cdot y_d(j) + \sum_{j=0}^{k-1} A_{Tk}{}^{k-j-1} \cdot e_{Tk} \cdot p_d(j)\right)$

$t_f := 6 \qquad k_f := \dfrac{t_f}{T} \qquad k := 0 .. k_f \qquad n_f := 400 \qquad n := 0 .. n_f \qquad t := 0, \dfrac{t_f}{n_f} .. t_f$

Entrada Continuo

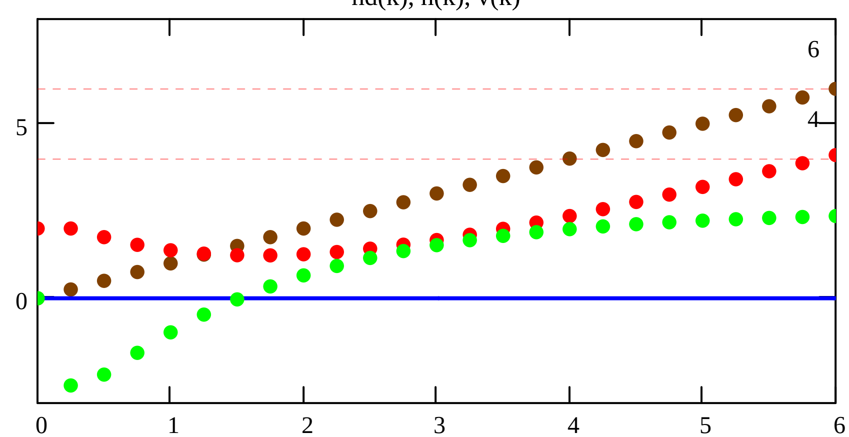
$v(k) := c_{ck} \cdot \xi(k)_2 \qquad f_e(t) := c_{ck} \cdot \xi\left(\text{trunc}\left(\dfrac{t}{T}\right)\right)_2$

Simulación Continua

$D(t, x) := A_t \cdot x_1 + b_t \cdot f_e(t) + e_t \cdot f_s(t) \qquad CI := 2$

$Z_{al} := \text{rkfixed}\left(CI, 0, t_f, n_f, D\right)$

hd(k), h(k), v(k)



hd(t), h(t), v(t), fs(t)