The Intertemporal Approach to the Current Account: Evidence for Chile

Sergio A. Salas Landeau*

August 2002

Abstract

In this paper the notion of "excessiveness" of current account imbalances is investigated for the Chilean case between 1960-1999, using a recently expanded model in the line of present value tests that allows for variable interest rate and exchange rate variations. Despite the simplicity of the model, most current account imbalances can be explained by it. Results suggest that the forward looking rational agent is validated. Besides, the relevance of variable interest rate and exchange rates is evident. Notwithstanding, in some periods the magnitude of current account imbalances implied by the model differs from the actual one underlining the importance of borrowing constraints and non-idiosyncratic shocks. We argue that the modest performance of the theoretical model in explaining the magnitude of the current account imbalances is related with the hypothesis of "excess sensitivity" of consumption.

Keywords: Current Account, Present value models, Excess sensitivity.
JEL Classification: F32.

*Department of Economics. University of Chile. E-mail:ssalas@econ.uchile.cl. I would like to thank to the seminar participants at the Department of Economics of the University of Chile for their comments and suggestions.

1. Introduction

In the intertemporal approach of the current account, we view imbalances in it as a result of forward looking dynamic saving and investment decisions.

" Consumption smoothing" plays a central role in determining the magnitudes of current account imbalances. Theory developed in this area has emphasized the Permanent Income Hypothesis. According to this hypothesis, changes in consumption are explained by revisions in expected discounted future income. When an agent with rational expectations gathers more information which leads him to expect a rise in future income he would use the credit market to rise consumption before the actual change in income, this is the notion of " consumption smoothing" which requires non borrowing constraints. A nation, under the representative agent paradigm, is viewed as using international borrowing markets (capital inflows and outflows) to smooth consumption and this would explain current account imbalances.

In this intertemporal approach of the current account, it is also possible to measure the external sustainability of it and this issue has major importance for many countries. Evaluating whether a country has a current account deficit that is sustainable is however a hard task as discussed in Milesi-Ferreti and Razin (1996). Sustainability is related with solvency. An economy is solvent if the present discounted value of future trade surpluses equals current external indebtedness, this is satisfied when the country meets its intertemporal budget constraint. The practical applicability of this theoretical definition is reduced by the fact that it relies on future events or policies[1]. Hence the relevance of the notion of sustainability, a country would be sustainable if the continuation of current account imbalance into the future and no changes in main features of macroeconomic environment does not violate solvency.

Another approach towards measuring the ability of countries to meet their obligations is studying the notion of excessiveness. Excessiveness can be measured finding what current account would a model consistent with intertemporal optimization subject to a budget constraint would predict and comparing it with actual data, this kind of approach has as its main focus the rational expectations Permanent Income Hypothesis discussed at the beginning of this section.

The concept of "excessiveness" imposes a more rigid test in finding evidence of inability of countries to meet their obligations, because this measure is based on deviations from an " optimal" benchmark, derived under the assumption of perfect capital mobility and efficient markets.

This paper follows an approach originally devised by Campbell (1987) to derive an optimal current account of an optimizing agent and find evidence of the

[1]For example in the case of fiscal imbalances, virtually any deficit path can be consistent with intertemporal solvency postulating large future surpluses

Permanent Income Hypothesis and test wether Chile's current account is excessive or not.

The simple intertemporal model implies that the current account surplus of a country should be equal to the present value of expected future declines in output, net of investment and government purchases. A VAR involving current account and output is usually estimated to compute what the optimal current account should be according to the model[2]. This can be compared and tested formally to check if it is equal to the actual current account. This simple model has as a main ingredient the notion of consumption smoothing.

Further extensions of this simple model deal with incorporating certain features that characterize small open economies[3]. Specifically, the role of external shocks are analyzed in this kind of models. It is expected that these shocks affect the economy mainly through interest rate movements or changes in the exchange rate. Interest rate movements would have the role of " unsmoothing " consumption. Moreover an anticipated rise in the relative price of internationally traded goods can rise the cost of borrowing from the rest of the world, when interest is paid in units of these goods. This can cause intertemporal substitution as well. Besides, anticipated changes in this relative price can have intratemporal effects by inducing substitution toward nontraded goods. In this paper this extensions are taken into account for analyzing the excessiveness of current account imbalances for Chile.

2. The Model and Estimation Techniques

This section describes a model presented in Obstfeld and Rogoff (1996) and extended further in Bergin and Sheffrin (2000). The model describes the behavior of a representative agent with rational expectations. This agent aims to maximize utility taking not only intertemporal optimization but also intratemporal optimization. The reason of intratemporal optimization is that the model considers two goods, traded and nontraded goods. So, in each period, the agent must choose optimally to allocate consumption expenditure between the two goods. Of course, as usual, the agent must choose consumption, or real consumption (an index that aggregates both type of goods) so as to maximize intertemporal utility as well, using external assets to this end. Specifically, the agent maximizes:

$$\max_{C_t^*} E_0[\sum_{t=0}^{\infty} \beta^t U(C_t^*)] \tag{1}$$

[2] This is the approach taken by Campbell (1987), followed by Gosh and Ostry (1995) and extended further by Bergin and Sheffrin (2000).

[3] An important advance along this lines comes from Bergin and Sheffrin(2000).

subject to:

$$Y_t - (C_{T,t} + P_t C_{N,t}) - I_t - G_t + r_t B_{t-1} = B_t - B_{t-1} \tag{2}$$

Where C_T and C_N are consumption of traded and nontraded goods respectively. Y_t denotes the value of current output,I_t is investment expenditure, and G_t is government spending on goods and services, since there is no money in this model, all variables are measured in terms of traded goods. The relative price of nontraded goods in terms of traded goods is P_t. B_t is the stock of external assets at the beginning of the period t. r_t is the net world real interest rate in terms of traded goods. By convention r_t is the interest rate calculated over external assets, from period $t-1$, to period t.The left hand side of the budget constraint (2) is by definition the current account. Also we can express total consumption expenditure in terms of traded goods as $C_t = C_{T,t} + P_t C_{N,t}$. $C^* = \Theta(C_T, C_N)$ is a linear homogeneous function of C_T and C_N. This function is interpreted as an index of total consumption, which is called real consumption. We specialize this function to a Cobb-Douglas function: $C^* = C_T^a C_N^{1-a}$. Assuming further a CES specification for the utility function $U(\bullet)$, we can rewrite (1) as:

$$\max_{C_{T,t}, C_{N,t}} E_0[\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}(C_{T,t}^a C_{N,t}^{1-a})^{1-\sigma}]$$

$$\sigma > 0, \quad 0 < a < 1.$$

σ is the coefficient of relative risk aversion, and a is the share of traded consumption in the real consumption index.

In the Appendix A, we show the derivation of the following Euler equation related with the consumption optimization:

$$1 = E_t[\beta(1 + r_{t+1})(\frac{C_t}{C_{t+1}})^{\sigma}(\frac{P_t}{P_{t+1}})^{(1-\sigma)(1-a)}] \tag{3}$$

Where $\gamma = 1/\sigma$ is the intertemporal elasticity of substitution. Assuming joint log normality for C_t and P_t and constant variances and covariances equation (3) may be written as[4]:

$$E_t \Delta c_{t+1} = \gamma E_t r_{t+1}^* \tag{4}$$

Where r_{t+1}^* is a *consumption based real interest rate* defined by:

$$r_t^* = r_t + [\frac{1-\gamma}{\gamma}(1-a)]\Delta p_t + \textit{constant terms} \tag{5}$$

[4]See Appendix A for this derivation.

Also, we define $\Delta c_{t+1} = LnC_{t+1} - LnC_t$,$\Delta p_{t+1} = LnP_{t+1} - LnP_t$, and $(1/\sigma) = \gamma$ equals to the intertemporal elasticity of substitution. The *constant terms* will be irrelevant for the estimation since we demean the series later.

Equation (4) shows the main ingredients in the optimal behavior of the representative agent. For example if it is expected that the real interest rate will rise, then current consumption is more expensive, so this leads to lower current consumption relative to the future with elasticity γ. It is possible another intertemporal effect concerning a change in the relative price of the nontraded good. If the price of traded goods is currently low and expected to rise (this means that Δp_t is negative), then the future repayment of a loan is relatively high. The consumption based interest rate r^*_{t+1} rises above the conventional interest rate, and lowers the current total consumption expenditure relative to the future with elasticity $\gamma(1-a)$. There is also an intratemporal effect that arise from the expected change in the relative price of nontraded goods. Again, if the price of traded goods is temporarily low relative to nontraded goods, the representative agent will substitute toward traded goods by the intratemporal elasticity (unity under the Cobb-Douglas specification). This rises total current consumption expenditure by elasticity $(1-a)$. This intratemporal effect will be dominated by the intertemporal effect if the intertemporal elasticity γ is grater than unity.

Equation (4) tells also something important. If we do not consider the consumption based real interest rate, unexpected temporal shocks changes the current account because of the desire to "smooth" consumption, to this end, the agent would trade external assets. Once changes in the terms of such borrowing or lending are allowed as equation (4) expressed, the agent could be willing to " unsmooth" consumption as described in the last paragraph.

