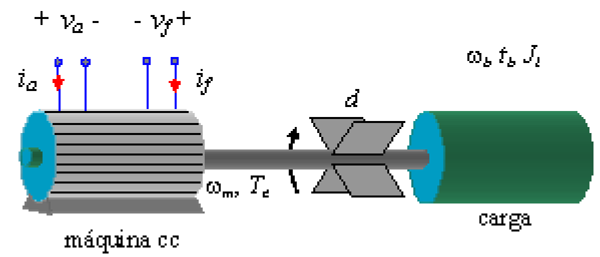
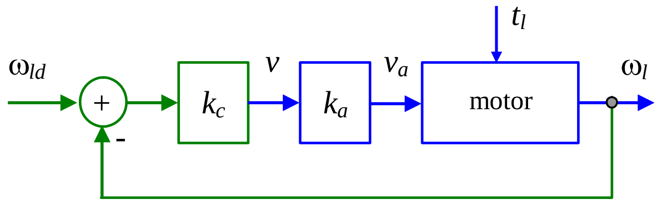
Control Discreto en Plantas Continuas

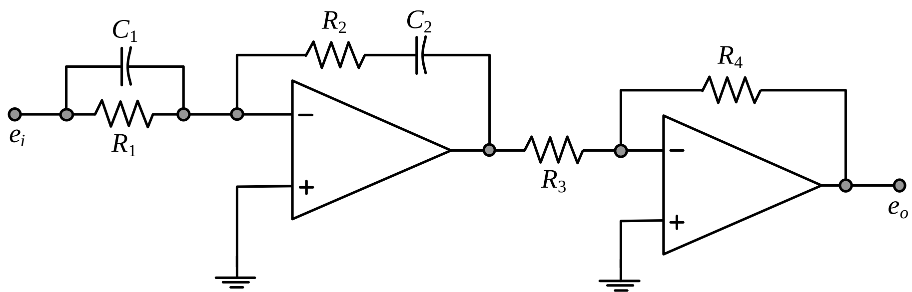


Problema Presentar el controlador discreto en un sistema continuo.

Control Análogo



Controlador Análogo PID

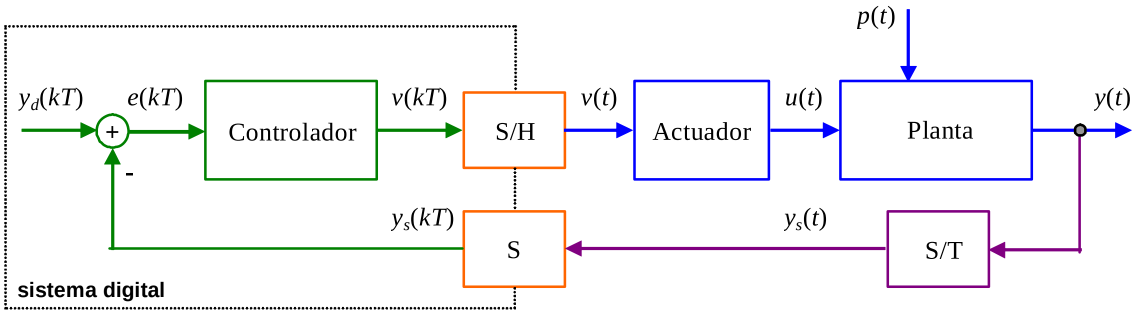


El controlador está implementado con componentes análogas.

F. de T. del Controlador Análogo PID

$$\frac{e_o}{e_i} = \frac{R_4}{R_3}\frac{R_1C_1 + R_2C_2}{R_1C_2}\left\{1 + \frac{1}{(R_1C_1 + R_2C_2)s} + \frac{R_1C_1R_2C_2}{R_1C_1 + R_2C_2}s\right\}$$

Control Discreto



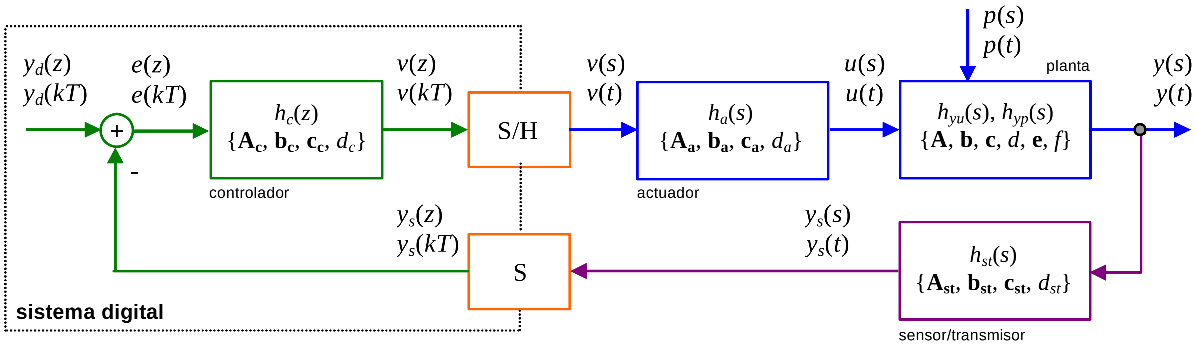
Controlador Digital

El controlador está implementado con componentes digitales (sample & hold, A/D, D/A).

Controladore-s Discretos

$$\begin{bmatrix}\zeta_1(kT+T)\\ \zeta_2(kT+T)\end{bmatrix} = \begin{bmatrix}-0.2 & 0\\ 0 & -0.5\end{bmatrix}\begin{bmatrix}\zeta_1(kT)\\ \zeta_2(kT)\end{bmatrix} + \begin{bmatrix}1\\ 0.2\end{bmatrix} e(kT), \qquad v(kT) = [1 \quad 0]\begin{bmatrix}\zeta_1(kT)\\ \zeta_2(kT)\end{bmatrix} + e(kT)$$

Formas de Representación



$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) + \mathbf{e}p(t), \quad y(t) = \mathbf{c}\mathbf{x}(t) + du(t) + fp(t)$$ planta

$$\dot{\boldsymbol{\eta}}(t) = \mathbf{A_a}\boldsymbol{\eta}(t) + \mathbf{b_a}v(t), \quad u(t) = \mathbf{c_a}\boldsymbol{\eta}(t) + d_a v(t)$$ actuador

$$\dot{\boldsymbol{\gamma}}(t) = \mathbf{A_{st}}\boldsymbol{\gamma}(t) + \mathbf{b_{st}}y(t), \quad y_s(t) = \mathbf{c_{st}}\boldsymbol{\gamma}(t) + d_{st}y(t)$$ sensor/transmisor

Controladores Discretos

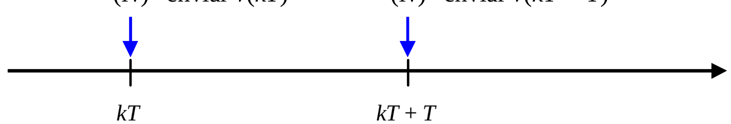
$$\begin{bmatrix} \zeta_1(kT+T) \\ \zeta_2(kT+T) \end{bmatrix} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} \zeta_1(kT) \\ \zeta_2(kT) \end{bmatrix} + \begin{bmatrix} 1 \\ 0.2 \end{bmatrix} e(kT), \qquad v(kT) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \zeta_1(kT) \\ \zeta_2(kT) \end{bmatrix} + e(kT)$$

$$h_c(z) = \frac{v(z)}{e(z)} = \frac{z^2 + 0.2}{z(z^2 + 0.5z)}$$

Algoritmo Ideal

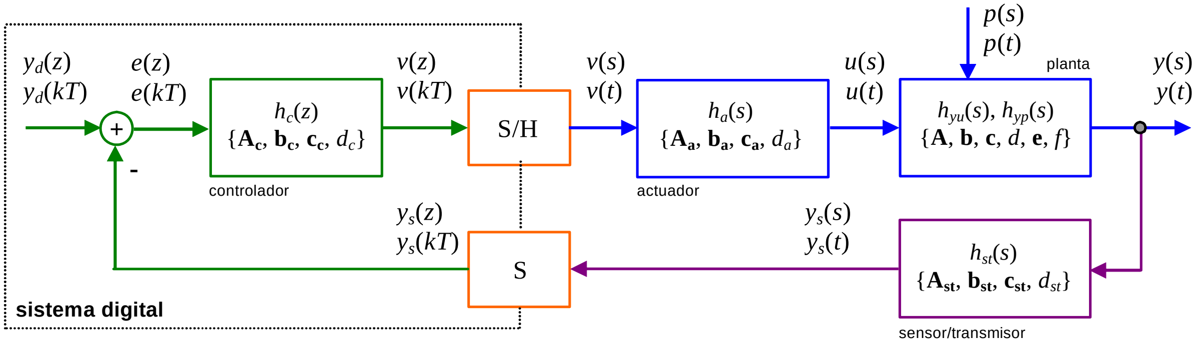
a)

(i) muestrear $y_s(kT)$	(i) muestrear $y_s(kT + T)$
(ii) calcular $e(kT)$	(ii) calcular $e(kT + T)$
(iii) calcular $v(kT)$	(iii) calcular $v(kT + T)$
(iv) enviar $v(kT)$	(iv) enviar $v(kT + T)$

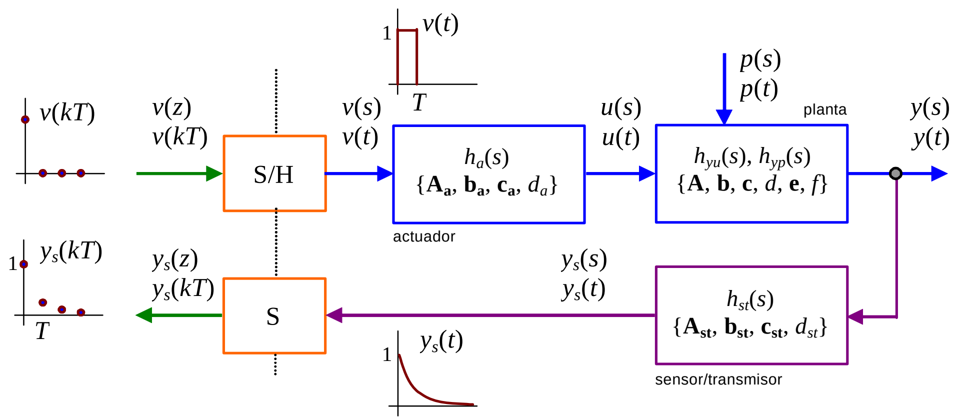


Conviene una representación uniforme. Se opta por pasar todo a discreto.

Problema Encontrar la F. de T. equivalente de la parte análoga del esquema de control.



