Efficiency and Productivity in the Chilean Banking Industry

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Abstract

This paper investigates the nature and extent of efficiency and productivity growth in Chilean banking over the period 1989 to 1999. Technical efficiency measures are computed using the Free Disposal Hull methodology. A Malmquist index approach is used to measure productivity. The results indicate that after a period of rapid productivity growth the banking sector experienced lower and relatively stable rates of productivity change. Over the entire period there was a 4.8 per cent annual productivity growth. Small banks appear to be the group that contributed the least to productivity growth. At the same time, it is the group with the greater number of inefficient banks. Although there is no evidence of large differences in average technical efficiency over the years, the average values tend to be greater from 1996 onward. Overall, the findings suggest that there are no large technical inefficiencies in the Chilean banking industry.

I INTRODUCTION

The Chilean banking system has undergone major changes in the last thirty years. Market-oriented macroeconomic policies lead up to financial liberalization and fast expansion during the seventies. A severe recession, in the early eighties, revealed the weakness of banks' financial positions and their inability to adjust to the new conditions. As a result, Chile experienced a sharp financial crisis involving nearly 25 percent of its banking institutions. Government rescue, structural changes, and subsequent modifications in prudential regulation and supervision laid out the framework that will prevail in the following years. In the 1990s, macroeconomic stability and sustained economic growth created a propitious environment for the development and consolidation of the banking system[1]. During this period, however, the market experienced a gradual reduction in the numbers of banks. As with many financial systems in developed and developing economies, mergers and acquisitions have characterized the consolidation stage. At the same time, no new bank has entered the market. The increments in concentration have caused obvious concern among banking authorities who see a potential deterioration in efficiency and competitive conditions. Thus, among other possible inquiries, the extent to which changes in efficiency measures or variations in productivity have occurred in the Chilean banking industry can be considered as relevant empirical questions. This paper attempts to answer these questions.

The majority of efficiency studies have been primarily concerned with US and European financial markets, while far less applications have focused on developing economies, particularly in Latin American countries. In the case of Chile, and despite the importance of efficiency in banking markets, few studies have investigated productive efficiency and no studies appear to have provided productivity measures. Early studies on efficiency in the Chilean banking industry, such as Marshall (1986), Nauriyal (1995), and Shirota (1996), relied solely on parametric techniques in analyzing data from banks. In contrast, this paper uses a nonparametric methodology to evaluate both

[1]In the 1990s, the average growth rate of the Chilean economy was 6%.

technical efficiency and productivity.

II METHODOLOGY

The nonparametric method employs data on inputs and outputs to construct a production possibilities set whose frontier is used to obtain measures of technical efficiency (TE). Regularity assumptions are imposed on the empirical production set so that its elements can be characterized as efficient or inefficient. However, including more assumptions does not necessarily mean that we obtain a better approximation of the true production set. In fact, there is a trade-off between specification and finite sample error in any nonparametric application. For example, a method assuming convexity and that exhibits variable returns (VRS) can reduce finite sample error because it yields a larger subset of the true production set, if the assumptions are satisfied (Banker et al. (1984), Post (1999)). But, a bank can be classified as inefficient because it was compared with an efficient bank, defined by the convexity condition, that actually does not exist. Certainly, this is an undesirable property for some empirical applications.

An alternative model, that does not assume convexity, is the Free Disposal Hull (FDH). It is an approach less sensitive to the presence of outliers and robust under certain economic conditions (Tulkens (1993), De Borger et al. (1994), Cherchye et al. (2000)). Although it may be subject to finite sample error, especially if the number of observations is small relative to the number of inputs and outputs, this is the preferred model to carry out the empirical analysis of TE. For comparison purposes, TE is also computed using the approach with VRS.

In a recent survey article Diewert (1992) argued that the Fisher ideal index is one of the best alternatives for measuring productivity. This index requires information both on prices and quantities. A less demanding option is the Malmquist index which only requires data on quantities. Fortunately, Balk (1993) has shown that the Malmquist index type is a reasonable first order approximation to the Fisher ideal index. Hence, the Malmquist index approach is used in this article to obtain measures of productivity[2]. In order to verify the accuracy of the approximation we use the procedure proposed by Kuosmanen and Post (2000) to compute the interval approximation of the Fisher ideal index.