To derive a testable implication of the model, some work must be done in the intertemporal budget constraint. We can write the dynamic constraint (2) as follows:

$$
\begin{aligned}
CA_t &= Y_t - (C_{T,t} + P_t C_{N,t}) - I_t - G_t + r_t B_{t-1} && (6)\\
&= NO_t - C_t + r_t B_{t-1} && (7)
\end{aligned}
$$

Where net output is defined as: $NO_t \equiv Y_t - I_t - G_t$. We also define a market discount factor for date s consumption, so that:

$$R_s = \frac{1}{\prod_{j=1}^{s}(1+r_j)}$$

Now summing over all periods of the infinite horizon, and imposing the following transversality condition:

$$\lim_{t\to\infty} E_0(R_t B_t) = 0$$

we can write an intertemporal budget constraint:

$$\sum_{t=0}^{\infty} E_0(R_t C_t) = \sum_{t=0}^{\infty} E_0(R_t NO_t) + B_0 \tag{8}$$

Where B_0 is initial net foreign assets. This last equation can be log linearized as follows[5]:

$$-\sum_{t=1}^{\infty} \beta^t [\Delta no_t - \Delta c_t] = no_0 - c_0 \tag{9}$$

Where lower case letters represent the logs of upper case counterparts. The procedure of linearisation necessarily implies the assumption that the steady state (around which to linearise) of the net foreign assets is zero, see appendix B for the derivation.

Now we can take expectations in (9) and combine it with the Euler equation (4) to obtain (using the law of iterated expectations):

$$-E_t[\sum_{s=t+1}^{\infty} \beta^s (\Delta no_s - \gamma r_s^*)] = no_t - c_t \tag{10}$$

Notice that the right hand side of this equation is approximately the same as the right hand side of (7) which in turn is equal to the current account, then we label this transformed current account as CA_t^*, which is the current account derived from the postulated model:

$$CA_t^* = -E_t[\sum_{s=t+1}^{\infty} \beta^s (\Delta no_s - \gamma r_s^*)], \qquad CA_t^* \equiv no_t - c_t \tag{11}$$

This last equation tells us the dynamics of the current account. If net output is expected to fall, the current account will rise because the representative agent wants to smooth consumption. However, he could also " unsmooth " consumption, because of changes in the consumption based real interest rate. If the consumption based real interest rate is expected to rise, the current account will rise also. So if for example, the agent has expectations of lower future output, this would lead him to "save for rainy days", but if the consumption based real interest rate falls, this can temper the agent desire to lend because of the low opportunity cost of present consumption.

Equation (11) characterizes the " optimal " current account, the problem with estimating (11), is that we do not know how the agent forms expectations of future realizations of net output and consumption based real interest rate. Campbell (1987) first address this issue by noticing that under the null hypothesis of (11),

[5] Appendix B gives a detailed derivation first developed in Huang and Lin(1993).

the current account itself should incorporate all of the consumer's information on future values of the linear combination of consumption based interest rate and net output changes specified in that equation. Also because empirically there could be some persistence in the macroeconomic series, lagged values of net output and the consumption based real interest rate can be useful in predicting that combination. All of this leads us to estimate a VAR for representing consumers forecasts:

$$\begin{bmatrix} \Delta no \\ CA^* \\ r^* \end{bmatrix}_t = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} + \begin{bmatrix} \Delta no \\ CA^* \\ r^* \end{bmatrix}_{t-1} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_t$$

This VAR can also be written more compactly as $y_t = Fy_{t-1} + u_t$. Also it would be the case that: $E_t[y_{t+i}] = F^i y_t$. Of course this can be generalized for a higher order VAR. Notice that equation (11) can be written using the VAR as:

$$CA_t^* = -\sum_{i=1}^{\infty} \beta^i (g_1 - \gamma g_2) F^i y_t \tag{12}$$

where $g_1 = [1\ 0\ 0], g_2 = [0\ 0\ 1]$, and $h = [0\ 0\ 1]$ (this can also be generalized for a higher order VAR)

Now if the VAR is stationary, it is possible to write (12) as:

$$CA_t^* = [-(g_1 - \gamma g_2)\beta F(I - \beta F)^{-1}]y_t \tag{13}$$

With the estimated parameters of the VAR and some values for the parameters β, γ and a, it is possible to find the estimated optimal current account[6]:

$$\widehat{CA_t^*} = ky_t \tag{14}$$

where

$$k = -(g_1 - \gamma g_2)\beta \widehat{F}(I - \beta \widehat{F})^{-1} \tag{15}$$

and $\widehat{F}$ is the matrix of estimated parameters from the VAR.

$\widehat{CA_t^*}$ can be compared with the actual (modified) data on current account (equation 11), as an indication of how well the restrictions of the theory (and method of forecasting) are satisfied. Moreover, formal test of this equality can be conducted by calculating a χ^2 statistic for the null hypothesis that $[0\ 1\ 0] = k$, under this null: $CA_t^* = \widehat{CA_t^*}$. Let $\widetilde{k}$, be the difference between the actual k and the hypothesized value, then the following test:

$$\widetilde{k}'[(\partial k/\partial F)V(\partial k/\partial F)']^{-1}\widetilde{k} \tag{16}$$

[6]We call this the optimal current account, but we mean optimal in the sense that it is the current account the model implies, together with the VAR as a forecast tool.

will be distributed chi-squared with three degrees of freedom (or the number of restrictions in a higher order VAR). Where V is the variance-covariance matrix of the estimated parameters in the VAR, and $\partial k/\partial F$ is the matrix of derivatives of the k vector with respect to the parameters of the VAR.

3. Data description

The availability of data for Chile, allows us to take into account quarterly data from 1960:1 to 1999:4. All data are taken from the Central Bank databases.

A measure of the world real interest rate has been computed, following the method of Barro and Sala-i-Martin (1990). First, we collected short term nominal interest rates, T-bill rates or equivalent, on the G-7 economies from the IFS (International Financial Statistics). We need to adjust this rate for inflation expectations which are more reliable forecasts for short periods of time to the future, hence the reason to use short term nominal rates. Inflation in each country is measured using that country's consumer price index, and expected inflation is constructed by estimating first an appropriate ARMA(p,q) process for each country, and then calculating a one step ahead forecast[7]. The nominal interest rate in each country then is adjusted by inflation expectations to compute an ex-ante real interest rate. Finally, an average ex-ante real interest rate was computed using time varying weights for each country based on its share of real GDP in the G-7 total, this gives us the series r_t.

The net output series, NO_t was constructed by subtracting investment and government purchases from GDP, adjusted by population, we use this in logged an differenced form Δno_t. The series for the current account variable CA_t^*, was constructed for each country by subtracting the log of consumption, adjusted for population from the log of net output.

We use as a proxy for P_t the real exchange rate of Chile as computed in the database of the Central Bank. We follow Rogoff (1992) and Bergin and Sheffrin (2000) in this approach. An ex-ante expected exchange rate appreciation is computed, using again an appropriate ARMA process, and calculating a one step ahead forecast[8]. Finally, we compute the consumption based real interest rate r_t^* using the calculated world real interest rate and the exchange rate appreciation as expressed in (5). Because we are interested in the dynamic implications of the intertemporal model, the three series, $\Delta no_t, CA_t^*$ and r_t^* are all demeaned.

We also need values for the parameters β, a and γ. For the first of this param-

[7] All process were estimated with seasonal dummies. For Canada the process is an ARMA(4,7), for France an ARMA(8,8), for Italy an ARMA(8,6), for Japan an ARMA(5,5), for United Kingdom an ARMA(5,3), for United States an ARMA(1,4) and for Germany an ARMA(8,8).

[8] The process were estimated with seasonal dummies, and the order is ARMA(8,7).

eters the model itself might be useful to find out its value. In the steady state, the Euler equation (3) implies that $\beta = 1/(1+\bar{r})$. Where $\bar{r}$ represents the steady state value of the world interest rate. Taking the mean of this variable in our data set, we find that $\beta \approx 0{,}95$. Regarding the share of traded goods in private final consumption we follow the strategy of trying different values of the parameters, we also take as a guide some empirical estimations for United States. We choose to take two different values $a = 1/2$ and $a = 2/3$. We show that there is a slight variation in the results from using this two values.

The intertemporal elasticity γ was traditionally the most complicated parameter, the literature seems to assign a value of 0.5 or bigger for this parameter, we use this value. Also, we use a value of this parameter chosen so as the optimal current account match the variance of the actual series, this value turns out to be grater than 0.5 but not so different.

4. Results

In order to evaluate the performance of the model, we are going to rely mainly in two measures: As a guidance, a visual comparison of the predicted optimal current account with the actual current account, we seek for matching turning points and volatility of the series. We also calculate confidence intervals for the optimal current account to check if the actual current account falls in it. The other measure comprehends more formal tests to asses whether the estimated optimal current account is different from the actual. First the chi squared test was computed as expressed in (16). In all of the estimations the optimal lag of the VAR was defined to be 2 according to usual tests[9]. Under the null ($\widehat{CA}_t = CA_t$) this test is distributed asymptotically as a χ^2 with six degrees of freedom. This result tough, hold under the hypothesis of normality. We perform then a normality test on VAR's residuals which yields to strongly reject normality raising doubts on the accuracy of the tests for all the estimations[10]. We then resort to bootstrapping to find critical chi squared values for rejection of the null hypothesis. Furthermore, we calculate a more accurate measure of bootstrap for the tests using block bootstrap which bootstraps the series itself (and not the residuals of the VAR) to correct for possible bias raising from omission of auto correlations in the traditional bootstrap[11]. Given our finding of non-normality, the confidence intervals for the optimal current account were also computed by bootstrap and block bootstrap.