Se busca la respuesta a impulso del sistema. Luego se toma la T. Z. de ésta, la cual corresponde a la F. de T. discreta equivalente de la parte análoga del sistema.



$$v(s) = \mathcal{L}(u(t) - u(t-T)) = \frac{1-e^{-sT}}{s} \qquad u(s) = \frac{1-e^{-sT}}{s} h_a(s) \qquad y(s) = \frac{1-e^{-sT}}{s} h_a(s) h_{yu}(s)$$

$$y_s(s) = \frac{1-e^{-sT}}{s} h_a(s) h_{yu}(s) h_{st}(s) \qquad y_s(t) = \mathcal{L}^{-1}\{y_s(s)\} = \mathcal{L}^{-1}\left\{\frac{1-e^{-sT}}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}$$

$$y_s(kT) = y_s(t)\Big|_{t=kT} = \mathcal{L}^{-1}\left\{\frac{1-e^{-sT}}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Bigg|_{t=kT}$$

$$\frac{y_s(z)}{v(z)} = \mathcal{Z}\{y_s(kT)\} = \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1-e^{-sT}}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\}$$

$$= \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\} - \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{e^{-sT}}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\}$$

$$= \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\} - z^{-1}\mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\}$$

$$= (1-z^{-1})\mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\}$$

$$= \frac{z-1}{z}\mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{\frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s)\right\}\Big|_{t=kT} \right\}$$

F. de T. buscada.

Ejemplo 1

$h_{yu}(s) := \frac{1}{s}$ $\qquad h_a(s) := 1$ $\qquad h_{st}(s) := 1$

$$h_d(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l} \text{invlaplace}, s \\ \text{substitute}, t = k\cdot T \\ \text{ztrans}, k \\ \text{collect}, z \end{array} \rightarrow T\cdot\frac{z}{(z-1)^2} \right] \rightarrow \frac{1}{z-1}\cdot T$$

$h_d(z)$ solve, z $\rightarrow$

cero de la F. de T. discreta.

Ejemplo 2

$h_{yu}(s) := \dfrac{s - a}{s + a}$ $\quad h_a(s) := 1$ $\quad h_{st}(s) := 1$ $\quad T := 1$

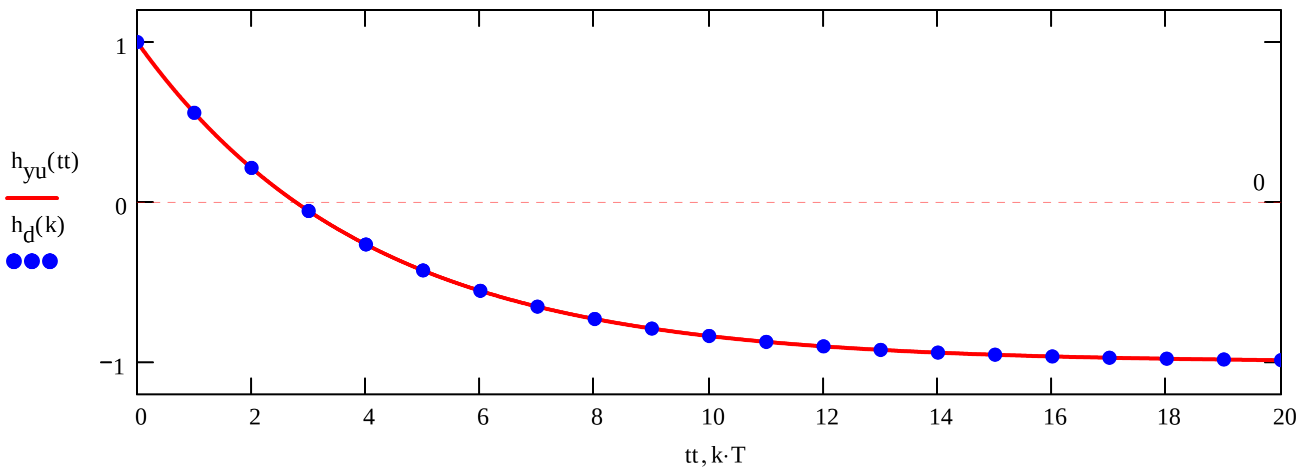
$$h_d(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l}\text{invlaplace}, s\\ \text{substitute}, t = k\cdot T\\ \text{ztrans}, k\\ \text{collect}, z\end{array} \rightarrow z\cdot\frac{z - 2 + \exp(-a)}{(z - \exp(-a))\cdot(z-1)}\right] \rightarrow \frac{z - 2 + \exp(-a)}{z - \exp(-a)}$$

$h_d(z)$ solve, z $\rightarrow \dfrac{2\cdot\exp(a) - 1}{\exp(a)}$ cero de la F. de T. discreta.

$a := 0.25$ $\quad t_f := 20$ $\quad tt := 0, 0.01 .. t_f$ $\quad k := 0 .. \dfrac{t_f}{T}$ $\quad \Delta(k) := \text{if}(k = 0, 1, 0)$

$$h_{yu}(t) := h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \text{ invlaplace}, s \rightarrow 2\cdot\exp(-a\cdot t) - 1$$

$$h_d(k) := h_d(z)\cdot\frac{z}{z-1} \;\middle|\; \begin{array}{l}\text{invztrans}, z\\ \text{substitute}, n = k\end{array} \rightarrow -1 + 2\cdot\left(\frac{1}{\exp(a)}\right)^k$$



Sistemas con Retardo

$$h_{yu}(s) = k_p \frac{1}{\tau s + 1} e^{-t_r s} = h_{yuo}(s) e^{-lTs}$$

$$\begin{aligned}
\frac{y_s(z)}{v(z)} = \mathcal{Z}\{y_s(kT)\} &= (1 - z^{-1}) \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{s} h_a(s) h_{yu}(s) h_{st}(s) \right\}\Big|_{t=kT} \right\} \\
&= (1 - z^{-1}) \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{s} h_a(s) h_{yuo}(s) e^{-lTs} h_{st}(s) \right\}\Big|_{t=kT} \right\} \\
&= (1 - z^{-1}) z^{-l} \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{s} h_a(s) h_{yuo}(s) h_{st}(s) \right\}\Big|_{t=kT} \right\} \\
&= \frac{z - 1}{z^{l+1}} \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{s} h_a(s) h_{yuo}(s) h_{st}(s) \right\}\Big|_{t=kT} \right\}
\end{aligned}$$

$h_{yu}(s) := \frac{s-b}{s+b}\cdot e^{-6\cdot s}$ $h_a(s) := 1$ $h_{st}(s) := 1$ $T := 2$

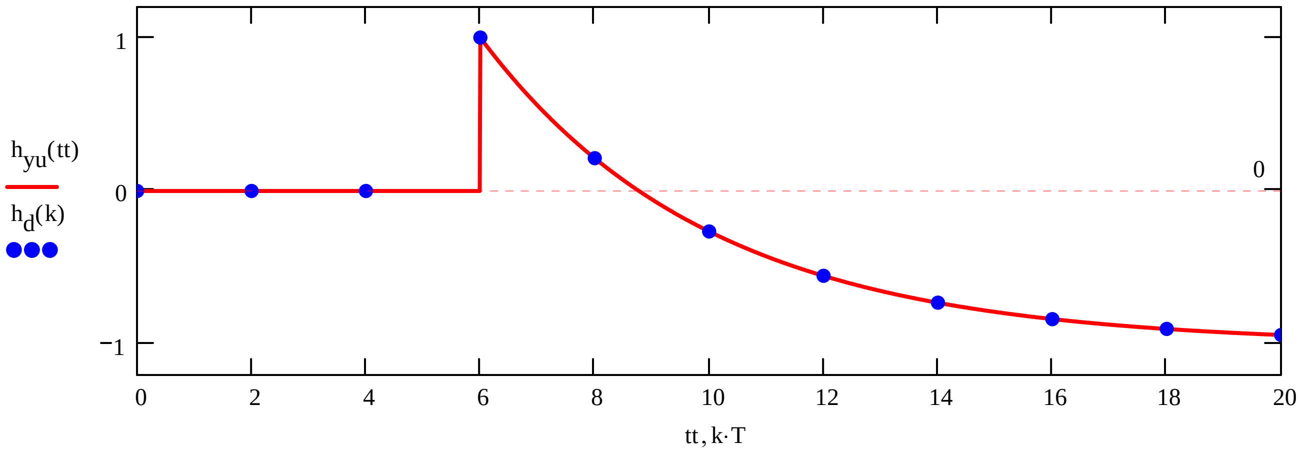
$$h_d(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s}\ \middle|\begin{array}{l}\text{invlaplace}, s\\ \text{substitute}, t = kk\cdot T\\ \text{ztrans}, kk\\ \text{collect}, z\end{array} \rightarrow \frac{z+\exp(-2\cdot b)-2}{(z-\exp(-2\cdot b))\cdot z^2\cdot(z-1)}\right] \rightarrow \frac{1}{z^3}\cdot\frac{z+\exp(-2\cdot b)-2}{z-\exp(-2\cdot b)}$$

$h_d(z)$ solve, z $\rightarrow -\exp(-2\cdot b)+2$ cero de la F. de T. discreta. $b := 0.25$

$a := 0.25$ $t_f := 20$ $tt := 0, 0.01 .. t_f$ $k := 0 .. \frac{t_f}{T}$ $\Delta(k) := \text{if}(k = 0, 1, 0)$

$$h_{yu}(t) := h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s}\ \text{invlaplace}, s \rightarrow 2\cdot\Phi(t-6)\cdot\exp[-b\cdot(t-6)] - \Phi(t-6)$$

$$h_d(k) := h_d(z)\cdot\frac{z}{z-1}\ \middle|\begin{array}{l}\text{invztrans}, z\\ \text{substitute}, n = k\end{array} \rightarrow \Delta(k) - 2\cdot\Delta(k)\cdot\exp(b)^6 + \Delta(k-1) - 2\cdot\Delta(k-1)\cdot\exp(b)^4 + \Delta(k-2) - 2\cdot\Delta(k-2)\cdot\exp(b)^2 + 2\cdot\left(\frac{1}{\exp(b)^2}\right)^k\cdot\exp(b)^6 - 1$$



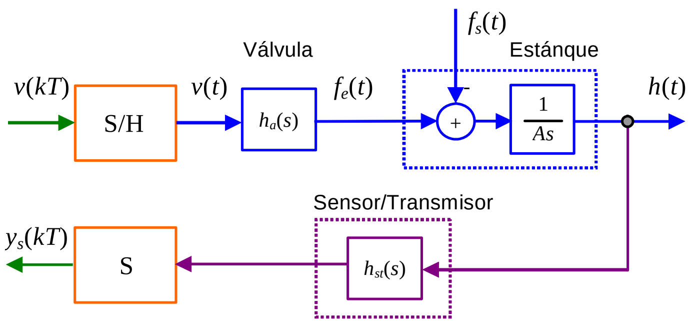
Ejemplo 4 Estanque.

