Probability Distributions in High-Density Dendroenergy Plantations

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Abstract: Six probability density functions were used to model diametric distributions of *Acacia melanoxylon, Eucalyptus camaldulensis,* and *Eucalyptus nitens* being investigated for dendroenergy purposes at three plantation densities (5,000, 7,500, and 10,000 trees ha^{-1}). The Weibull was the function with the best fit, followed in decreasing order by the beta, Johnson S_B, gamma, lognormal, and Johnson S_U functions. Planting density affected the shape and amplitude of the diametric distribution of all species. Increasing stocking made distributions more leptokurtic and narrower. Analyses over time during the first 28 months suggested a stronger effect on diameter distribution form, which was less evident during the early ages of the crop. FOR. SCI. 58(6): 663–672.

Keywords: probability density functions, short rotation crops, bioenergy, stocking, Weibull

DECISIONMAKING IN FORESTRY is often based largely on a plantation's growth and yield (Parresol 2003), and the prediction of these variables has provided a constant focus for studies. At present, mathematical modeling is used to predict the growth and yield of a given plantation, considering stand variables such as stocking, basal area, dominant height, and diametric frequency distribution per unit area (Gove and Patil 1998). Knowing the diametric distribution of a plantation, as well as the distribution of other variables at the tree level (e.g., volume and biomass), constitutes a fundamental tool for decisionmaking in forest management (Zhang et al. 2003, Cao 2004) and is one of the main characteristics used to determine stand state variables such as basal area, volume, or biomass per unit area (Mehtätalo 2004).

Different theoretical probability density functions (PDFs) have been used to describe the diametric distribution of plantations, including the beta (Clutter and Bennett 1965, Lenhart and Clutter 1971, Zohrer 1972, Li et al. 2002), gamma (Nelson 1964), lognormal (Bliss and Reinker 1964), Weibull (Bailey and Dell 1973, Rennolls et al. 1985), and Johnson (Hafley and Schreuder 1977, Zhou and McTague 1996, Kamziah et al. 1999). Clutter and Bennett (1965) were pioneers introducing the diametric distribution methodology in growth and yield models. These authors used the four-parameter beta PDF to describe the distribution of the number of trees per unit area for classes of diameter at breast height (dbh). Since then, PDFs have been widely used to model plantation growth and yield (Lindsay et al. 1996). Bailey and Dell (1973) were the first to use the Weibull PDF as a dbh frequency distribution model, noting that this function had certain advantages over the beta function; because the Weibull PDF has a closed form, it can be expressed as a cumulative density function that can be evaluated without numerical integration and only requires the estimation of three parameters. Hafley and Schreuder (1977) introduced the Johnson S_B distribution (system bounded) (Johnson 1949) to the methodology of diametric distributions. Alzaid and Sultan (2009) analyzed the gamma and lognormal PDFs and recommended the use of the gamma distribution, because of its higher flexibility.

The analysis and selection of the right PDF for a data set has been widely discussed in different research areas. Several authors (Cox 1961, 1962, Chambers and Cox 1967, Atkinson 1969, 1970, Dyer 1973, Chen 1980) highlighted the importance of selecting the optimum PDF because of the better utility of these functions. Moreover, several studies have focused on describing differences among PDFs (Jackson 1969, Dumonceaux and Antle 1973, Bain and Engelhard 1980, Fearn and Nebenzahl 1991, Wiens 1999, Gupta and Kundu 2003a, 2003b, 2004, Alzaid and Sultan 2009). Although numerous research cases in forestry address this topic, they focus nearly exclusively on using PDFs to describe the diametric distributions of traditional plantations (i.e., stands for sawtimber or pulp) and on modeling the diametric distribution at a specific site without comparing PDFs in terms of fit quality and precision (Hafley and Schreuder 1977, Reynolds 1984, Newberry and Burk 1985, Reynolds et al. 1988, Lindsay et al. 1996, Zhang et al. 2003, Cao 2004, Lei 2008). Studies published on this area have not considered plantations for biomass production for dendroenergy grown at high stockings for short rotation harvesting periods.