Moreover, we have computed sign tests for the model. In the context of the current account is important to check the match between the sign of the optimal

[9] For example checking withe noise in vectorial sense and performing Information Criterion Tests.

[10] The normality test was based in Doornik and Hansen (1994).

[11] The reference for this kind of bootstrap is Berkowitz and Killian (1996).

current account and the actual series, since a divergence in sings would imply that our theoretical model says that for example the country borrows to finance consumption, and the actual series is saying that the country is lending resources abroad. The test sign computes the probability that both the model and actual series share the same current account sign. It is also possible to compute conditional probabilities that the optimal current account is positive when the actual series is positive and conditional probabilities that the optimal current account is negative given that the actual series is negative respectively. The null hypothesis of this this test is that the probability is 0,5. This would imply that there is no useful structure in the theoretical model to explain the current account in signs and matches just happen by coincidences, the probability estimations can be done by Maximum Likelihood Estimation. Finally, we also compute the variance ratio of the optimal current account to the actual one. If the model predicts actual current account fluctuations, this ratio should be close to one.

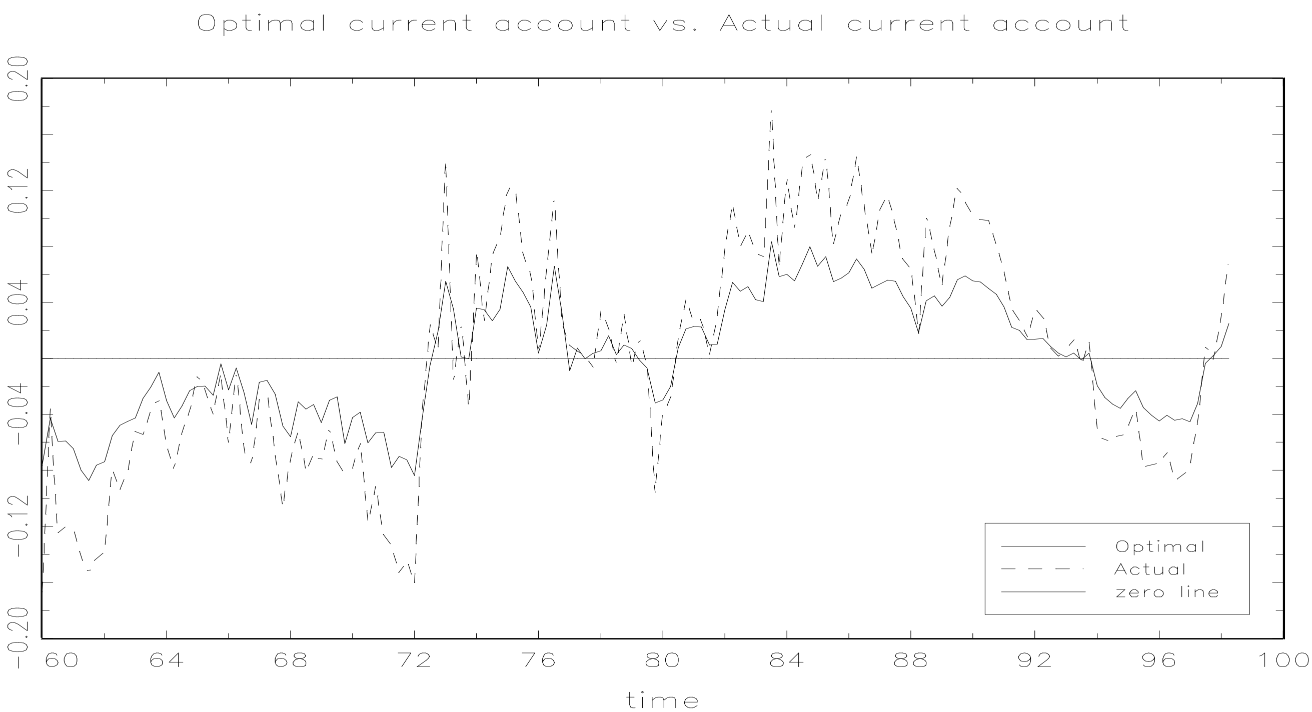
All this information is summarized in figures 1 to 5 and in table 1 for several estimations of the " optimal " current account (labelled case 1, case 2, etc.). The reason of having several estimations is that our aim is not only to find the correct model that best explains current account imbalances, but also, by taking full advantage of the variables included in the theoretical model, to check the relevance of some variables in explaining current account movements. Moreover, all the tests (formal and informal) can help us to assign values to some parameters the theoretical model use, for example the intertemporal elasticity of substitution γ, and the share of consumption of traded goods a.

We begin therefore, to calculate a predicted current account not considering the consumption based real interest rate r^* (see equation 5 and 11), in this case the current account in the theoretical model should be equal to the negative of the expected present value of changes in net output. That is, the possibility of " unsmooth " consumption, is not considered here.

We also calculate the eigenvalues for the matrix F, and check that all of them lies outside the unit circle[12]. Then the VAR is stationary and we proceed to calculate the matrix k in (15) and find, using this vector, what current account the model implies as postulated in (14). In figure 1(a) we present the optimal current account along with the actual current account.

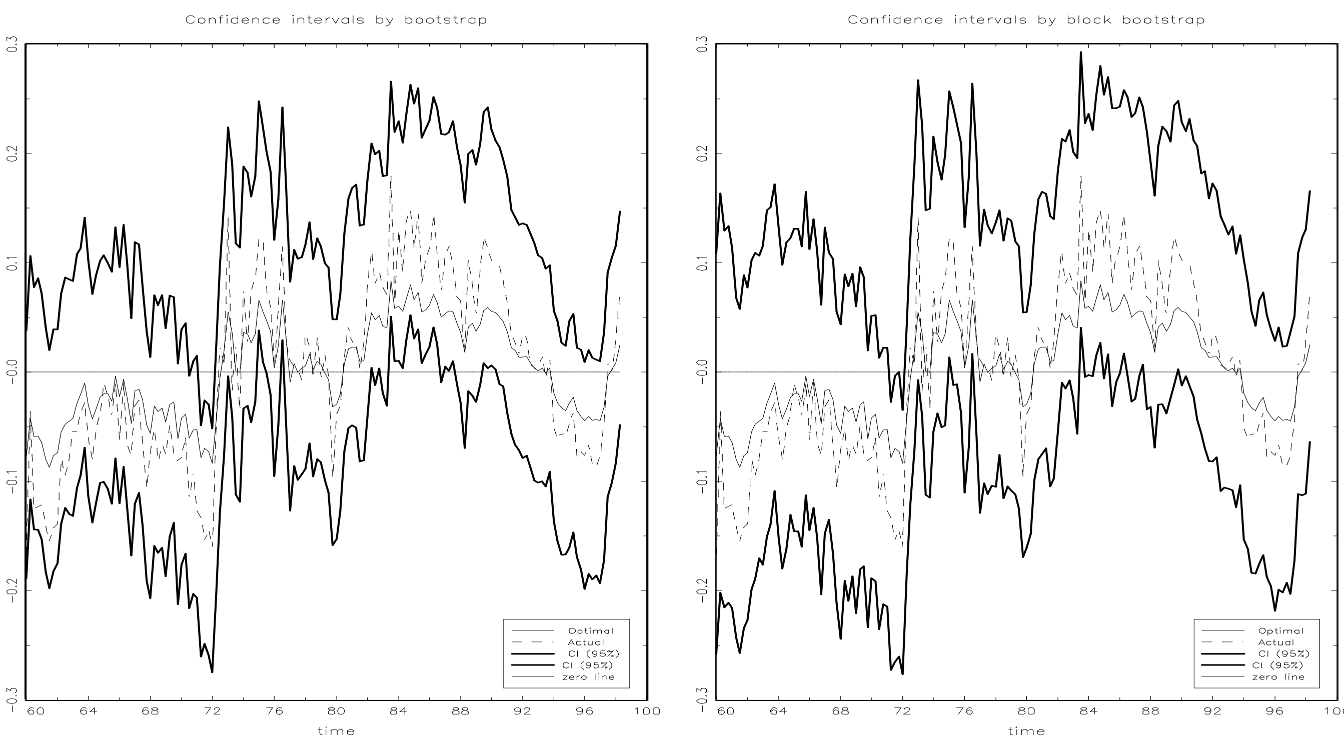
[12] This result hold in all of the estimations performed.

Figure 1(a) Case 1 (Without r^*)



The graphical performance of this simple model is surprisingly remarkable, considering that the consumption based real interest r^* has not been taken into account. Remember that by not including the consumption based real interest rate we are precluding this country of consumption unsmoothing by disregarding the effects of changes in the world interest rate and expected changes in the exchange rate. Also, as it can be seen in the first panel of table 1, the overall performance of the model is not bad. Both **Failure** and **Failure (BB)** are zero and the formal χ^2 test cannot reject the null hypothesis that the vector k is [0 0 1 0], so the restrictions of the theory are satisfied. Also, both the conditional and unconditional probabilities of success are approximately 0.94, different from 0.5. But we should be cautious in this case because it appears that confidence intervals are too wide, as can be seen in figure 1(b), so that the accuracy of them is not appropriate. Moreover, the ratio of variances is only 0.28 in tabel 1 , and this is also something that is evident from the figure. In this case the " optimal " current account is underestimating the magnitude of the actual current account fluctuations.