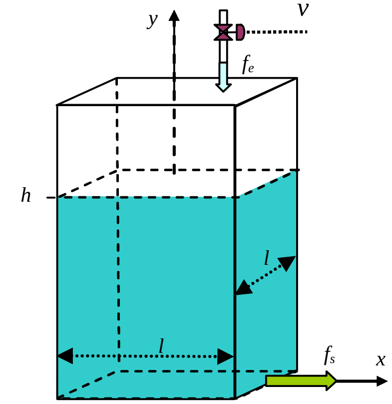
$f_s(t) := 0 \qquad T := .25 \qquad h_a(s) := k_a \cdot e^{-4 \cdot T \cdot s} \qquad h_{st}(s) := k_{st}$

Modelo Continuo

$\frac{d}{dt}h = \frac{1}{A_e}\cdot\left(f_e - f_s\right) \qquad A_e := A_e \qquad h_{yu}(s) := \frac{1}{s \cdot A_e}$

$$h_{vys}(s) := \left(h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s) \rightarrow k_a \cdot \frac{\exp(-1.00 \cdot s)}{s \cdot A_e} \cdot k_{st}\right) \rightarrow k_a \cdot \frac{\exp(-1.00 \cdot s)}{s \cdot A_e} \cdot k_{st}$$





$$\text{del}_T(t) := \frac{1}{T}\cdot\left(\Phi(t) - \Phi(t - T)\right)$$

Modelo discreto

$$h_{vys}(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l} \text{invlaplace}, s \\ \text{substitute}, t = kk \cdot T \\ \text{ztrans}, kk \\ \text{collect}, z \\ \text{factor} \end{array} \rightarrow \frac{1}{4}\cdot k_a \cdot \frac{k_{st}}{A_e \cdot (z-1)^2 \cdot z^3}\right] \rightarrow \frac{1}{4 \cdot (z-1) \cdot z^4} \cdot k_a \cdot \frac{k_{st}}{A_e}$$

Salida para Escalón con Retardo 2T

$$h_{vys}(k) := h_{vys}(z)\cdot\frac{z}{(z-1)\cdot z^2} \;\middle|\; \begin{array}{l} \text{invztrans}, z \\ \text{substitute}, n = k \\ \text{factor} \end{array} \rightarrow \frac{1}{4}\cdot k_a \cdot k_{st} \cdot \frac{\Delta(k-5) + 2\cdot\Delta(k-4) + 3\cdot\Delta(k-3) + 4\cdot\Delta(k-2) + 5\cdot\Delta(k-1) + 6\cdot\Delta(k) - 6 + k}{A_e}$$

Parámetros $A_e := 2.5$ $k_a := 1$ $k_{st} := 1$

Modelo Continuo $A_t := 0$ $b_t := \frac{1}{A_e}$ $e_t := -\frac{1}{A_e}$ $c_t := 1$

Entrada.

$v(k) := \Phi(k-2)$

Salida.

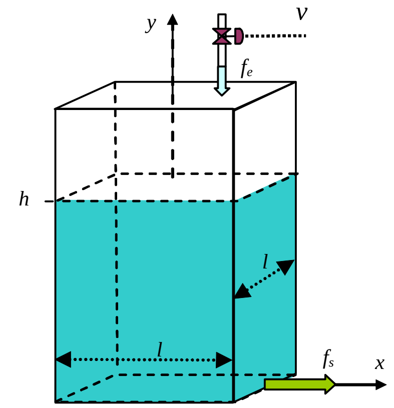
$h(k) := \frac{1}{4} \cdot k_a \cdot k_{st} \cdot \frac{\Delta(k-5) + 2\cdot\Delta(k-4) + 3\cdot\Delta(k-3) + 4\cdot\Delta(k-2) + 5\cdot\Delta(k-1) + 6\cdot\Delta(k) - 6 + k}{A_e}$

Cáculo de v(t) $v_c(t) := \sum_{i=0}^{t\cdot T^{-1}} v(i)\,del_T(t - i\cdot T)\cdot T$

Entrada Continua $f_e(t) := v_c(t - 4\cdot T)$

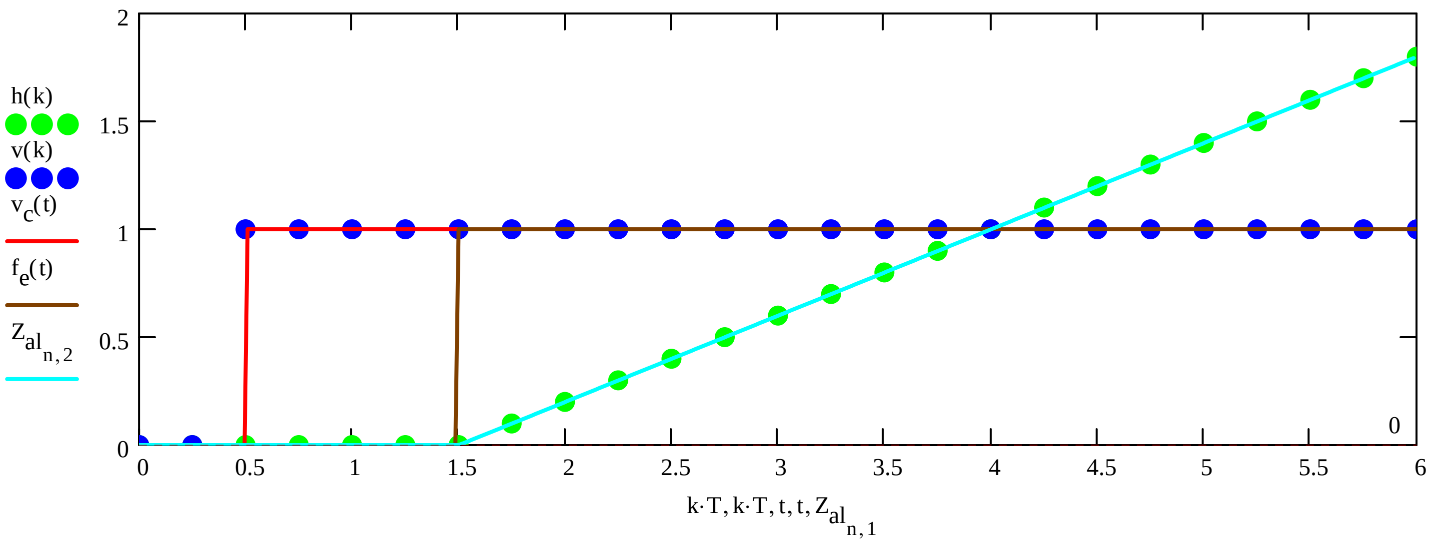
$t_f := 6$ $k_f := \frac{t_f}{T}$ $k := 0 .. k_f$

$n_f := 400$ $n := 0 .. n_f$ $t := 0, \frac{t_f}{n_f} .. t_f$



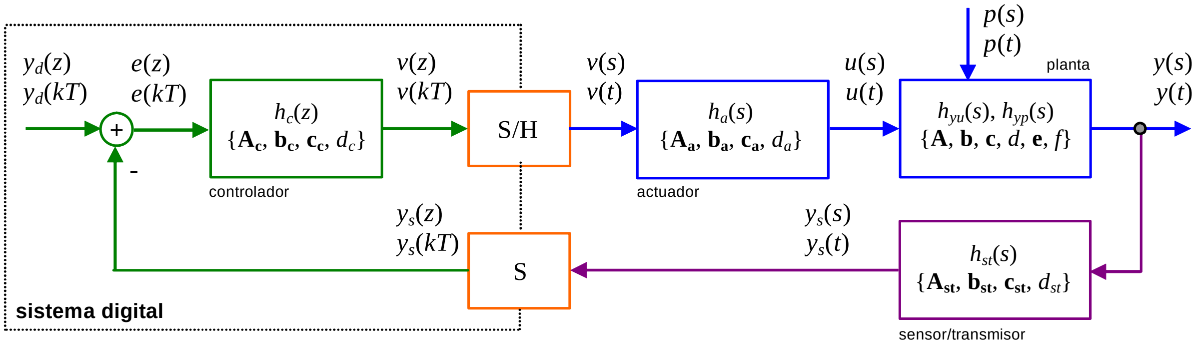
Simulación Continuo $D(t,x) := A_t\cdot x_1 + b_t\cdot f_e(t) + e_t\cdot f_s(t)$

$CI := 0$ $Z_{al} := rkfixed(CI, 0, t_f, n_f, D)$



Problema Encontrar una representación en variables de estado entre la entrada $v(kT)$ y la salida $y_s(kT)$.

Sin Retardo



$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) + \mathbf{e}p(t), \quad y(t) = \mathbf{c}\mathbf{x}(t) + du(t) + fp(t)$$ planta

$$\dot{\boldsymbol{\eta}}(t) = \mathbf{A_a}\boldsymbol{\eta}(t) + \mathbf{b_a}v(t), \quad u(t) = \mathbf{c_a}\boldsymbol{\eta}(t) + d_a v(t)$$ actuador

$$\dot{\boldsymbol{\gamma}}(t) = \mathbf{A_{st}}\boldsymbol{\gamma}(t) + \mathbf{b_{st}}y(t), \quad y_s(t) = \mathbf{c_{st}}\boldsymbol{\gamma}(t) + d_{st}y(t)$$ sensor/transmisor

se agrupan las variables de estado de manera de tener una nueva representación con $v(t)$ y $p(t)$ como entradas e $y_s(t)$ como salida.

$$\begin{bmatrix} \dot{\boldsymbol{\eta}}(t) \\ \dot{\mathbf{x}}(t) \\ \dot{\boldsymbol{\gamma}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A_a} & \mathbf{0} & \mathbf{0} \\ \mathbf{b}\mathbf{c_a} & \mathbf{A} & \mathbf{0} \\ \mathbf{b_{st}}d\mathbf{c_a} & \mathbf{b_{st}}\mathbf{c} & \mathbf{A_{st}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}(t) \\ \mathbf{x}(t) \\ \boldsymbol{\gamma}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{b_a} \\ \mathbf{b}d_a \\ \mathbf{b_{st}}dd_a \end{bmatrix} v(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{e} \\ \mathbf{b_{st}}f \end{bmatrix} p(t) \qquad y_s(t) = [d_{st}d\mathbf{c_a} \quad d_{st}\mathbf{c} \quad \mathbf{c_{st}}] \begin{bmatrix} \boldsymbol{\eta}(t) \\ \mathbf{x}(t) \\ \boldsymbol{\gamma}(t) \end{bmatrix} + d_{st}dd_a v(t) + d_{st}fp(t)$$

esta representación se puede escribir definiendo nuevas matrices y vectores de parámetros como,

$$\dot{\boldsymbol{\psi}}(t) = \mathcal{A}\boldsymbol{\psi}(t) + \mathcal{b}v(t) + \mathcal{e}p(t), \quad y_s(t) = \mathcal{c}\boldsymbol{\psi}(t) + \mathcal{d}v(t) + \mathcal{f}p(t) \qquad \boldsymbol{\psi}(t) = [\boldsymbol{\eta}(t)^T\ \mathbf{x}(t)^T\ \boldsymbol{\gamma}(t)^T]^T$$

esta ecuaciones tienen un equivalente discreto considerando el S/H de la entrada dado por,

$$\boldsymbol{\psi}(kT+T) = \mathbf{A_d}\boldsymbol{\psi}(kT) + \mathbf{b_d}v(kT) + \mathbf{e_d}p(kT), \quad y_s(kT) = \mathbf{c_d}\boldsymbol{\psi}(kT) + d_d v(kT) + f_d p(kT)$$

en donde las nuevas matrices están dadas por,

$$\mathbf{A_d} = \Phi(T) = e^{\mathcal{A}T}, \ \mathbf{b_d} = \left\{\int_0^T e^{\mathcal{A}(T-\sigma)}\mathcal{b}d\sigma\right\}, \ \mathbf{e_d} = \left\{\int_0^T e^{\mathcal{A}(T-\sigma)}\mathcal{e}d\sigma\right\}, \ \mathbf{c_d} = \mathcal{c}, \ d_d = \mathcal{d} \ \text{y} \ f_d = \mathcal{f}$$