TECHNICAL EFFICIENCY

Assume that a bank produces s outputs $(y = (y_1, y_2, ..., y_s) \in \Re^s_+)$ employing m inputs $(x = (x_1, x_2, ..., x_m) \in \Re^m_+)$. There are n banks in the banking industry. A nonparametric model constructs an efficient frontier (surface) that envelopes all data. It utilizes a sequence of linear programs to establish which of n banks determines the 'best practice' frontier, and calculates efficiency measures relative to such frontier.

The efficient frontier is computed by solving the following output based Linear Programming problem (LP) (1):

$$\frac{1}{D^p_o(y^q, x^q)} = \underset{\theta,\lambda}{\text{Max}} \;\; \theta$$

$$\begin{aligned} \text{s.t. } & -\theta y^q_{js} + \textstyle\sum_{j=1}^{J} \lambda_j \, y^p_{js} \geq 0, \quad s = 1, 2, \ldots, S \\ & x^q_{jm} - \textstyle\sum_{j=1}^{J} \lambda_j \, x^p_{jm} \geq 0, \quad m = 1, 2, \ldots, M \\ & \lambda \geq 0 \end{aligned} \tag{1}$$

where θ is a scalar, and λ is a vector of constants $\lambda = (\lambda_1, \lambda_2, ..., \lambda_J) \in \Re^n_+$, that denotes the intensity levels at which each of the n activities are carried out. The superscripts p and q are temporal indexes, with $p = q$ when efficiency is calculated from data for the same period of time. A particular bank being analyzed is efficient if and only if $\theta^* = 1$. Failure to achieve

[2]Productivty measures are calculated assuming constant returns of scale. According to Grifell-Tatje and Lovell (1995) the Malmquist index provides an inaccurate measure of productivity change in the presence of VRS. It will be left for future research the application of the FDH methodology for computing productivity measures.

efficiency occurs when $\theta^* < 1$.

Note that the linear programming problem must be solved n times, once for each firm in the sample. A value of θ is then obtained for each bank. The scalar variable θ is the (proportional) reduction applied to all inputs of firm i, the bank being evaluated, to improve efficiency by movement toward the frontier.

LP (1) assumes a constant returns of scale technology. A weaker assumption on returns of scale can be obtained if we include the convexity constraint $\mathbf{1}\lambda = 1$ in LP (1). Thus, we obtain a technical efficiency measure which assumes a VRS technology.

The LP (1) can be modified to obtain a FDH efficiency measure if we constrain λ to take two integer values, i.e., $\lambda \in \{0, 1\}$.

MALMQUIST PRODUCTIVITY INDEX

A productivity index compares the performance at t and $t+1$ relative to the technology at t. Let (y^0, x^0) and (y^1, x^1) denote the combinations of outputs and inputs produced in $t = 0$ and $t = 1$, respectively. Färe et al. (1994) defined a geometric mean of the two output-based Malmquist indexes to yield the following Malmquist-type measure of productivity:

$$M_o(y^{0,1}, x^{0,1}) = \left[\frac{D_o^0(y^1, x^1)}{D_o^0(y^0, x^0)} \frac{D_o^1(y^1, x^1)}{D_o^1(y^0, x^0)} \right]^{\frac{1}{2}}, \tag{2}$$

from which we can obtain

$$M_o(y^{0,1}, x^{0,1}) = \frac{D_o^1(y^1, x^1)}{D_o^0(y^0, x^0)} \left[\frac{D_o^0(y^1, x^1)}{D_o^1(y^1, x^1)} \frac{D_o^0(y^0, x^0)}{D_o^1(y^0, x^0)} \right]^{\frac{1}{2}} \tag{3}$$

The Malmquist index (3) represents a decomposition of (2) between efficiency change and technical change. The first term measures the change in the output-oriented measure of technical efficiency between years 0 and 1. The expression within brackets captures the shift in technology between the two periods evaluated at the input level x^1 and at the input level realized at x^0. Similar applications of this approach can be found, for example, in Noulas (1997), Devaney and Weber (2000), or Drake (2001).