Figure 1(b) Case 1 (Without r^*)

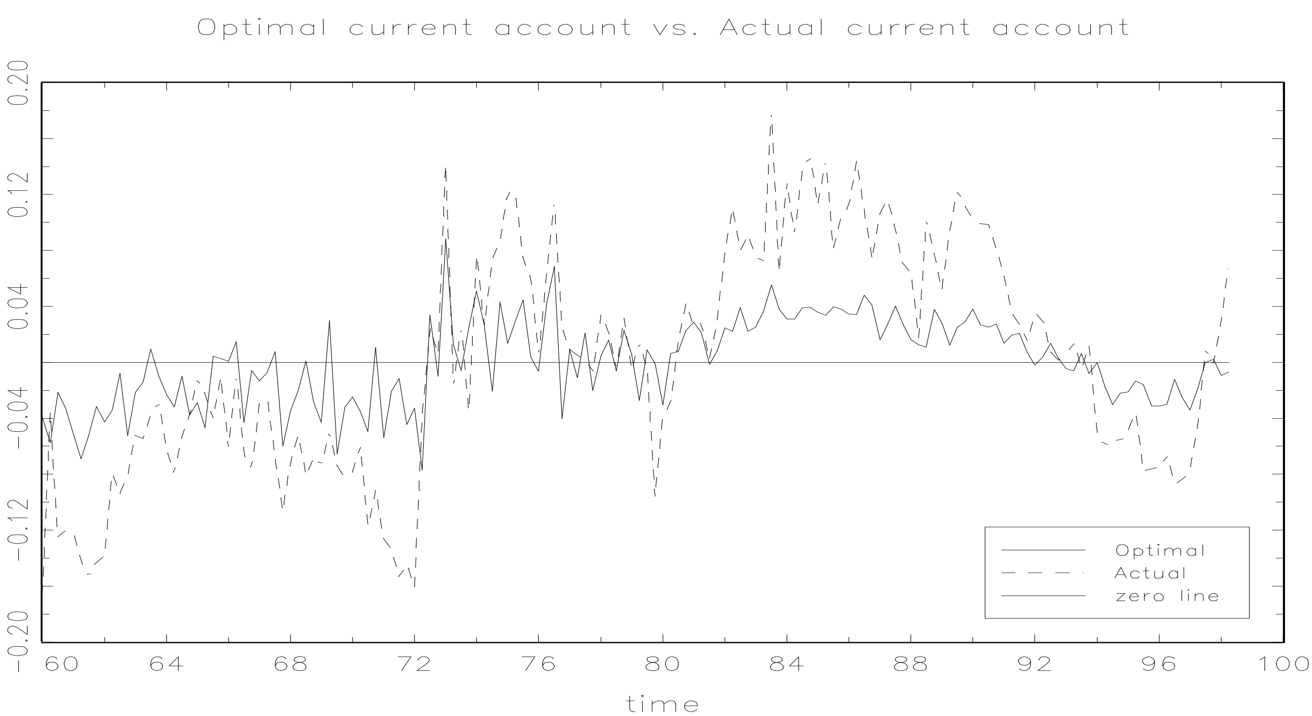


We can verify in figure 1(b), that all of the observations lie within the interval, but the interval itself is two wide. Moreover, the confidence interval includes zero in most of the time as well as a wide range of positive and negative values for the current account.

It is very likely that the model improves a lot with the incorporation of the consumption based real interest rate.

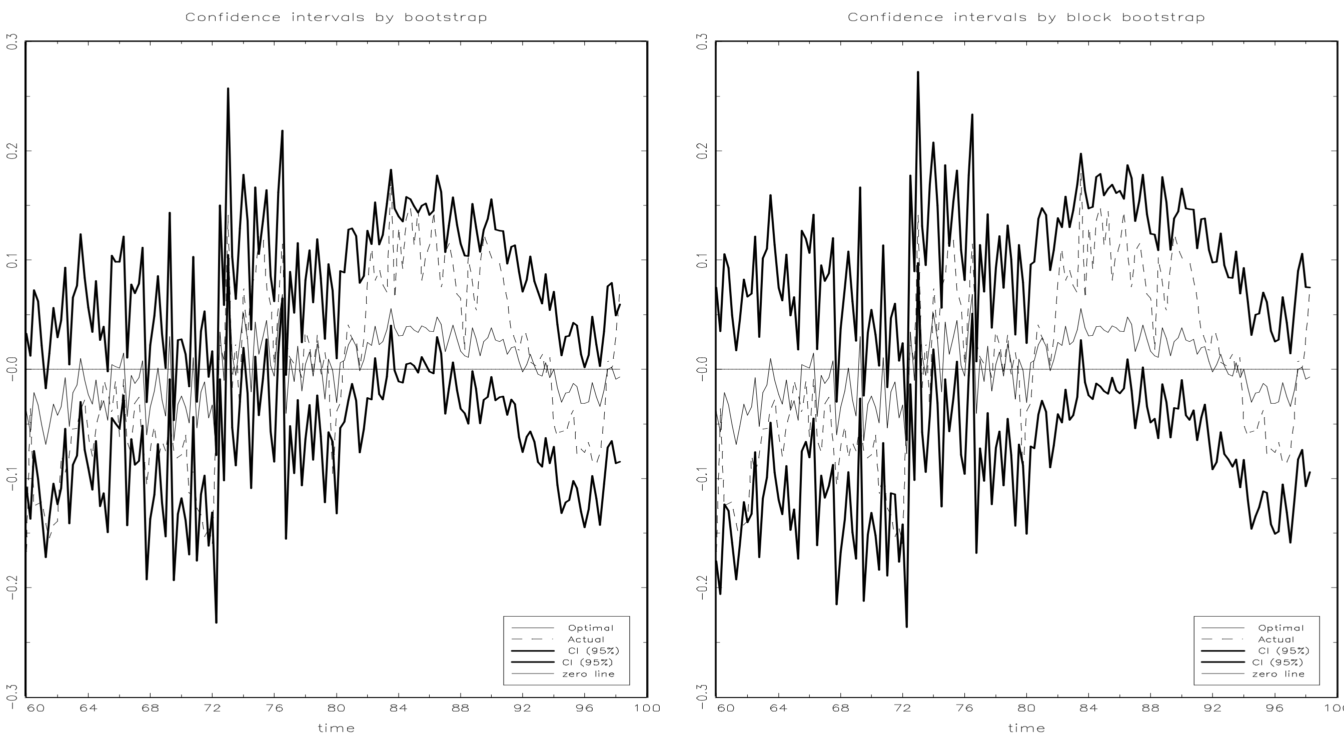
We take then into account the consumption based real interest rate, notice that effects from changes in this rate may come from two sources, changes in the world real interest rate, or expected changes in the exchange rate. In order to identify which one is the source that mainly explains current account movements we first estimate the model without world interest rate (and including exchange rate) in figure 2, and without exchange rate (and including world real interest rate) in figure 3. In both cases the vector k should not be statistically different from [0 0 1 0 0 0], for the model not to be rejected.

Figure 2(a) Case 2 (Without world interest rate, $\gamma = 0{,}5$**,** $a = 2/3$**)**



From this figure it is possible to see that little improvement has been made. In general, the optimal current account changes little, there is more volatility but the same underestimation of variance is observed, in fact the variance ratio has lowered to 0.14 compared with the first estimation as can be seen in the second panel of table 1. Moreover, the probabilities of success have also fallen to the range of 0.79-0.82, far from 0.5 though. Some improvement is observed however in some periods of the sample. The volatility and magnitude of the model match better actual series in the seventies, suggesting the importance of the expected appreciation in the current account.

Figure 2(b) Case 2 (Without world interest rate, $\gamma = 0{,}5$**,** $a = 2/3$**)**