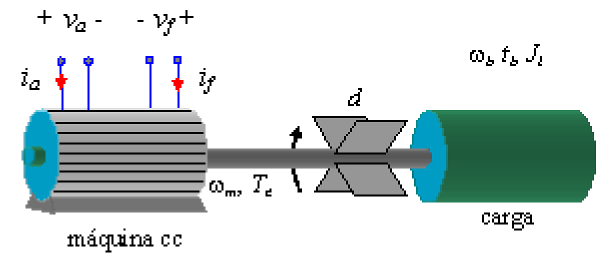
Caso I — Motor de Corriente Continua.

Parámetros

$d := 0.08$ $\quad R := 1.2$ $\quad h_a(s) := 1$ $\quad h_{st}(s) := 1$

$k_m := 0.6$ $\quad L := 50\cdot 10^{-3}$ $\quad J_l := 0.135$

$t_f := 10$ $\quad n_f := 1000$ $\quad n := 0 .. n_f$



Modelo Continuo

$$A_c := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \qquad b_c := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \qquad e_c := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix}$$

Variables de Estado

$x_1 = i_a \quad x_2 = \omega$

Simulación Continua

$T := 0.5$ $\quad t_l(t) := \Phi(t-7)$ $\quad v(k) := k\cdot T\cdot\Phi(k\cdot T) - (k\cdot T - 6\cdot T)\cdot\Phi(k\cdot T - 6\cdot T$ $\quad \mathrm{del}(k) := \mathrm{if}(k = 0, 1, 0)$

$$\mathrm{del}_T(t) := \frac{1}{T}\cdot(\Phi(t) - \Phi(t-T)) \qquad v(t) := \sum_{i=0}^{t\cdot T^{-1}} v(i-2)\,\mathrm{del}_T(t - i\cdot T)\cdot T \qquad v_a(t) := v(t)$$

$$D(t,x) := A_c\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b_c\cdot v_a(t) + e_c\cdot t_l(t) \qquad CI := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad Z_{al} := \mathrm{rkfixed}(CI, 0, t_f, n_f, D) \qquad x_o := CI$$

Modelo Discreto

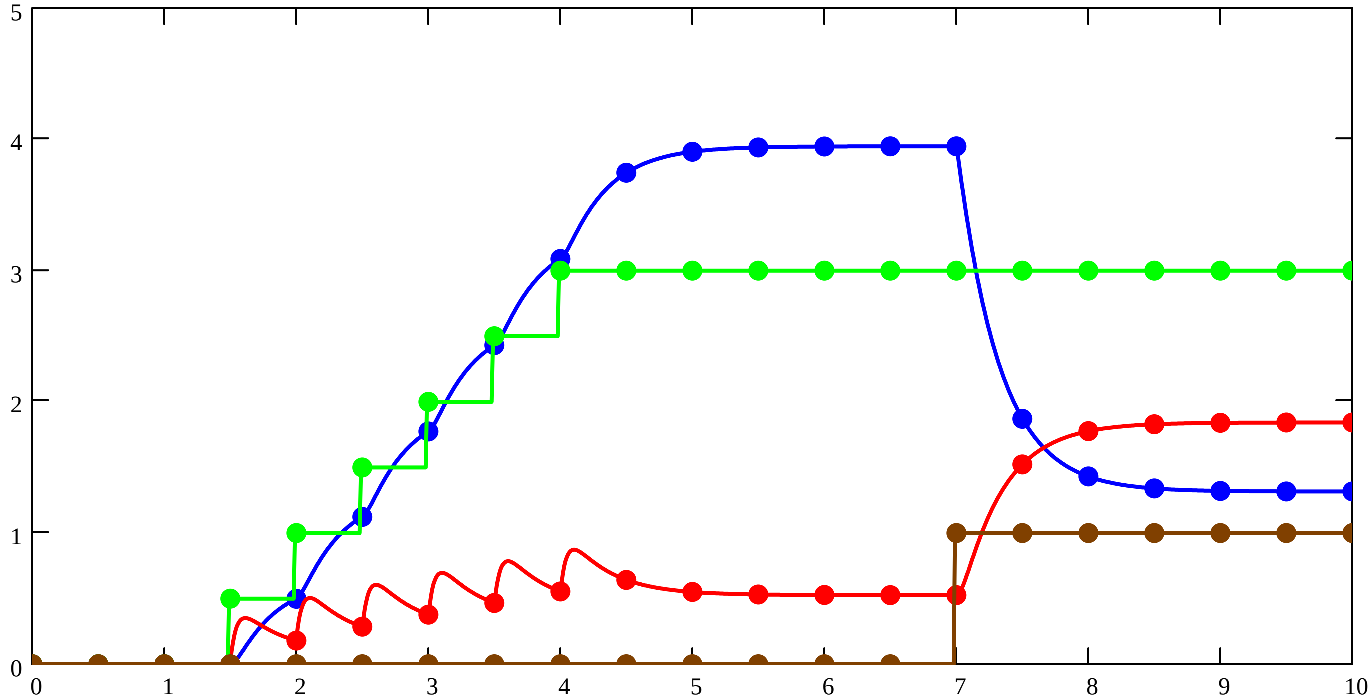
$$\Phi_c(t) := \mathrm{eigenvecs}(A_c)\cdot\begin{pmatrix} \exp(\mathrm{eigenvals}(A_c)_1\cdot t) & 0 \\ 0 & \exp(\mathrm{eigenvals}(A_c)_2\cdot t) \end{pmatrix}\cdot\mathrm{eigenvecs}(A_c)^{-1}$$

$$A_d := \Phi_c(T) \qquad b_d := \begin{bmatrix} \int_0^T (\Phi_c(T-\tau)\cdot b_c)_1\, d\tau \\ \int_0^T (\Phi_c(T-\tau)\cdot b_c)_2\, d\tau \end{bmatrix} \qquad e_d := \begin{bmatrix} \int_0^T (\Phi_c(T-\tau)\cdot e_c)_1\, d\tau \\ \int_0^T (\Phi_c(T-\tau)\cdot e_c)_2\, d\tau \end{bmatrix}$$

Simulación Discreta

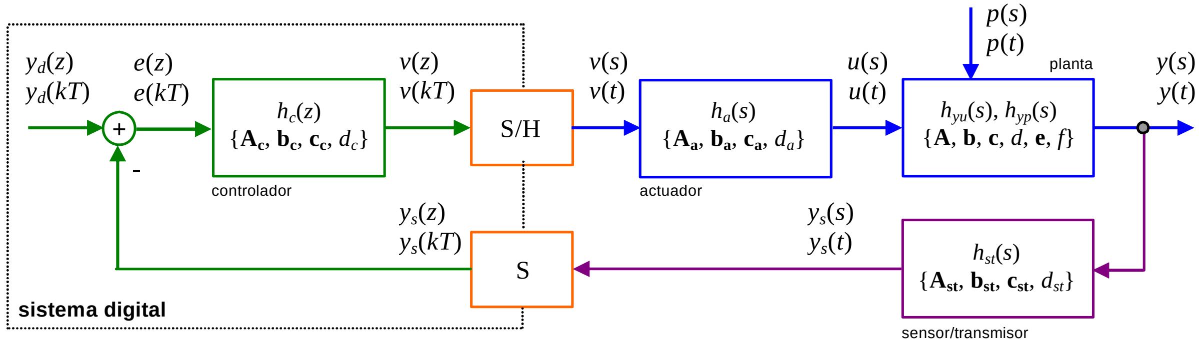
$v(k) := v_a(k\cdot T)$ $\quad t_{ld}(k) := t_l(k\cdot T)$

$$y_d(k) := \mathrm{if}\left(k = 0, x_o, A_d{}^k\cdot x_o + \sum_{j=0}^{k-1} A_d{}^{k-j-1}\cdot b_d\cdot v(j) + \sum_{j=0}^{k-1} A_d{}^{k-j-1}\cdot e_d\cdot t_{ld}(j)\right) \qquad n_{fd} := \frac{t_f}{T} \qquad k := 0 .. n_{fd}$$



5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10

Con Retardo



$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) + \mathbf{e}p(t), \quad y(t) = \mathbf{c}\mathbf{x}(t) + du(t) + fp(t)$$ planta

$$\dot{\boldsymbol{\eta}}(t) = \mathbf{A_a}\boldsymbol{\eta}(t) + \mathbf{b_a}v(t-t_r), \quad u(t) = \mathbf{c_a}\boldsymbol{\eta}(t) + d_a v(t-t_r)$$ actuador

$$\dot{\boldsymbol{\gamma}}(t) = \mathbf{A_{st}}\boldsymbol{\gamma}(t) + \mathbf{b_{st}}y(t), \quad y_s(t) = \mathbf{c_{st}}\boldsymbol{\gamma}(t) + d_{st}y(t)$$ sensor/transmisor

se agrupan las variables de estado de manera de tener una nueva representación con $v(t - t_r)$ y $p(t)$ como entradas e $y_s(t)$ como salida.