In order to calculate the productivity of a bank between 0 and 1 based on equation (3), it is necessary to solve four different sets of linear programming problems. The first two sets can be solved using LP (1) with suitable time superscripts 0 and 1, respectively. The last two sets require a mixed-period representation, i.e., the frontier is constructed from data in one period, whereas the observation to be evaluated is from another period. Accordingly, the third set of linear programs requires to set $p = 0$ and $q = 1$. Finally, the last set is obtained when these superscripts are reversed.

THE APPROXIMATION OF THE FISHER IDEAL INDEX

The Malmquist index may provide a poor approximation of the Fisher ideal index when prices and/or technologies changes have occurred during the sample period. A procedure that improves the approximation for this particular case has been suggested by Kuosmanen and Post (2000). They use a modified distance function ($\widetilde{D}_o^p(y^q, x^q)$) to replace the output distance function ($D_o^p(y^q, x^q)$) in Equation (2), so that a modified Malmquist index ($\widetilde{M}_o(y^{0,1}, x^{0,1})$ is obtained. Additionally, they propose an interval approximation of the Fisher ideal index[3]. This interval, defined by its upper and lower bound, can be computed by the procedure shown below. Furthermore, a geometric mean between the two extreme points can also be computed so that a point estimate is obtained. Thus, the interval and the point estimate provide a useful way to verify the consistency of the results obtained using the standard Malmquist index.

A two step procedure is used to calculate the modified distance functions $\widetilde{D}_o^p(y^q, x^q)$. We first calculated the distance functions $C_o^p(y^q, x^q)$ with $p = q = 0, 1$, using LP (4)

[3]This methodology is more suitable when the shadow prices are non-unique, see Kuosmanen and Post (2000).

$$C_o^p(y^q, x^q) = \underset{\theta,\lambda,\xi}{\text{Min}} \ \theta$$

$$\begin{aligned} \text{s.t. } -y_{js}^q + \textstyle\sum_{j=1}^J \lambda_j \, y_{js}^p \geq 0, \quad s = 1, 2, \ldots, S \\ \theta x_{jm}^q - \textstyle\sum_{j=1}^J \lambda_j \, x_{jm}^p \geq 0, \quad M = 1, 2, \ldots, M \\ \lambda \geq 0 \end{aligned} \tag{4}$$

Once the distance functions from LP (4) are computed, we can calculate the modified distance function using LP (5). The four sets of LP problems that we need to calculate utilize the same superscript structure as was explained above.

$$\widetilde{D}_o^p(y^q, x^q) = \underset{\theta,\lambda,\xi}{\text{Min}} \ \theta$$

$$\begin{aligned} \text{s.t. } -y_{js}^q + \textstyle\sum_{j=1}^J \lambda_j \, y_{js}^p + \xi y_{js}^p \geq 0, \quad s = 1, 2, \ldots, S \\ \theta x_{jm}^q - \textstyle\sum_{j=1}^J \lambda_j \, x_{jm}^p - \xi x_{jm}^p C_o^p(y^p, x^p) \geq 0, \quad m = 1, 2, \ldots, M \\ \lambda \geq 0 \end{aligned} \tag{5}$$

The expressions that define an upper and lower bound for the Fisher index are, respectively,

$$U_F(y^{0,1}, x^{0,1}) = \left(\frac{C^1(y^1, x^1)}{C^0(y^0, x^0)} \frac{\widetilde{D}^0(y^1, x^1)}{\widetilde{E}^1(y^0, x^0)} \right)^{\frac{1}{2}} ; \ L_F(y^{0,1}, x^{0,1}) = \left(\frac{C^1(y^1, x^1)}{C^0(y^0, x^0)} \frac{\widetilde{E}^0(y^1, x^1)}{\widetilde{D}^1(y^0, x^0)} \right)^{\frac{1}{2}}$$

The distance function $\widetilde{E}_o^p(y^q, x^q)$ is computed using LP (5) with the output set of constraints substituted by $y_{jm}^q + \sum_{j=1}^J \lambda_j \, y_{jm}^p + \xi y_{jm}^p \geq 0, \ m = 1, 2, \ldots, M$.