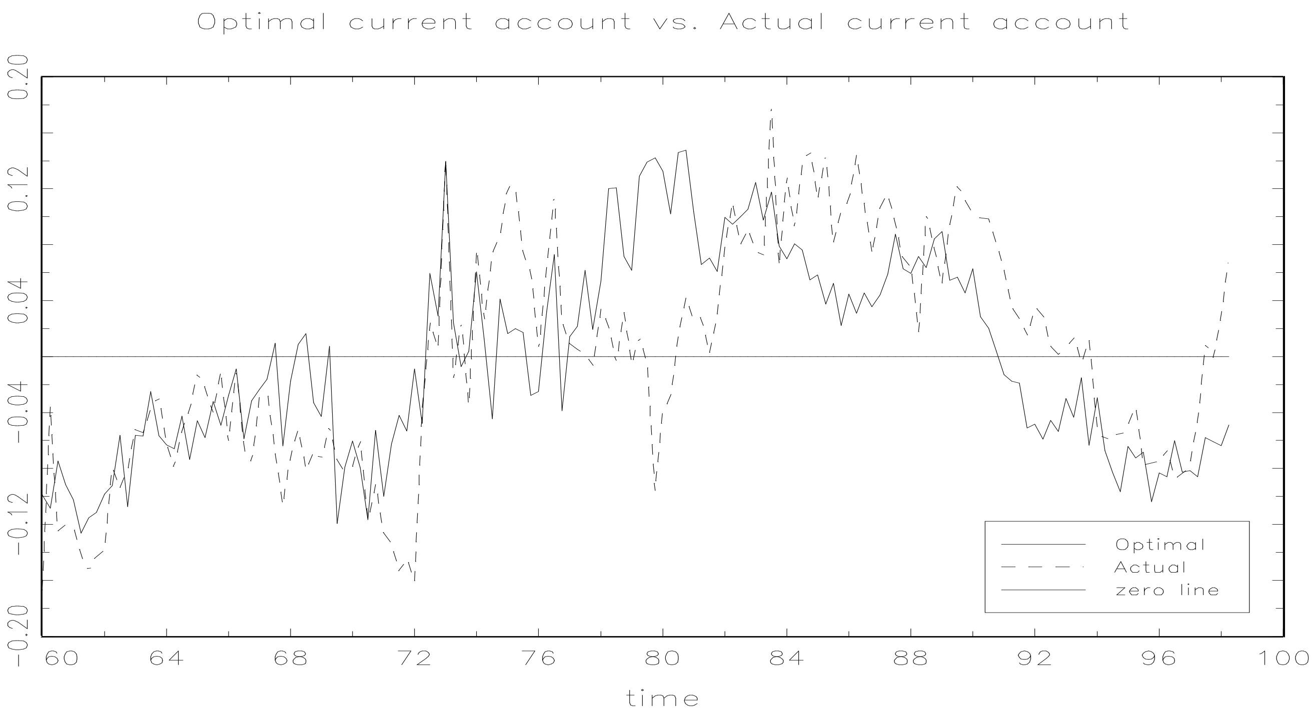


In figure 2(b) it is possible to see that there has been some improvement in the estimation of confidence intervals. Now, some observations lie outside the interval but this is precisely because this last result. Both confidence intervals leads to similar results, according to the bootstrap, 0.17 observations lie outside the interval, and 0.08 according the block bootstrap which can be seen in table

1. Also, the formal χ^2 test does not reject the null hypothesis. Notice that in this case the value chosen for the parameters in the estimation were $\gamma = 0{,}5$ and $a = 2/3$, which turn out to be in the range of reasonable values according to estimations in the complete model as we are going to see later.

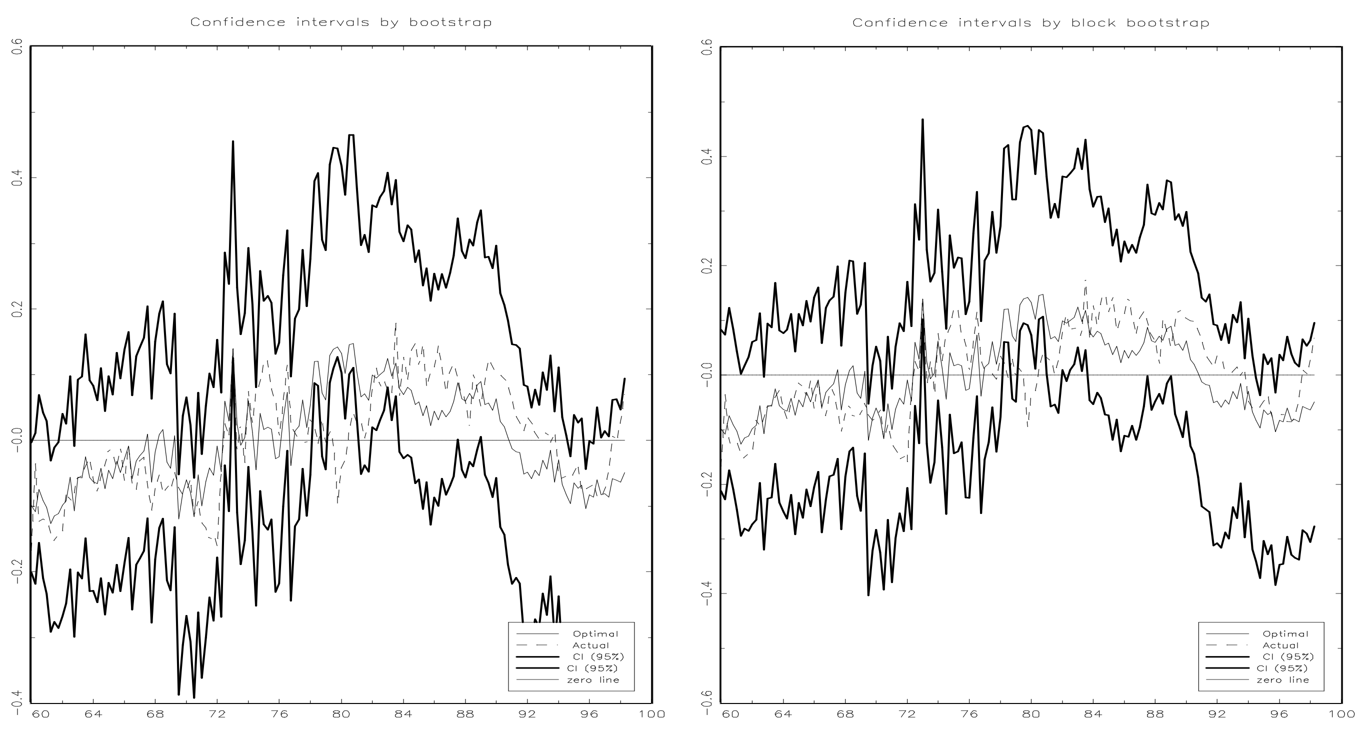
We turn now, to the estimation that considers the world interest rate and not the expected exchange appreciation, the series can be seen in figure 3(a).

Figure 3(a) Case 3 (Without exchange rate, $\gamma = 0{,}5$, $a = 1$**)**



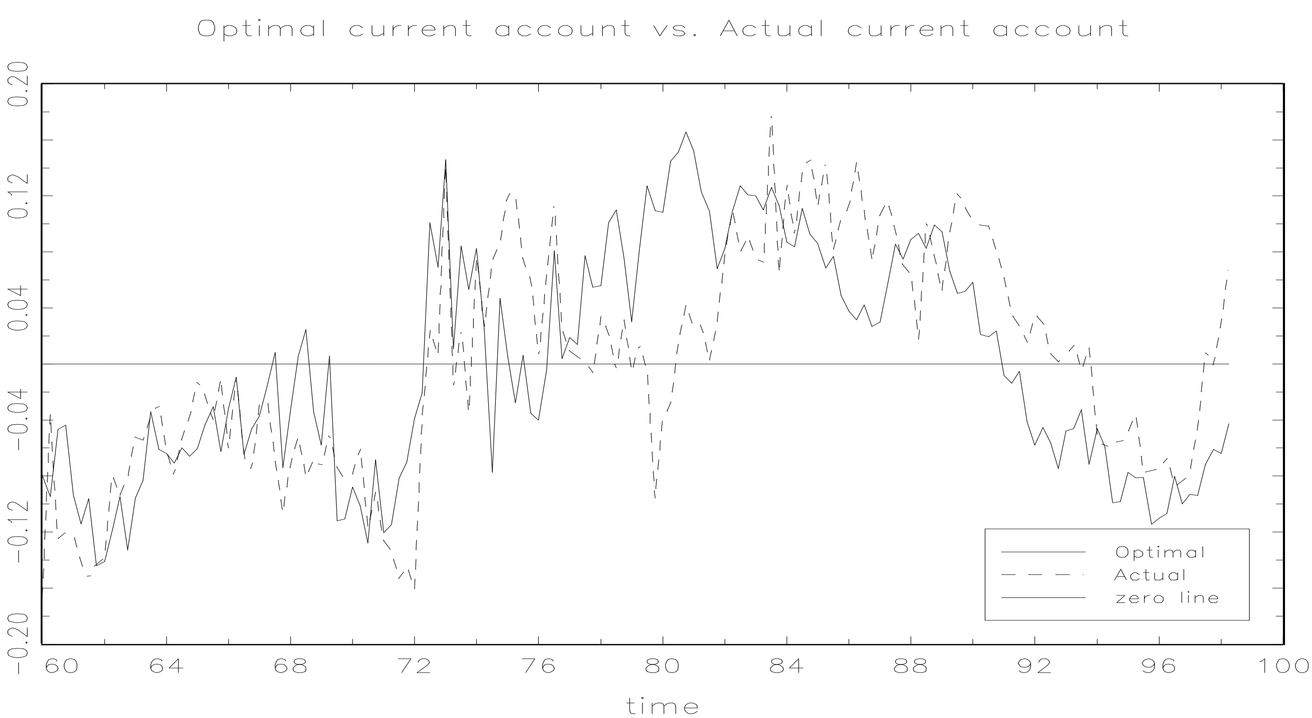
It is clear that the variable that most explain current account imbalances is the world real interest rate. This is expected since we are considering a case where the country is small. Notwithstanding, the model performs worse for the late seventies and early eighties. Table one also give some useful measures for comparison with the previous case (that not incorporate this variable). Again neither model is rejected given the bootstrap chi squared critical values. But the variance ratio is much better for the case of considering only the world interest rate: 0.78. Moreover the percentage of failure in bootstrap confidence intervals gives more reliability for the model that considers the real interest rate (0.07 and 0.06 for the bootstrap and block bootstrap respectively), although confidence intervals are a little bigger for this model than the previous case, this is corroborated in table 1, since both measures of failure, by bootstrap and block bootstrap leads to fewer observations outside the intervals. The probabilities of success have not changed much either. However, one period where exchange rate variations seems to be quite important is in 1980, when the model including just exchange rate variations is capable of explaining to some extent, the deep imbalance of that period.