$$\begin{bmatrix} \dot{\boldsymbol{\eta}}(t) \\ \dot{\mathbf{x}}(t) \\ \dot{\boldsymbol{\gamma}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A_a} & \mathbf{0} & \mathbf{0} \\ \mathbf{b}\mathbf{c_a} & \mathbf{A} & \mathbf{0} \\ \mathbf{b_{st}}d\mathbf{c_a} & \mathbf{b_{st}}\mathbf{c} & \mathbf{A_{st}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_{\mathbf{tr}}(t) \\ \mathbf{x}(t) \\ \boldsymbol{\gamma}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{b_a} \\ \mathbf{b}d_a \\ \mathbf{b_{st}}dd_a \end{bmatrix} v(t-t_r) + \begin{bmatrix} \mathbf{0} \\ \mathbf{e} \\ \mathbf{b_{st}}f \end{bmatrix} p(t) \qquad y_s(t) = [d_{st}d\mathbf{c_a} \quad d_{st}\mathbf{c} \quad \mathbf{c_{st}}] \begin{bmatrix} \boldsymbol{\eta}(t) \\ \mathbf{x}(t) \\ \boldsymbol{\gamma}(t) \end{bmatrix} + d_{st}dd_a v(t-t_r) + d_{st}fp(t)$$

esta representación se puede escribir definiendo nuevas matrices y vectores de parámetros como,

$$\dot{\boldsymbol{\psi}}(t) = \mathcal{A}\boldsymbol{\psi}(t) + \mathcal{b}v(t-t_r) + \mathcal{e}p(t), \quad y_s(t) = \mathcal{c}\boldsymbol{\psi}(t) + \mathcal{d}v(t-t_r) + \mathcal{f}p(t) \qquad \boldsymbol{\psi}(t) = [\boldsymbol{\eta}(t)^T \, \mathbf{x}(t)^T \, \boldsymbol{\gamma}(t)^T]^T$$

esta ecuaciones tienen un equivalente discreto considerando el S/H de la entrada dado por,

$$\boldsymbol{\psi}(kT+T) = \mathbf{A_d}\boldsymbol{\psi}(kT) + \mathbf{b_d}v(kT-lT) + \mathbf{e_d}p(kT) \qquad y_s(kT) = \mathbf{c_d}\boldsymbol{\psi}(kT) + d_d v(kT-lT) + f_d p(kT)$$

en donde las nuevas matrices están dadas por,

$$\mathbf{A_d} = \Phi(T) = e^{\mathcal{A}T}, \; \mathbf{b_d} = \left\{\int_0^T e^{\mathcal{A}(T-\sigma)}\mathcal{b}\,d\sigma\right\}, \; \mathbf{e_d} = \left\{\int_0^T e^{\mathcal{A}(T-\sigma)}\mathcal{e}\,d\sigma\right\}, \; \mathbf{c_d} = \mathcal{c}, \; d_d = \mathcal{d} \text{ y } f_d = \mathcal{f}$$

se definen las variables de estado discretas auxiliares dadas por,

$$
\begin{aligned}
w_1(kT) &= v(kT - lT) & \qquad w_1(kT+T) &= v(kT - lT + T) &= w_2(kT)\\
w_2(kT) &= v(kT - lT + T) & \qquad w_2(kT+T) &= v(kT - lT + 2T) &= w_3(kT)\\
&\vdots & &\vdots & \\
w_{l-1}(kT) &= v(kT - 2T) & \qquad w_{l-1}(kT+T) &= v(kT - T) &= w_l(kT)\\
w_l(kT) &= v(kT - T) & \qquad w_l(kT+T) &= v(kT) &
\end{aligned}
$$

lo que permite escribir el sistema original como sigue,

$$
\begin{bmatrix} \boldsymbol{\psi}(kT+T) \\ w_1(kT+T) \\ w_2(kT+T) \\ \vdots \\ w_{l-1}(kT+T) \\ w_l(kT+T) \end{bmatrix}
=
\begin{bmatrix}
\mathbf{A_d} & \mathbf{b_d} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & 0 & 1 & \cdots & 0 & 0 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 1 \\
\mathbf{0} & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\begin{bmatrix} \boldsymbol{\psi}(kT) \\ w_1(kT) \\ w_2(kT) \\ \vdots \\ w_{l-1}(kT) \\ w_l(kT) \end{bmatrix}
+
\begin{bmatrix} \mathbf{0} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v(kT)
+
\begin{bmatrix} \mathbf{e_d} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} p(kT)
$$

$$
y_s(kT) = \begin{bmatrix} \mathbf{c_d} & d_d & 0 & \cdots & 0 & 0 \end{bmatrix}
\begin{bmatrix} \boldsymbol{\psi}(kT) \\ w_1(kT) \\ w_2(kT) \\ \vdots \\ w_{l-1}(kT) \\ w_l(kT) \end{bmatrix}
+ f_d\, p(kT)
$$

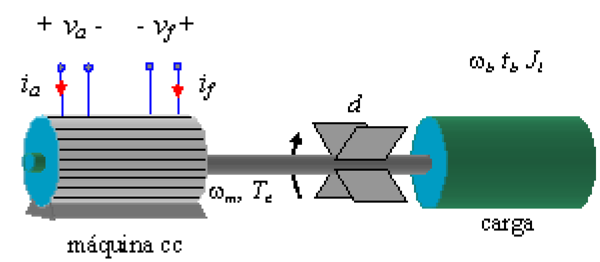
Caso II Motor de Corriente Continua con Retardo en el Actuador

Parámetros

$d := 0.08$ $R := 1.2$ $h_a(s) := 1$ $h_{st}(s) := 1$

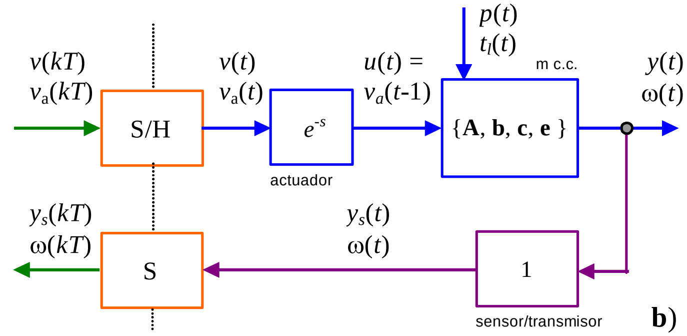
$k_m := 0.6$ $L := 50 \cdot 10^{-3}$ $J_l := 0.135$

$t_f := 10$ $n_f := 1000$ $n := 0 .. n_f$



Variables de Estado

$x_1 = i_a \quad x_2 = \omega$



Modelo Continuo

$$A_c := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \qquad b_c := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \qquad e_c := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix}$$

Simulación Continua

$$t_l(t) := \Phi(t-7) \qquad v(k) := k \cdot T \cdot \Phi(k \cdot T) - (k \cdot T - 6 \cdot T) \cdot \Phi(k \cdot T - 6 \cdot T) \qquad \mathrm{del}(k) := \mathrm{if}(k = 0, 1, 0)$$

$$\mathrm{del}_T(t) := \frac{1}{T} \cdot (\Phi(t) - \Phi(t-T)) \qquad v(t) := \sum_{i=0}^{t \cdot T^{-1}} v(i-2)\, \mathrm{del}_T(t - i \cdot T) \cdot T \qquad v_a(t) := v(t-1)$$

$$D(t,x) := A_c \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b_c \cdot v_a(t) + e_c \cdot t_l(t) \quad CI := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad Z_{al} := \mathrm{rkfixed}(CI, 0, t_f, n_f, D) \qquad x_o := CI$$

Modelo Discreto

$$\Phi_c(t) := \mathrm{eigenvecs}(A_c) \cdot \begin{pmatrix} \exp(\mathrm{eigenvals}(A_c)_1 \cdot t) & 0 \\ 0 & \exp(\mathrm{eigenvals}(A_c)_2 \cdot t) \end{pmatrix} \cdot \mathrm{eigenvecs}(A_c)^{-1}$$

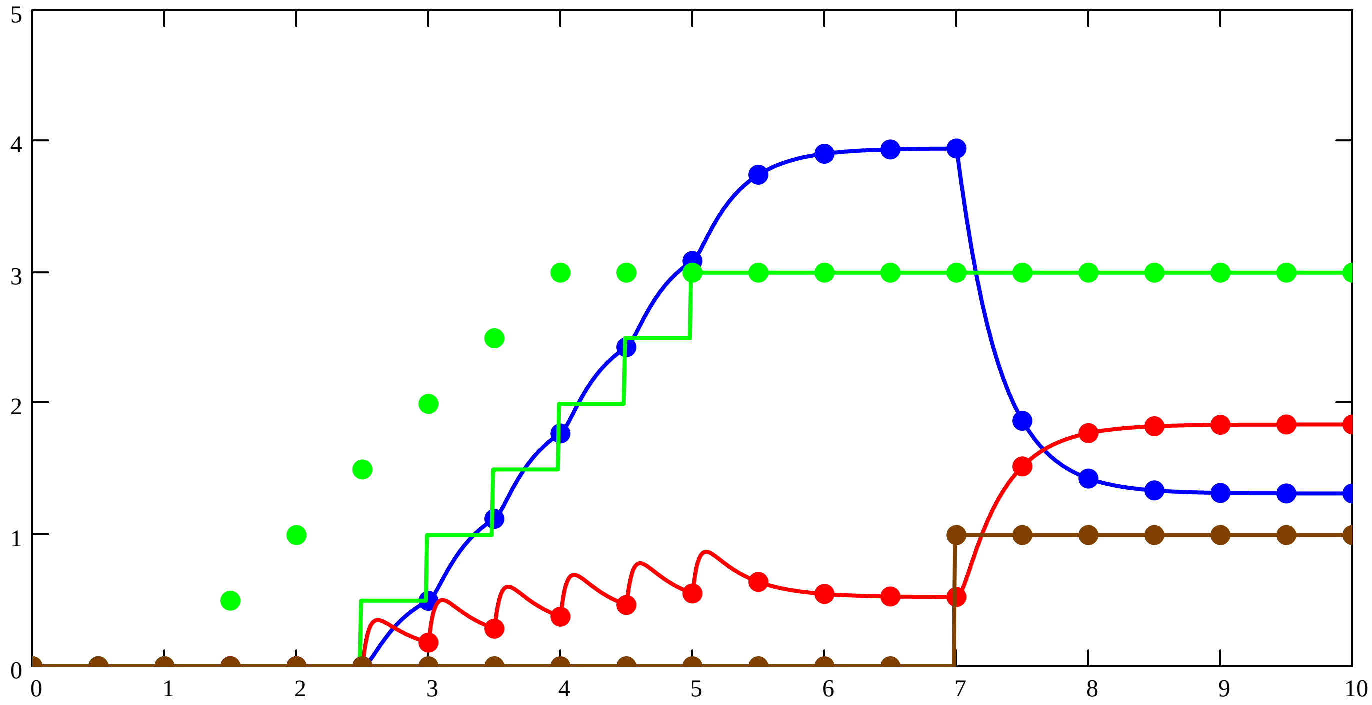
$$A_d := \Phi_c(T) \qquad b_d := \begin{bmatrix} \int_0^T \left(\Phi_c(T-\tau) \cdot b_c\right)_1 d\tau \\ \int_0^T \left(\Phi_c(T-\tau) \cdot b_c\right)_2 d\tau \end{bmatrix} \qquad e_d := \begin{bmatrix} \int_0^T \left(\Phi_c(T-\tau) \cdot e_c\right)_1 d\tau \\ \int_0^T \left(\Phi_c(T-\tau) \cdot e_c\right)_2 d\tau \end{bmatrix}$$