DATA AND VARIABLES

The data used for the analysis were retrieved from the Balance Sheets of Assets and Liabilities of each individual institution in the Chilean banking system. These are published in monthly bulletins, Informacion Financiera, by the Superintendencia de Bancos e Instituciones Financieras.

Several controversial issues coexist in the banking literature regarding the definition of outputs and inputs and how they can be measured. In general, three approaches are used to obtain empirical measurement of banks' outputs and inputs: the intermediation, the value-added and the user cost approach. For this paper we use the value-added method which suggest to include loans and deposits as outputs[4]. Accordingly, the output variables are loans (y_1), investments (y_2) and deposits (y_3). The set of inputs are labor (x_1) and physical capital (x_2). Labor is measured as number of employees. The number of branches is used as a proxy for physical capital. In the case of measuring productivity the analysis is carried out with real values of the variables, obtained by deflating the nominal values by the consumer price index (December 1998=100). Descriptive statistics for the outputs and inputs are included in Table 1.

In the analysis below, the banks have been grouped on the basis of average total assets, following the approach suggested in Franken (2001). In particular, the group of large banks have average total assets in excess of $3 billion US dollars[5]. The medium size banks have average total assets between $1 and $3 billions. The remaining banks with average total assets below $1 billion form the group of small banks.

[4]We follow here the same approach adopted by Nauriyal (1995), and Shirota (1996).

[5]An approximate exchange rate of 462 pesos per dollar was used to convert 1998 Chilean pesos to dollars.

Table 1. Descriptive statistics (millions of pesos of December 1998)

	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
LOANS											
Mean	199625	191167	196981	243208	308983	322286	387727	469002	543011	568592	635947
sd	279949	271235	280845	328378	396338	410537	486630	593369	739901	785192	829457
INVESTMENTS											
Mean	109985	124415	125498	110841	113270	121973	135556	147381	157919	155099	206548
sd	207925	223835	220127	218032	215958	240411	281839	273446	306007	273547	333062
DEPOSITS											
Mean	214275	219083	248561	277896	323881	349376	433052	523029	556272	631552	741450
sd	317118	332537	348690	391267	443181	478058	590692	696998	785929	879153	963001
LABOR											
Mean	874	921	935	981	1123	1252	1220	1410	1475	1327	1412
sd	1448	1509	1502	1517	1614	1694	1704	1851	1958	1821	1877
CAPITAL											
Mean	24	24	25	27	30	32	35	39	41	42	47
sd	36	36	36	37	40	42	47	52	55	56	64

III EMPIRICAL FINDINGS

For each year two frontiers were computed, VRS and FDH. The latter frontier envelope the data more closely, so that greater values for FDH efficiency scores are to be expected. Table 2 contains the average technical efficiency results. There appears to be no trend in the FDH's mean values from 1989 to 1995. However, from 1996 onward there is a slight increase in productive efficiency which is partially confirmed by the standard deviations. In fact, standard deviations seem relatively stable over the period. Average efficiency varies between 0.684 and 0.767 for VRS, and 0.828 and 0.931 for the FDH model. In either case, the intertemporal variation is less than 10 percent. These results should be approached with caution. They represent changes in relative efficiency, not in absolute efficiency. That is, an increase in the mean value, between two years, means a reduction of the gap between standard practice and best practice bank. Based on the FDH approach it appears that Chilean banks provide on average only about 83%-93% of the services provided by the best practice Chilean banks. In order to evaluate the degree of agreement between the banks' ranking generated under the two methodologies, the Spearman rank correlation (ρ_s) was computed for all years. The correlations are positive and statistically significant, and do not seem to suggest a high degree of disagreement between the rankings.