Figure 3(b) Case 3 (Without exchange rate, $\gamma = 0{,}5, \quad a = 1$**)**



Finally we considered two models, both fully take into account all the variables detailed in section 2. That is the consumption based real interest rate was included as defined in (5). For the value of parameters, we tried two different specifications. In figure 4 we take $a = 2/3$, and we chose γ, so as to match the variance of the actual series. This gives $\gamma = 0{,}604$. In figure 5, we take $a = 1/2$, and $\gamma = 0{,}5$. Details in the several measures of comparison can be observed in the fourth and fifth panels of table 1. Again, given that the order of the VAR is 2, the vector k should not be statistically different from [0 0 1 0 0 0] for the model not to be rejected.

Figure 4(a) Case 4 ($\gamma = 0{,}604$, $a = 2/3$)

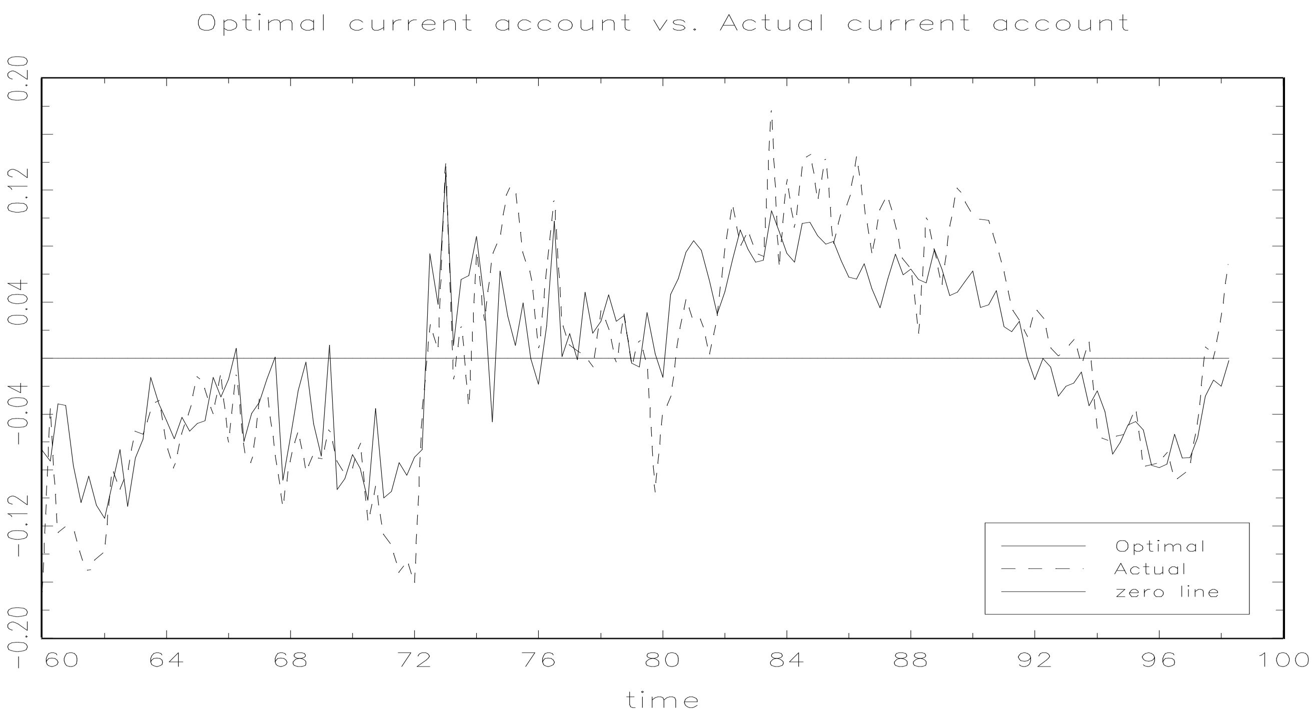


With the inclusion of both the world real interest rate and the expected appreciation of the exchange rate the performance of the model improves a lot, in this figure we are considering a traded goods share of $a = 2/3$ and the intertemporal elasticity that match the variance of actual series that is $\gamma = 0{,}604$. Still, there is some problem in matching some imbalances of the current account. In general the sings are correct, as it can be verified in the fourth panel of table 1. Besides, the probabilities of success ranges from 0.76 to 0.82 different from 0.5. Furthermore, the percentage of failure are again low, close to 0.08, but it is not easy to replicate the magnitude of the imbalances, because even in this case where the intertemporal parameter has chosen to match the variance, observation of the series implies some movements of the actual series be unexplained by the theoretical model.

Figure 4(b) Case 4 ($\gamma = 0{,}604, \quad a = 2/3$)

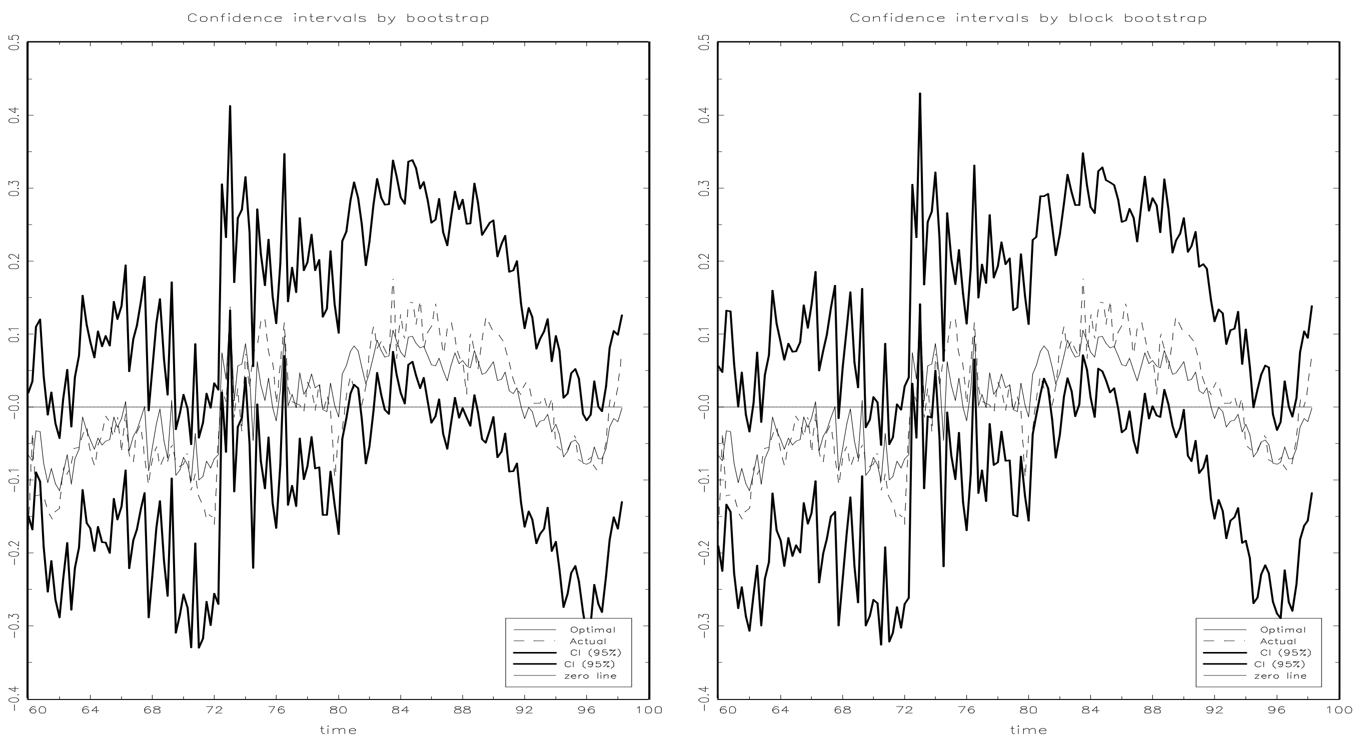
Confidence intervals are wide and the percentage of failure is low as expressed in table 1. However, in the period of a deep imbalance of the current account, 1980-1982, the model is unable to match the sign of the actual current account. Furthermore confidence intervals do not include the actual observation in that date. The formal tests leads us to not reject the model as it happened in every model we have analyzed.

Figure 5(a) Case 5 ($\gamma = 0{,}5, \ \ a = 1/2$)



The series are very similar to the previous figure, however, now the model captures the sign of the current account in the troubled period of the beginning of the eighties. Also by looking at the fifth panel of table 1, it is possible to see some improvements in different ways, both the failure measure and the probability of success improves, again the model cannot be rejected. The problem of variance persist, in this case the theoretical model's variance is a little bit more than half of the actual variance.

Figure 5(b) Case 5 ($\gamma = 0{,}5$, $a = 1/2$)



In this case we can see that the confidence intervals includes the imbalance of the beginnings of the eighties, besides that confidence intervals are about the same width of the previous figure. As the fifth panel of table 1 indicates, probabilities of success are beyond 0.80 and the percentage of failure below 0.04.

Observation of the figures 4 and 5, lead us to conclude that there is an improvement over past specifications, many changes in current account are matched in time by both models. There is some problem with the variance of the models however, since even in the model where γ was chosen so as to match variance (figure 4), it is not possible to match it in every period, that is even if the overall volatility is the same, the estimated optimal current account does not match many movements of the series.