$$A_d := \begin{pmatrix} A_{d_{1,1}} & A_{d_{1,2}} & b_{d_1} & 0 \\ A_{d_{2,1}} & A_{d_{2,2}} & b_{d_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad b_d := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad e_d := \begin{pmatrix} e_{d_1} \\ e_{d_2} \\ 0 \\ 0 \end{pmatrix} \quad x_o := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Simulación Discreta

$$v(k) := v(k \cdot T) \qquad t_{ld}(k) := t_l(k \cdot T)$$

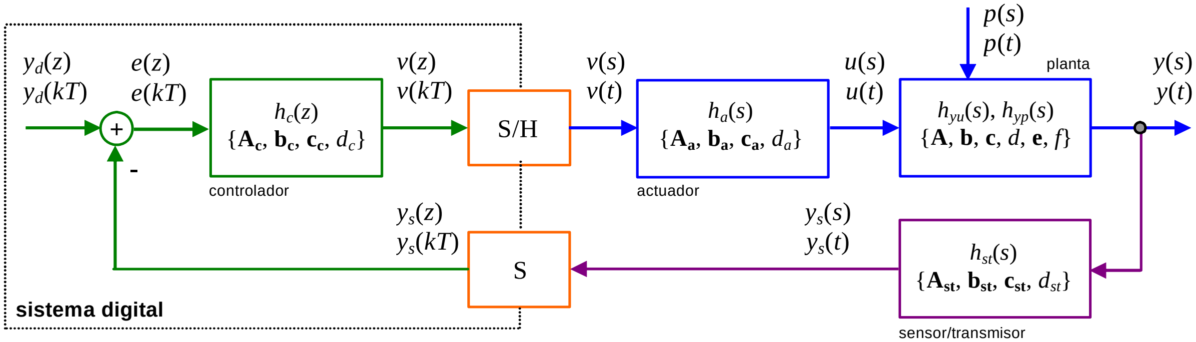
$$y_d(k) := \text{if}\left(k = 0, x_o, A_d^{\,k} \cdot x_o + \sum_{j=0}^{k-1} A_d^{\,k-j-1} \cdot b_d \cdot v(j) + \sum_{j=0}^{k-1} A_d^{\,k-j-1} \cdot e_d \cdot t_{ld}(j)\right) \qquad n_{fd} := \frac{t_f}{T} \qquad k := 0 .. n_{fd}$$



Retardo por Cálculo

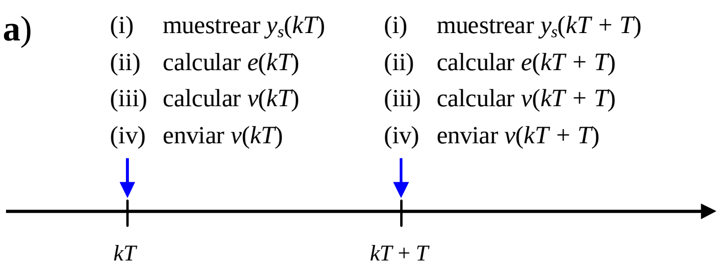
Problema Demostrar que el muestreo, cálculo y envío de información genera un retardo adicional.

Planta continua con controaldor discreto



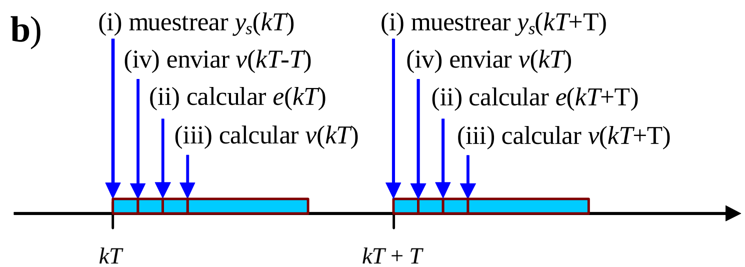
Cálculo idealizado

a)

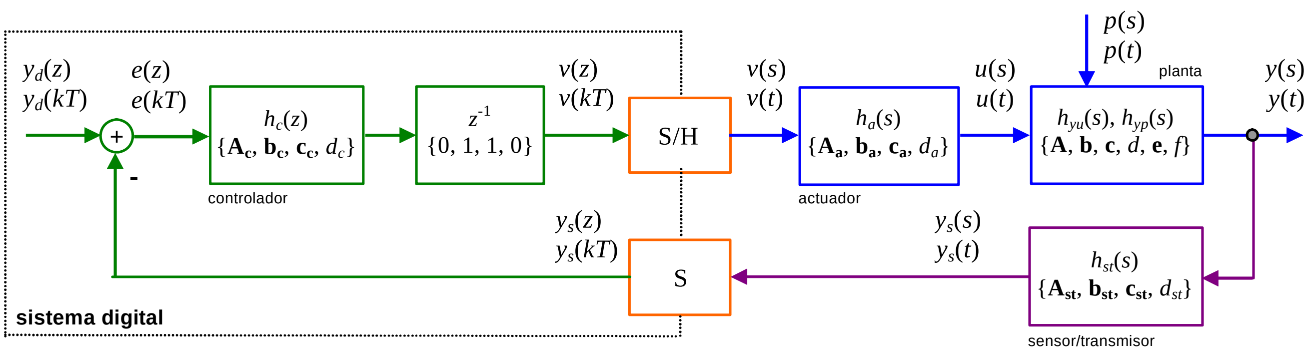


Cálculo realista

b)



Equivalente del retardo por cálculo



Polos Equivalentes

$t := t \qquad k := k \qquad T := T$

Problema Ilustrar la relación entre polos continuos y polos discretos.

Ejemplo 1

$$h_{yu}(s) := \frac{1}{s^2} \qquad h_a(s) := 1 \qquad h_{st}(s) := 1$$

$$h_{yu}(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l}\text{invlaplace}, s\\ \text{substitute}, t = k\cdot T\\ \text{ztrans}, k\\ \text{collect}, z\end{array} \to \frac{1}{2}\cdot T^2\cdot z\cdot\frac{z+1}{(z-1)^3}\right] \to \frac{1}{2\cdot(z-1)^2}\cdot T^2\cdot(z+1)$$

$$h_{yu}(z)^{-1} \text{ solve}, z \to \begin{pmatrix}1\\1\end{pmatrix}$$

polos de la F. de T. discreta.

Ejemplo 2

$$h_{yu}(s) := \frac{s-aa}{s+aa} \qquad h_a(s) := 1 \qquad h_{st}(s) := 1$$

$$h_{yu}(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l}\text{invlaplace}, s\\ \text{substitute}, t = k\cdot T\\ \text{ztrans}, k\\ \text{collect}, z\end{array} \to z\cdot\frac{z-2+\exp(-aa\cdot T)}{(z-\exp(-aa\cdot T))\cdot(z-1)}\right] \to \frac{z-2+\exp(-aa\cdot T)}{z-\exp(-aa\cdot T)}$$

$$h_{yu}(z)^{-1} \text{ solve}, z \to \exp(-aa\cdot T)$$

polos de la F. de T. discreta.

Ejemplo 3

$$h_{yu}(s) := \frac{s-aa}{s+aa}\cdot e^{-2\cdot s} \qquad h_a(s) := 1 \qquad h_{st}(s) := 1 \qquad T := 1$$

$$h_{yu}(z) := \frac{z-1}{z}\cdot\left[h_a(s)\cdot h_{yu}(s)\cdot h_{st}(s)\cdot\frac{1}{s} \;\middle|\; \begin{array}{l}\text{invlaplace}, s\\ \text{substitute}, t = k\cdot T\\ \text{ztrans}, k\\ \text{collect}, z\end{array} \to \frac{z-2+\exp(-aa)}{(z-\exp(-aa))\cdot z\cdot(z-1)}\right] \to \frac{1}{z^2}\cdot\frac{z-2+\exp(-aa)}{z-\exp(-aa)}$$

$$h_{yu}(z)^{-1} \text{ solve}, z \to \begin{pmatrix}0\\0\\ \dfrac{1}{\exp(aa)}\end{pmatrix}$$

polos de la F. de T. discreta.

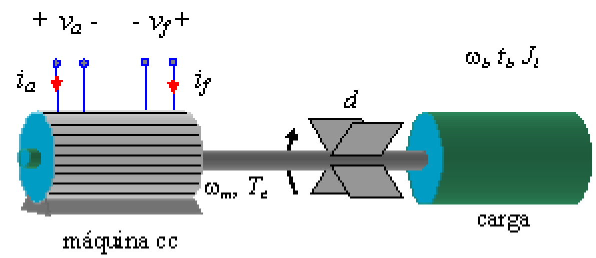
Ejemplo 4 Motor de Corriente Continua.

Parámetros

$d := 0.08$ $R := 1.2$ $h_a(s) := 1$ $h_{st}(s) := 1$

$k_m := 0.6$ $L := 50 \cdot 10^{-3}$ $J_l := 0.135$

$t_f := 10$ $n_f := 1000$ $n := 0 .. n_f$



Modelo Continuo

$$A_c := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \quad b_c := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \quad e_c := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix}$$

Variables de Estado

$x_1 = i_a \quad x_2 = \omega$

Simulación Continua

$T := 0.5$

Modelo Discreto

$$\Phi_c(t) := \text{eigenvecs}(A_c) \cdot \begin{pmatrix} \exp(\text{eigenvals}(A_c)_1 \cdot t) & 0 \\ 0 & \exp(\text{eigenvals}(A_c)_2 \cdot t) \end{pmatrix} \cdot \text{eigenvecs}(A_c)^{-1}$$

$A_d := \Phi_c(T)$

Matriz A del Modelo Continuo

$$A_c = \begin{pmatrix} -24 & -12 \\ 4.444 & -0.593 \end{pmatrix} \quad \text{eigenvals}(A_c) = \begin{pmatrix} -21.442 \\ -3.151 \end{pmatrix}$$

valores propios de **A**

Matriz A del Modelo Dsicreto

$$A_d = \begin{pmatrix} -0.029 & -0.136 \\ 0.05 & 0.236 \end{pmatrix} \quad \text{eigenvals}(A_d) = \begin{pmatrix} 2.208 \times 10^{-5} \\ 0.207 \end{pmatrix}$$

valores propios de $\mathbf{A_d}$

$$e^{\text{eigenvals}(A_c) \cdot T} = \begin{pmatrix} 2.208 \times 10^{-5} \\ 0.207 \end{pmatrix}$$

relación entre los valores propios de A y de $\mathbf{A_d}$

Ejemplo 5 Motor de Corriente Continua con retardo de 2T.

$$b_d := \begin{bmatrix} \int_0^T \left(\Phi_c(T-\tau)\cdot b_c\right)_1 d\tau \\ \int_0^T \left(\Phi_c(T-\tau)\cdot b_c\right)_2 d\tau \end{bmatrix}$$