Table 2. Annual productive efficiency and Spearman rank order correlation

Method	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
VRS											
mean	0.689	0.725	0.719	0.684	0.706	0.715	0.729	0.755	0.722	0.746	0.767
stdev	0.273	0.305	0.300	0.276	0.274	0.276	0.286	0.289	0.293	0.291	0.256
FDH											
mean	0.864	0.857	0.828	0.858	0.886	0.833	0.864	0.895	0.887	0.896	0.931
stdev	0.219	0.250	0.246	0.221	0.192	0.239	0.265	0.234	0.247	0.237	0.178
ρ_s	0.633^a	0.734^a	0.832^a	0.722^a	0.734^a	0.805^a	0.677^a	0.718^a	0.740^a	0.680^a	0.635^a

^{a}Significant at 1% level.

Table 3 displays the number of banks, grouped by size, operating on the FDH frontier. As a group, small banks are more inefficient than large and medium size banks. Examination of Table 3 indicates that, except for the medium size group, there are two types of structural distributions of efficient banks within each group. First, is the period from 1989 to 1993 in which the percentage of efficient large banks and small banks is on average 70 and 47, respectively. Second, is the period from 1994 to 1999 when these percentages change to 100 and 57, respectively[6]. In contrast, within the medium size

[6]A warning is called for here. The reduction of the sample size may have some influence on the increasing proportion of banks being classified as efficient. Decreasing the sample size decreases the possibility of being inefficient for any given bank. In general, a bank is declared inefficient (or dominated)

group, the number of efficient banks remains fairly stable at about 85% during the whole period.

Table 3. Number of banks FDH efficient

Banks	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Large size	6	6	6	6	6	6	6	5	5	5	5
efficient	4	4	4	5	6	6	6	5	5	5	5
%	66.7	66.7	66.7	83.3	100	100	100	100	100	100	100
Medium size	11	11	11	11	11	11	11	11	10	10	10
efficient	10	10	9	9	9	9	9	11	8	9	8
%	90.9	90.9	81.8	81.8	81.8	81.8	81.8	100	80	90	80
Small size	23	23	23	23	21	20	17	17	17	17	14
efficient	11	12	10	10	8	7	10	10	10	12	11
%	47.8	52.2	43.5	43.5	38.1	35	58.8	58.8	58.8	70.6	78.6
Total	40	40	40	40	38	37	34	33	32	32	29
efficient	23	26	23	24	23	22	25	26	23	26	24
%	57.5	65	57.5	60	60.5	59.5	73.5	78.8	71.9	81.3	82.8

Productivity change results are summarized in Table 4. In one of eleven years the geometric mean of the Chilean banks Malmquist indexes is under unity and for the overall period the geometric mean of the annual Malmquist indexes suggests a 4.8 percent annual productivity growth. The decomposition of this productivity change indicates that Chilean banks exhibit evidence of positive frontier shifts in 1991-93, 1995, 1997 and 1998. Technical progress improved productivity by 5.6% per year over the entire period. On the other hand, there occurred productive efficiency improvement in five years. But this catching up with best practice was not enough to provide an overall productivity improvement. In fact, a slight deterioration of 1% is the result for technical efficiency change for the sample period.

Three different stages can be distinguished from the results in Table 4. First, the year 1990 which exhibits productivity regress. Second, a rapid

if it is possible to find at least one other bank which produces the same or more outputs but uses strictly less of at least one input, or which uses the same or less inputs to produce strictly more of at least one output.

productivity growth during the period 1991-1993. Finally, from 1994 onward there is a normal period in productivity growth that shows a slight tendency to a slower growth rate at the end. The reason to label this last stage as normal and not, say, as a slowdown in productivity is that the second phase seems like an unusual event rather than something permanent, after which productivity growth returned to lower and relatively stable rates.