The general conclusion with regard of the behavior of the theoretical model is that it is quite successful in explaining most of the imbalances, but it has some shortcomings trying to explain the magnitude of those imbalances which is related with the low performance in matching the variance of the actual current account.

Table 1[†] **Case 1 (Without r^*)**

Vector k:

$[no_t \quad no_{t-1} \quad CA_t^* \quad CA_{t-1}^* \quad r_t^* \quad r_{t-1}^*]$

$[-0{,}05 \quad 0{,}37 \quad -0{,}11 \quad 0{,}18 \quad -- \quad --]$

Failure	**Chi squared test**
0	666
Failure (BB)	**Critical Chi**
0	905
Unconditional probability of success	**Critical Chi (BB)**
0.94*	888
Conditional probability of success (+)	**Variance ratio**
0.94*	0.28
Conditional probability of success (-)	
0.93*	

Case 2 (Without world interest rate,$\gamma = 0{,}5$, $a = 2/3$)

Vector k:

$[no_t \quad no_{t-1} \quad CA_t^* \quad CA_{t-1}^* \quad r_t^* \quad r_{t-1}^*]$

$[-0{,}37 \quad 0{,}22 \quad -0{,}22 \quad -0{,}08 \quad 0{,}11 \quad -0{,}03]$

Failure	**Chi squared test**
0.17	140
Failure (BB)	**Critical Chi**
0.08	242
Unconditional probability of success	**Critical Chi (BB)**
0.80*	196
Conditional probability of success (+)	**Variance ratio**
0.82*	0.14
Conditional probability of success (-)	
0.79*	

Case 3 (Without exchange rate,$\gamma = 0{,}5$, $a = 1$)

Vector k:

$[no_t \quad no_{t-1} \quad CA_t^* \quad CA_{t-1}^* \quad r_t^* \quad r_{t-1}^*]$

$[-0{,}35 \quad 0{,}27 \quad 2{,}91 \quad -0{,}1 \quad 0{,}15 \quad -0{,}47]$

Failure	**Chi squared test**
0.07	55
Failure (BB)	**Critical Chi**
0.06	113
Unconditional probability of success	**Critical Chi (BB)**
0.79*	116
Conditional probability of success (+)	**Variance ratio**
0.76*	0.78
Conditional probability of success (-)	
0.83*	

Case 4 ($\gamma = 0{,}604$, $a = 2/3$)

Vector k**:**

$[no_t \;\; no_{t-1} \;\; CA^*_t \;\; CA^*_{t-1} \;\; r^*_t \;\; r^*_{t-1}]$

$[-0{,}37 \;\; 0{,}25 \;\; 1{,}86 \;\; -0{,}13 \;\; 0{,}25 \;\; 0{,}74]$

Failure	**Chi squared test**
0.07	50
Failure (BB)	**Critical Chi**
0.09	106
Unconditional probability of success	**Critical Chi (BB)**
0.79*	107
Conditional probability of success (+)	**Variance ratio**
0.76*	1
Conditional probability of success (-)	
0.82*	

Case 5 ($\gamma = 0{,}5$, $a = 1/2$)

Vector k**:**

$[no_t \;\; no_{t-1} \;\; CA^*_t \;\; CA^*_{t-1} \;\; r^*_t \;\; r^*_{t-1}]$

$[-0{,}4 \;\; 0{,}36 \;\; 0{,}51 \;\; -0{,}14 \;\; 0{,}26 \;\; 0{,}27]$

Failure	**Chi squared test**
0.04	99
Failure (BB)	**Critical Chi**
0.03	193
Unconditional probability of success	**Critical Chi (BB)**
0.84*	166
Conditional probability of success (+)	**Variance ratio**
0.81*	0.54
Conditional probability of success (-)	
0.87*	

† **Failure** measures the proportion of observations of the actual current account that lies outside the confidence intervals.**Failure (BB)**, is the same measure but using the confidence intervals from the block bootstrap. **Unconditional probability of success** is the probability that the model follows actual current account in sign. **Conditional probability of success** (+) and (-), is the probability that the model leads to a positive current account when the actual series of current account is positive, and leads to a negative current account when the actual series of the current account is negative respectively. **Critical chi** and **Critical chi (BB)** is the critical chi squared calculated from bootstrap and block bootstrap respectively. **Variance ratio** is the ratio of the optimal variance of the current account over the actual series.

* Statistically different from 0.5.

5. Conclusions

Our search of a model that captures movements of the current account has led us to a specification where the importance of consumption " unsmoothing " has prevailed. We have shown that as a small country Chile's current account is greatly influenced by external shocks such as variations in the world interest rate and changes in exchange rate.

The model captures major current account imbalances at least in most of the time. Moreover, given that we find no statistical evidence that the current account predicted by the model differs from the actual current account, is not possible to say that there is " excessiveness " of current account imbalances. But as the evidence on the mismatch of variances suggested us, it appears that something is missing in explaining more accurately current account imbalances in magnitude in certain periods.

One point to highlight related to excessiveness in current account imbalances is that actual current account is in general grater (in absolute value) than the optimal one not only in deficits, but also in superavits. It can be possible that capital controls are playing a crucial role, this can be true given the period analyzed, since capital controls have been eliminated only very recently (1999).

In a more general level, not only capital controls may have been underlying our findings, but any kind of international financial imperfections like borrowing constraints. Moreover, the model does not incorporate a difference between aggregate and idiosyncratic shocks, it simply assumes that all shocks are idiosyncratic and so diversifiable, allowing consumption smoothing, but it is possible that there have been aggregate shocks that this model do not take into account.

The main finding in several of our specifications is that the " optimal " current account underestimates the magnitude of actual current account. This fact is consistent with the " excess sensitivity " of consumption which is related with the borrowing constraint explanation we just pointed out: Suppose consumers receive new information that causes an upward revision to expectations about future incomes, the Permanent Income Hypothesis calls for an upward revision of consumption before future arrives. This could be done with efficient financial markets by lending these consumers against their future income, but with financial constraints or sharing a common (aggregate) shock with potential lenders, consumers should wait until income improvement actually materializes, then the data would tell us that a big imbalance has realized in the actual current account, which is not true for our theoretical model which presumes that the financial markets has functioned well so as to smooth out these big oscillations in consumption.

Finally it would be desirable in future work to take into account some additional considerations such as investment dynamics or liquidity constraints, which are not incorporated in our model, or many other aspects such as the modelling of labor decisions.

References

- Campbell J. (1987) "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis". Econometrica Vol 55 (November).
- Barro R. and Sala-i-Martin J. (1990) "World Real Interest Rates". NBER Working Paper 3317 (April).
- Bergin P. and Sheffrin S. (2000) "Interest rates, Exchange rates and Present Value Models of the Current Account". The Economic Journal Vol 110 (April).
- Berkowitz J. and Killian L. (1996) Recent Developments in Bootstrapping Time Series"Working Paper Federal Reserve Board Washington.
- Doornik J. and Hansen H. (1994) .An Omnibus Test for Univariate and Multivariate Normality"Working Paper Nuffield College Oxford.
- Ghosh A. and Ostry J. (1995) "The Current Account in Developing Countries: a Perspective from the Consumption Smoothing Approach "The World Bank Economic Review Vol 9 No. 2 (June).
- Hall R. (1988) "Intertemporal Substitution in Consumption"Journal of Political Economy Vol 96 No.2 (April).
- Huang and Lin. (1993) "Deficits, Government expenditures and Tax smoothing in the United Sates: 1929-1988". Journal of Monetary Economics Vol 3 (April).
- Milesi-Ferreti G. and Razin A. (1996) "Sustainability of Persistent Current Account Deficits". NBER Working Paper 5467 (February).
- Obstfeld M. and Rogoff K. (1996) "Foundations of International Macroeconomics". MIT Press.
- Obstfeld M. and Rogoff K. (1995) "The Intertemporal Approach to the Current Account". Handbook of International Economics. Vol III Elsiever Science.
- Rogoff K. (1992) "Traded Goods Consumption and the Random Walk Behavior of the Real Exchange Rate". Bank of Japan and Economic Studies Vol 10 No. 2 (November).

Appendix A: Deriving the Euler equation

For the derivation of the euler equation (3), we must first solve the intratemporal optimization of the agent. Recall that we have defined real consumption as an index with the following specification: $C_t^* = C_{T,t}^a C_{N,t}^{1-a}$. It is possible to interpret this index as a (intratemporal) utility function which can be maximized under the restriction:$C_t = C_{T,t} + P_t C_{N,t}$. Doing so, we can easily find the optimal consumption of traded and nontraded goods:

$$C_{T,t} = aC_t \quad C_{N,t} = \frac{(1-a)}{P_t} C_t$$

That is, we have found the Marshalian demands for traded and nontraded goods.

In this point, we need to define the *consumption-based price index* P_t^*, which is defined as the minimum consumption expenditure $C_t = C_{T,t} + P_t C_{N,t}$ such that $C_t^* = 1$ given P_t.

Substituting the Marshalian demands in the real consumption index we get:

$$(aC_t)^a [(1-a)\frac{C_t}{P_t}]^{1-a} = C_t^*$$

by the definition of the consumption-based price index we can write this last expression as:

$$(aP_t^*)^a [(1-a)\frac{P_t^*}{P_t}]^{1-a} = 1$$

From which the solution:

$$P_t^* = P_t^{1-a}[a^{-a}(1-a)^{-(1-a)}]$$

follows.