Matriz **A** del Modelo Dsicreto

$$A_d := \begin{pmatrix} A_{d_{1,1}} & A_{d_{1,2}} & b_{d_1} & 0 \\ A_{d_{2,1}} & A_{d_{2,2}} & b_{d_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{eigenvals}(A_d) = \begin{pmatrix} 2.208 \times 10^{-5} \\ 0.207 \\ 0 \\ 0 \end{pmatrix}$$

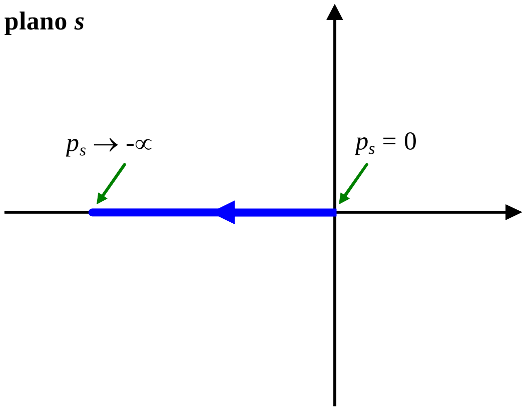
valores propios de $\mathbf{A_d}$

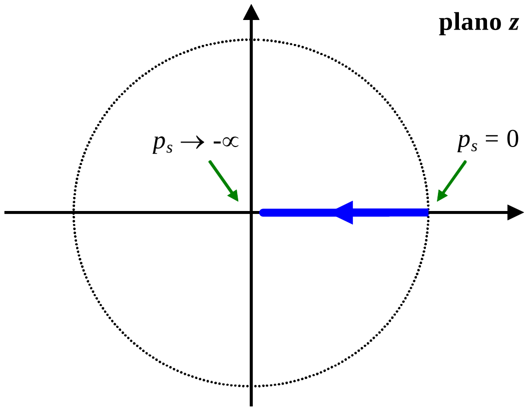
Mapeo de Polos.

Problema Ilustrar cómo opera el mapeo de polos.

Caso I Sistema de Primer Orden.

$h_{yu}(s) = \frac{1}{s + p_s}$ polo en s: $-p_s$ polo en z: $e^{-p_s \cdot T}$

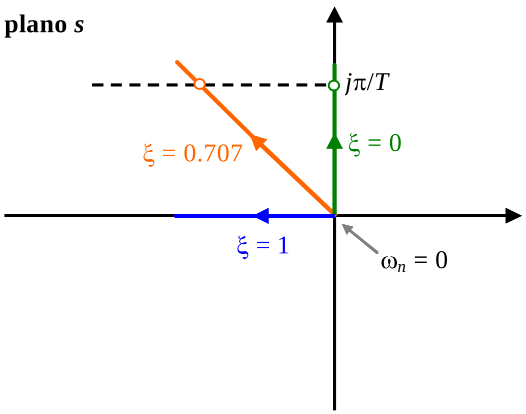


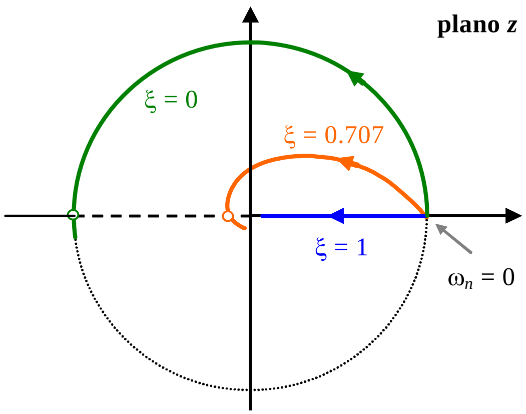


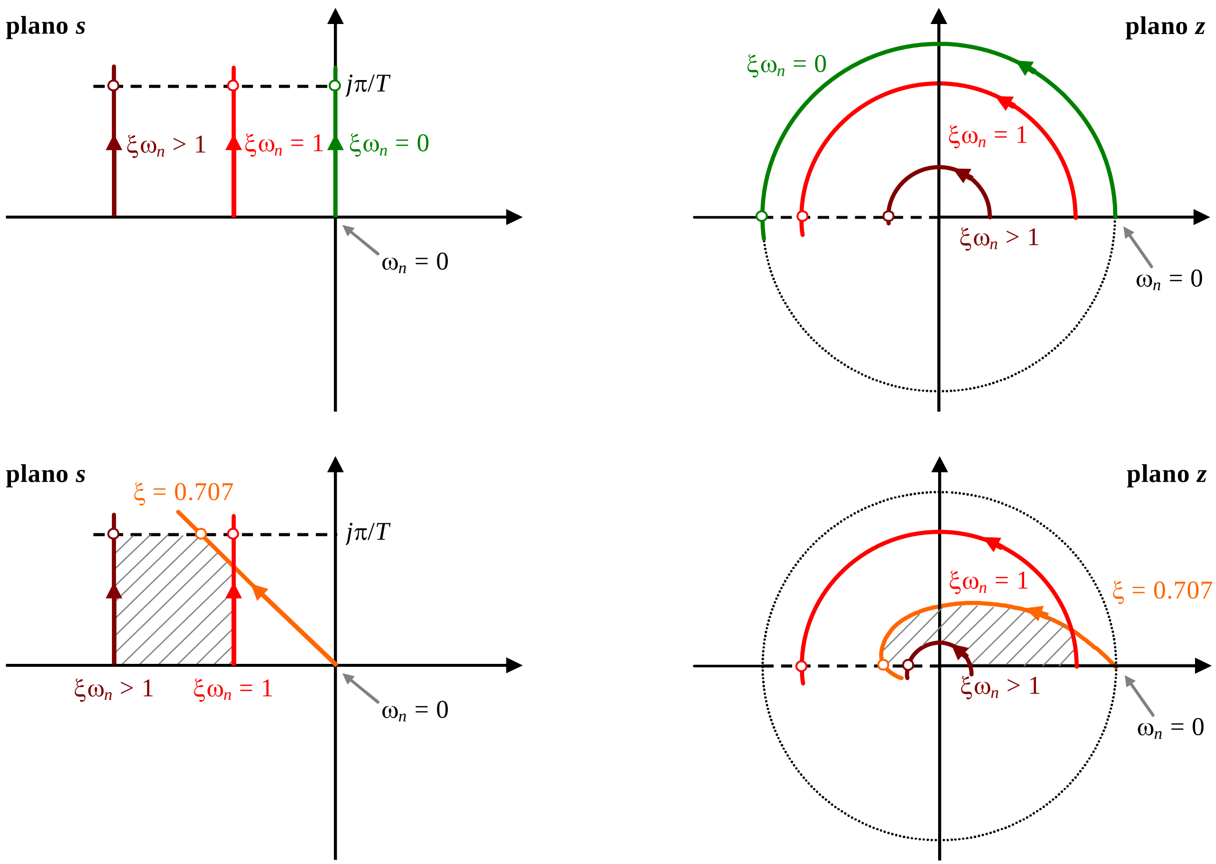
Caso II Sistema de Segundo Orden Oscilatorio.

$h_{yu}(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$ polo en s: $p_{s1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

polo en z: $p_{z1,2} = e^{\left\{-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}\right\}T} = e^{-\xi\omega_n T} e^{\pm j\omega_n T\sqrt{1-\xi^2}} = e^{-\xi\omega_n T}\left\{\cos\left(\omega_n T\sqrt{1-\xi^2}\right) + j\sin\left(\omega_n T\sqrt{1-\xi^2}\right)\right\}$






plano s
jπ/T
ξωn > 1
ξωn = 1
ξωn = 0
ωn = 0
plano z
ξωn = 0
ξωn = 1
ξωn > 1
ωn = 0
plano s
ξ = 0.707
jπ/T
ξωn > 1
ξωn = 1
ωn = 0
plano z
ξωn = 1
ξ = 0.707
ξωn > 1
ωn = 0

$T := 1$

$N := 1$

$p_{sa} := -1 + \frac{\pi}{2} \cdot j$ $\qquad p_{sa} = -1 + 1.571i$ $\qquad$ polo en s

$p_{sb} := -1 + \left(\frac{\pi}{2} + \frac{2 \cdot N \cdot \pi}{T}\right) \cdot j$ $\qquad p_{sb} = -1 + 7.854i$ $\qquad$ polo en s

$e^{p_{sa} \cdot T} = 0.368i$ $\qquad$ polo mapeado en z

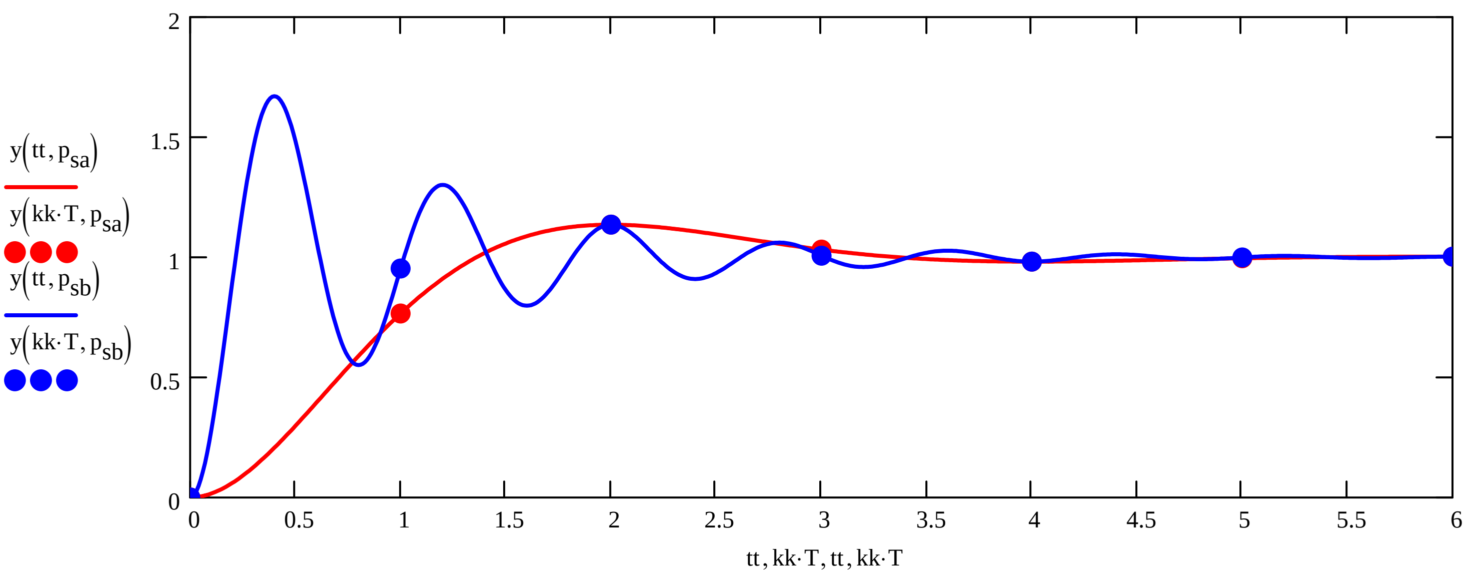
$e^{p_{sb} \cdot T} = 0.368i$ $\qquad$ polo mapeado en z

$$y(s, p_s) := \frac{p_s \cdot \overline{p_s}}{(s - p_s) \cdot (s - \overline{p_s})} \cdot \frac{1}{s}$$

$tt := 0, 0.01 .. 6$

$kk := 0 .. 6$

$$y(t, p_s) := p_s \cdot \overline{p_s} \cdot \left[\frac{1}{(p_s - \overline{p_s}) \cdot p_s} \cdot \exp(p_s \cdot t) - \frac{1}{(p_s - \overline{p_s}) \cdot \overline{p_s}} \cdot \exp(\overline{p_s} \cdot t) + \frac{1}{p_s \cdot \overline{p_s}}\right]$$



Problema Probar el mínimo tiempo de muestreo.