Table 4. Malmquist Index Summary of Annual Means

	Technical efficiency change	Technical change	Malmquist index
1990	1.022	0.942	0.963
1991	0.957	1.109	1.061
1992	0.885	1.233	1.091
1993	1.047	1.141	1.194
1994	1.111	0.902	1.002
1995	0.948	1.117	1.059
1996	1.057	0.994	1.05
1997	0.905	1.139	1.031
1998	0.964	1.086	1.047
1999	1.062	0.947	1.006
Mean	0.992	1.056	1.048

The accuracy of the Fisher index approximation by the Malmquist index is reviewed in Table 5. Fisher index lower and upper bound were calculated. The results highlight the substantial improvement in productivity experienced in 1991-1993, as the Fisher index lower bound is greater than one. The standard Malquimst index appears to be biased downwards since the overall geometric mean between both Fisher indexes is 5.3%. In 1999 the interval approximation becomes greater, indicating a case where the standard Malmquist index tends to be most inaccurate.

Table 5. Fisher index: upper and lower bound

year	Fisher index upper bound	Fisher index lower bound	Geometric Mean
1990	1.029	0.916	0.971
1991	1.115	1.005	1.058
1992	1.155	1.017	1.084
1993	1.262	1.123	1.190
1994	1.050	0.951	0.999
1995	1.127	0.995	1.059
1996	1.114	0.995	1.053
1997	1.102	0.941	1.018
1998	1.141	0.994	1.065
1999	1.219	0.908	1.052
Geometric mean	1.128	0.982	1.053

The structural distribution of productivity, either for growth or deterioration, can be appreciated from Table 6. Banks, grouped by size, were classified as obtaining productivity gains a loss. The three stages mentioned early are also borne out by Table 6. For example, looking out the column for 1990 one observes that productivity losses prevail. Thus, in 1990 most large and medium size banks exhibit productivity regress. Surprisingly, nearly 40 per cent of small banks exhibit productivity gains. Note that the opposite occurs during the rapid productivity growth period. In general, most large banks exhibit productivity growth throughout the normal period, and almost the same is observed for the case of medium size banks. However, the group of small banks contributes the least to productivity growth during this period.

It is worth noting a striking similarity between the years 1990 and 1999, a year with overall productivity regress and a year with practically nonexistent productivity growth, respectively. Table 6 shows that there is a substantial number of small banks that do exhibit productivity gains in 1999, an outcome that is even stronger in comparison with 1990. Additionally, nearly all medium size banks experienced productivity regress. Large banks, however, seem to have changed the pattern observed in 1990.

Table 6. Banks classified by size and productivity gain or loss

Banks	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Large size	6	6	6	6	6	6	5	5	5	5
gains	1	5	6	5	4	4	3	4	4	5
loss	5	1	0	1	2	2	2	1	1	0
Medium size	11	11	11	11	11	11	11	10	10	10
gains	2	7	11	10	6	10	9	5	8	1
loss	9	4	0	1	5	1	2	5	2	9
Small size	22	23	23	21	20	17	17	17	17	14
gains	9	12	16	17	9	8	8	11	8	9
loss	13	11	7	4	11	9	9	6	9	5
Total	39	40	40	38	37	34	33	32	32	29
gains	12	24	33	32	19	22	20	20	20	15
loss	27	16	7	6	18	12	13	12	12	14

IV SUMMARY

Chile has experienced a high rate of economic growth during the period considered in this study. This performance was accompanied by an increment in average productivity of 4.8% of its banking system. The main cause of the productivity improvement correspond to a shift in the production possibilities frontiers rather than changes in productive efficiency. The results indicate that after a period of rapid productivity growth the banking sector experienced lower and relatively stable rates of productivity change. Small banks appear to be the group that contribute the least to productivity improvement. In addition, this is the group with more banks away from the best practice frontier.

The results provided in this paper seem to suggest that there is no evidence of large technical inefficiency in the Chilean banking system. In fact, when compared with previous years the average technical efficiency values tend to be greater from 1996 to 1999. The findings also suggest that the differences in average technical efficiency over the years are relatively small.

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