This last expression allows us to rewrite the budget constraint (2) as[13]

$$Y_t - P_t^* C_t^* - I_t - G_t + r_t B_{t-1} = B_t - B_{t-1}$$

and the utility function as: $U(C_t^*) = [1/(1-\sigma)](C_t^*)^{1-\sigma}$. Following well known methods of optimization we get the euler equation:

$$1 = E_t[\beta(1 + r_{t+1})(\frac{P_t^*}{P_{t+1}^*})(\frac{C_t^*}{C_{t+1}^*})^\sigma]$$

For empirical estimation purposes we can rewrite this last equation in terms of consumption expenditure and the relative price of nontraded goods:

$$1 = E_t[\beta(1 + r_{t+1})(\frac{C_t}{C_{t+1}})^\sigma (\frac{P_t}{P_{t+1}})^{(1-\gamma)(1-a)}]$$

[13] This follows from solving for C_t in the equation: $(aC_t)^a[(1-a)\frac{C_t}{P_t}]^{1-a} = C_t^*$

Which is the euler equation (3).

We can also express the euler equation (3) as[14]

$$\frac{C_t^{-\sigma} P_t^{(\sigma-1)(1-a)}}{1+r_{t+1}} = \beta E_t[C_{t+1}^{-\sigma} P_{t+1}^{(\sigma-1)(1-a)}]$$

Next, we can assume joint log normal distribution between the variables:

$$\begin{bmatrix} -\sigma LnC_{t+1} \\ -[(\sigma-1)(1-a)]LnP_{t+1} \end{bmatrix} \sim N\{\begin{bmatrix} -\sigma E_t[LnC_{t+1}] \\ -[(\sigma-1)(1-a)]E_t[LnP_{t+1}] \end{bmatrix}, \begin{bmatrix} \sigma^2\sigma_c^2 & \sigma[(\sigma-1)(1-a)]\sigma_{cp}^2 \\ \sigma[(\sigma-1)(1-a)]\sigma_{cp}^2 & [(\sigma-1)(1-a)]^2\sigma_p^2 \end{bmatrix}\}$$

We can state that:

$$e^{-\sigma LnC_t} e^{(\sigma-1)(1-a)LnP_t} e^{(1+r_{t+1})} = \beta e^{-\sigma E_t[LnC_{t+1}]+[(\sigma-1)(1-a)]E_t[LnP_{t+1}]} \\ e^{\frac{1}{2}[\sigma^2\sigma_c^2+[(\sigma-1)(1-a)]^2\sigma_p^2+2\sigma[(\sigma-1)(1-a)]\sigma_{cp}}$$

Taking logs and rearranging:

$$\begin{aligned} \sigma E_t[LnC_{t+1} - LnC_t] = E_t[Ln(1+r_{t+1})] + [(\sigma-1)(1-a)]E_t[LnP_{t+1} - LnP_t] \\ + \frac{1}{2}[\sigma_r^2 + \sigma^2\sigma_c^2 + [(\sigma-1)(1-a)]^2\sigma_p^2 \\ +2\sigma[(\sigma-1)(1-a)]\sigma_{cp} - 2[(\sigma-1)(1-a)]\sigma_{rp}] + Ln\beta \end{aligned}$$

And defining $\Delta c_{t+1} = LnC_{t+1} - LnC_t$,$\Delta p_{t+1} = LnP_{t+1} - LnP_t$, and $(1/\sigma) = \gamma$, we find that[15]:

$$E_t\Delta c_{t+1} = \gamma E_t[r_{t+1} + \frac{1-\gamma}{\gamma}(1-a)\Delta p_{t+1}] + constant\ terms$$

Defining the terms in brackets,as the *consumption based real interest rate*: r_{t+1}^*, we finally arrived to equation (4).

Appendix B:Log-linearisation of the budget constraint

Given the budget constraint (8),it is possible to show that: $\sum_{t=0}^{\infty} R_t C_t = \Phi_0$, and $\sum_{t=0}^{\infty} R_t NO_t = \Psi_0$, where Φ_0 and Ψ_0 come from a difference equation such

[14] The gross real interest rate is known by convention as of time t, then the conditional expectation operator does not apply to it. However for empirical estimation, expected inflation is considered, then the conditional expectation over r_{t+1} is taken into account, equation (4) is unchanged in this case.

[15] Also we have been used the approximation $Ln(1+r_{t+1}) \simeq r_{t+1}$.

as: $\Phi_{t+1} = (1 + r_{t+1})(\Phi_t - C_t)$, and $\Psi_{t+1} = (1 + r_{t+1})(\Psi_t - NO_t)$. We are going to show this for the case of consumption expenditure:

We can write the former equation as: $\Phi_t = C_t + \frac{\Phi_t}{1+r_{t+1}}$, iterating forward this equation from period 0, we find that:

$$\Phi_0 = \sum_{t=0}^{T} R_t C_t + R_{T+1} \Phi_{T+1}$$

Now, when T goes to infinity, we find that this equation is:

$$\Phi_0 = \sum_{t=0}^{\infty} R_t C_t$$

as claimed.

We need also to work with the difference equation, notice that it can be written as:

$$\frac{\Phi_{t+1}}{\Phi_t} = (1 + r_{t+1}) + [1 - \frac{C_t}{\Phi_t}]$$

Takings logs this equation becomes:

$$\phi_{t+1} - \phi_t = r_{t+1} + Ln[1 - e^{c_t - \phi_t}] \tag{17}$$

Where $Ln\Phi_t = \phi_t$, $LnC_t = c_t$ and the approximation $Ln(1 + r_{t+1}) \approx r_{t+1}$ have been used.

We need to log-linearise the term $Ln[1 - e^{c_t - \phi_t}]$ around the steady state values of c and ϕ, doing so we find that:

$$Ln[1 - e^{c_t - \phi_t}] \approx k + (1 - \frac{1}{\rho})(c_t - \phi_t)$$

Where $k = Ln\rho - (1 - \frac{1}{\rho})Ln(1 - \rho)$, and $\rho = 1 - e^{(\overline{c} - \overline{\phi})} = 1 - \frac{\overline{C}}{\overline{\Phi}}$, $\overline{c}$ and $\overline{\phi}$ are the steady states values of c and ϕ respectively. Now (10) can be written as:

$$\phi_{t+1} - \phi_t = r_{t+1} + k + (1 - \frac{1}{\rho})(c_t - \phi_t) \tag{18}$$

We need further to use a trick, notice that:

$$\phi_{t+1} - \phi_t \equiv \Delta c_{t+1} + (c_t - \phi_t) - (c_{t+1} - \phi_{t+1})$$

So, we can write (11) as:

$$\Delta c_{t+1} + (c_t - \phi_t) - (c_{t+1} - \phi_{t+1}) \approx r_{t+1} + k + (1 - \frac{1}{\rho})(c_t - \phi_t)$$

We can rewrite this expression as:

$$c_t - \phi_t = \rho(c_{t+1} - \phi_{t+1}) - \rho\Delta c_{t+1} + \rho k + \rho r_{t+1}$$

Iterating forward from period 0, we find that:

$$c_0 - \phi_0 = \rho^T(c_T - \phi_T) + \sum_{t=1}^{T} \rho^t(r_t + \Delta c_t) + \rho k + \rho^2 k + \rho^3 k + ...$$

And assuming that $0 < \rho < 1$ and $T \longrightarrow \infty$, this equation can be written as[16]:

$$c_0 - \phi_0 = \sum_{t=1}^{T} \rho^t(r_t + \Delta c_t) + k_1 \tag{19}$$

Notice that in a completely analogous way, it is possible to derive an equation similar to (19) for net output NO_t[17]:

$$no_0 - \psi_0 = \sum_{t=1}^{\infty} \rho^t(r_t + \Delta no_t) + k_1 \tag{20}$$

Notice that (8) can be written as: $\Psi_0 = \Phi_0 - B_0$, which is the same as: $\frac{\Psi_0}{\Phi_0} = 1 - \frac{B_0}{\Phi_0}$. Taking logs and linearising around the steady state, we find that:

$$\phi_0 - \psi_0 = (1 - \frac{1}{\Omega})(b_0 - \psi_0) + k_1 \tag{21}$$

Where $\Omega = 1 + \frac{\overline{B}}{\Psi}$ is a constant grater than one.

Substitution of (19) and (20) into (21) leads to the following equation[18]:

$$\sum_{t=1}^{\infty} \beta^t[r_t(1 - \frac{1}{\Omega}) - \Delta c_t + \frac{1}{\Omega}\Delta no_t] = c_0 - k_1(1 - \frac{1}{\Omega}) - b_0(1 - \frac{1}{\Omega}) - \frac{1}{\Omega}no_0$$

Finally we must assume that the steady state of net foreign assets at which we are linearising ($\overline{B}$) is zero, hence $\Omega = 1$ and the above equation can be written as:

$$-\sum_{t=1}^{\infty} \beta^t[\Delta no_t - \Delta c_t] = no_0 - c_0$$

Which is equation (9) in the text.

[16] k_1 is an unimportant constant equal to $\frac{\rho k}{1-\rho}$

[17] We assume for simplicity, the same value ρ for the derivation.

[18] The assumption $\rho = \beta$ is imposed in this step.