Caso I Sistema de Segundo Orden Oscilatorio.

$k_p := 1 \quad \xi := 0.3 \quad \omega_n := \frac{2\cdot\pi}{\sqrt{1-\xi^2}} \quad \omega_n = 6.587$

Modelo Continuo en Ecuaciones de Estado

$A_c := \begin{pmatrix} 0 & 1 \\ -\omega_n^{\,2} & -2\cdot\xi\cdot\omega_n \end{pmatrix} \quad b_c := \begin{pmatrix} 0 \\ \omega_n^{\,2} \end{pmatrix} \quad x_o := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{eigenvals}(A_c) = \begin{pmatrix} -1.976 + 6.283i \\ -1.976 - 6.283i \end{pmatrix}$

Modelo Discreto en Ecuaciones de Diferencias

$\frac{1}{\text{Re}(\text{eigenvals}(A_c))} = \begin{pmatrix} -0.506 \\ -0.506 \end{pmatrix}$

$Tt := \text{eigenvecs}(A_c)^{-1} \quad \Phi_{Tt}(t) := \begin{pmatrix} \exp(\text{eigenvals}(A_c)_1\cdot t) & 0 \\ 0 & \exp(\text{eigenvals}(A_c)_2\cdot t) \end{pmatrix} \quad T := 0.5 \quad \frac{\pi}{T} = 6.283$

$\Phi_c(t) := Tt^{-1}\cdot\Phi_{Tt}(t)\cdot Tt \quad A_d := \Phi_c(T) \quad b_d := \left[\int_0^T (\Phi_c(T-\tau)\cdot b_c)_1\, d\tau \quad \int_0^T (\Phi_c(T-\tau)\cdot b_c)_2\, d\tau \right]^T$

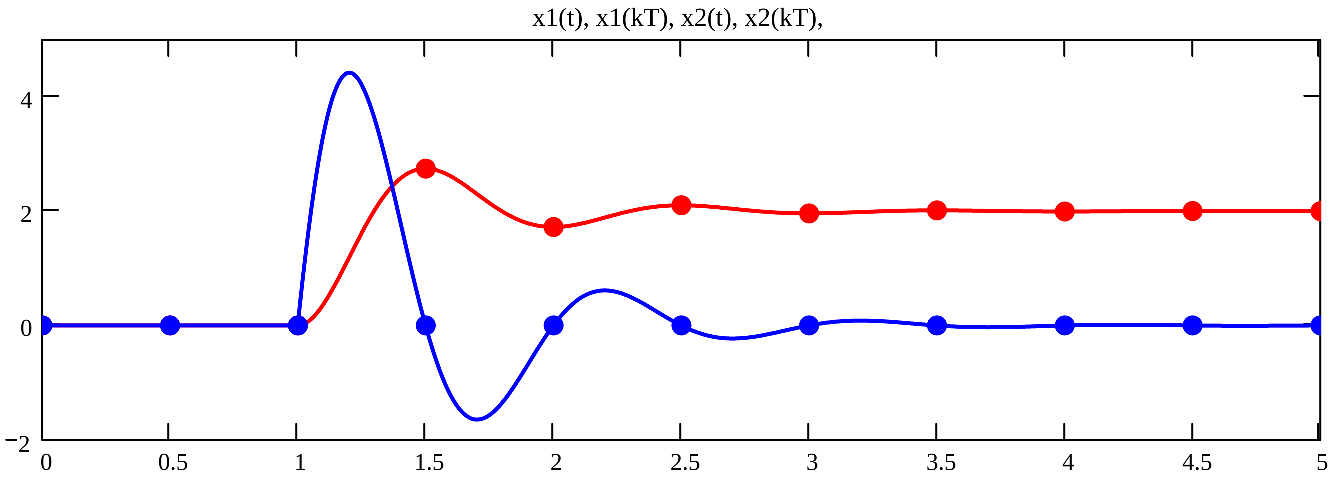
Simulación del Modelo Continuo en Ecuaciones de Estado

$t_f := 5 \quad n_f := 1000 \quad n := 0..n_f \quad u_c(t) := \Phi(t-1) \quad D(t,x) := A_c\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b_c\cdot u_c(t) \quad CI := x_o \quad Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$

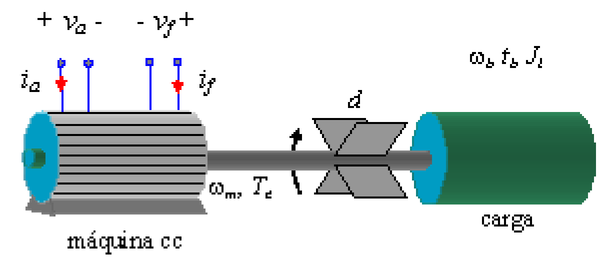
Simulación del Modelo Discreto en Ecuaciones de Diferencias

$v(k) := u_c(k\cdot T)$

$y_d(k) := \text{if}\left(k = 0, x_o, A_d^{\,k}\cdot x_o + \sum_{j=0}^{k-1} A_d^{\,k-j-1}\cdot b_d\cdot v(j)\right) \quad n_{fd} := \frac{t_f}{T} \quad k := 0..n_{fd}$



Caso II   Motor de Corriente Continua.



Parámetros

$d := 0.08 \qquad R := 1.2$

$k_m := 0.6 \qquad L := 50\cdot 10^{-3} \qquad J_l := 0.135$

$t_f := 2 \qquad n_f := 1000 \qquad n := 0 .. n_f$

Modelo Continuo

$$A_c := \begin{pmatrix} \frac{-R}{L} & \frac{-k_m}{L} \\ \frac{k_m}{J_l} & \frac{-d}{J_l} \end{pmatrix} \qquad b_c := \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \qquad e_c := \begin{pmatrix} 0 \\ \frac{-1}{J_l} \end{pmatrix}$$

Variables de Estado

$x_1 = i_a \qquad x_2 = \omega$

$$\text{eigenvals}(A_c)^{-1} = \begin{pmatrix} -0.047 \\ -0.317 \end{pmatrix}$$

Simulación Continua

$$t_l(t) := 0.5\cdot\Phi(t-6) \qquad v_a(t) := 3\cdot\Phi(t-0.5) \qquad D(t,x) := A_c\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b_c\cdot v_a(t) + e_c\cdot t_l(t) \qquad CI := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad Z_{al} := \text{rkfixed}(CI, 0, t_f, n_f, D)$$

Modelo Discreto

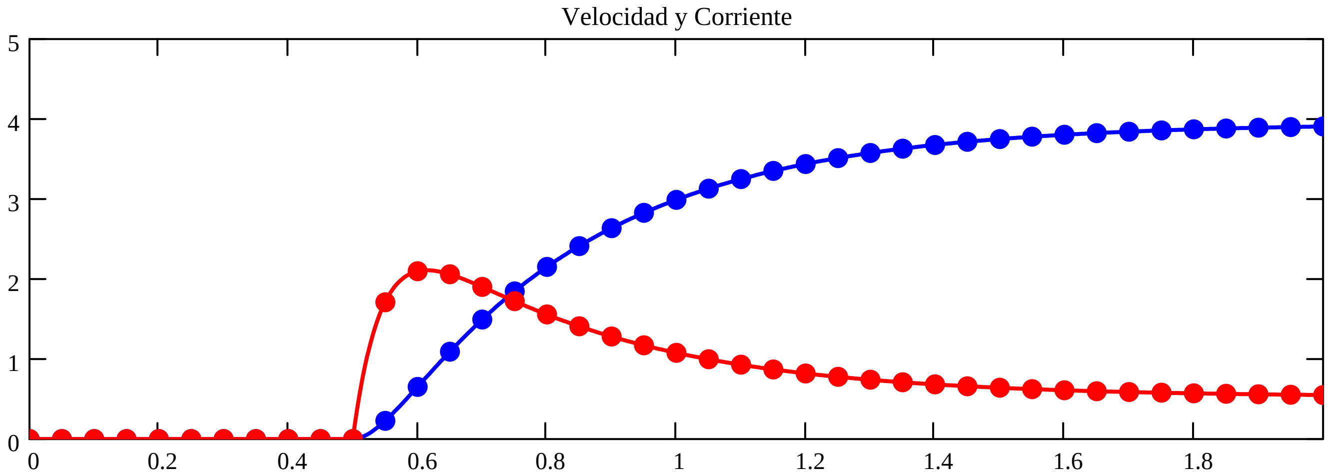
$$T := 0.05 \qquad Tt := \text{eigenvecs}(A_c)^{-1} \qquad \Phi_{Tt}(t) := \begin{pmatrix} \exp(\text{eigenvals}(A_c)_1\cdot t) & 0 \\ 0 & \exp(\text{eigenvals}(A_c)_2\cdot t) \end{pmatrix} \qquad \Phi_c(t) := Tt^{-1}\cdot\Phi_{Tt}(t)\cdot Tt$$

$$A_d := \Phi_c(T) \qquad b_d := \begin{bmatrix} \int_0^T (\Phi_c(T-\tau)\cdot b_c)_1\, d\tau \\ \int_0^T (\Phi_c(T-\tau)\cdot b_c)_2\, d\tau \end{bmatrix} \qquad e_d := \begin{bmatrix} \int_0^T (\Phi_c(T-\tau)\cdot e_c)_1\, d\tau \\ \int_0^T (\Phi_c(T-\tau)\cdot e_c)_2\, d\tau \end{bmatrix}$$

Simulación Discreta

$v(k) := v_a(k\cdot T) \qquad t_{ld}(k) := t_l(k\cdot T)$

$$y_d(k) := \text{if}\left(k = 0, x_o, A_d^{\,k}\cdot x_o + \sum_{j=0}^{k-1} A_d^{\,k-j-1}\cdot b_d\cdot v(j) + \sum_{j=0}^{k-1} A_d^{\,k-j-1}\cdot e_d\cdot t_{ld}(j)\right) \qquad n_{fd} := \frac{t_f}{T} \qquad k := 0 .. n_{fd}$$


Velocidad y Corriente
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8