In Chile, crops for biomass production for dendroenergy

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This article uses metric units; the applicable conversion factors are: millimeters (mm): 1 mm = 0.039 in.; centimeters (cm): 1 cm = 0.39 in.; meters (m): 1 m = 3.3 ft.; hectares (ha): 1 ha = 2.47 ac.; kilograms (kg): 1 kg = 2.2 lb.

have only been recently established in experimental areas, and the potential for establishing these crops on an operational scale is still unknown. However, future potential expansion suggests that development of growth and yield projection methods for these plantations will be important. The objective of our study was to model the changes in diametric distribution of a dendroenergy crop over time by testing six PDFs for three species (*Acacia melanoxylon* R. Br., *Eucalyptus camaldulensis* Dehnh., and *Eucalyptus nitens* Deane & Maiden) at three stockings (5,000, 7,500, and 10,000 trees ha^{-1}). These first functions are to be considered for the development of growth and yield models for dendroenergetic crops.

Materials and Methods

Assay Characteristics and Study Area

The assay was established in August 2007 on the Llohué site, located in the interior dryland of the Bio-Bio Region, Ninhue Township, Chile. The site presents nutritional and water limitations and is predominately characterized by low-yield forest plantations dedicated to pulp or sawtimber production. Soils are granitic and classified as the Cauquenes soil series, which show high susceptibility to erosion and have severe compaction potential. Surface soil horizons are low in organic matter and have a high gravel content that negatively affects initial establishment and plantation growth. The site has up to 5 months of drought per year (Carrasco et al. 1993, Dirección General de Aeronáutica Civil 2010), with an average annual rainfall of 700 mm and temperatures that fluctuate between 0 and 32°C. The terrain has a rolling to abrupt topography but the slope in the study area does not exceed 5%.

Before the beginning of the trial, the site was prepared by extracting the stumps of the previous crop (*Pinus radiata* D. Don) and subsoiled in a grid layout to 80-cm depth using a Caterpillar D8K tractor with 60-cm distance between rows. Weeds were controlled at pre- and postplanting using a chemical mixture containing 4 kg of glyphosate (Roundup Max), 1.5 kg of simazine, and 2.5 kg of atrazine. Postplanting fertilization included 15 g of boronatrocalcite, 75 g of diammonium phosphate, and 25 g of Sul-Po-Mag applied in a circle at 25 cm from the collar of each plant. A perimeter fence made of galvanized mesh 5014 was erected to protect the study area from animals; the fence was buried about 0.3 m and stood approximately 1.2 m above ground level.

The trial was established as a complete randomized block design with three replicates. Blocks were squares of 75 m at each side (5,625 m^2) consisting of nine experimental units of 25 m per side (625 m^2) with 49 measurement trees and a buffer zone to reduce edge effects. Three species (*A. melanoxylon, E. camaldulensis,* and *E. nitens*) were established in each block at three planting densities (5,000, 7,500, and 10,000 trees).

Tree Measurements

Individual tree measurements at each experimental unit were made in October and December 2007, July and December 2008, and July and December 2009. At each measurement time collar diameter (D) at 0.1 m above the ground, dbh once the trees were taller than 1.3 m, crown diameter, and total height of all the trees were measured for each experimental unit. In this study we used only D for all analyses. Data from October 2007 measurements was excluded from the analysis because of the lack of diameter classes at this stage of stand development.

Data Analysis

Diameter data, time of measurement, and planting density were tabulated into classes of amplitude, two for each species. The relative frequency was used to fit the gamma, beta, Weibull, lognormal, Johnson S_B, and Johnson S_U PDFs for each one of these factors (Table 1).

For all models presented in Table 1, x is the diametric class, a is the parameter of location, b is the parameter of scale, and c and d are parameters of shape. As suggested by Frazier (1981), the parameter of location was restricted in all the functions to predict the minimum value of D on the

Table 1. Structures of the PDFs.

Function	PDFs
Gamma	$\frac{1}{\Gamma(c)b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left[-\left(\frac{x-a}{b}\right)\right]; x > a$
Beta	$\frac{(x-a)^{c-1}(a+b-x)^{d-1}}{b^{(c+d-1)}}\cdot\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}; a < x < (a+b)$
Weibull	$\left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1}\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]; x > a$
Lognormal	$\left(\frac{1}{c(x-a)\sqrt{2\pi}}\right)\exp\left[-\frac{(\log(x-a)-b)^2}{2c^2}\right]; x > a$
Johnson S_B	$\frac{c}{b\sqrt{2\pi}}\left[\left(\frac{x-a}{b}\right)\left(1-\frac{x-a}{b}\right)\right]^{-1}\exp\left[-\frac{1}{2}\left(d+c\log\left(\frac{x-a}{a+b-x}\right)\right)^2\right]; a < x < (a+b)$
Johnson S_U	$\frac{c}{b\sqrt{2\pi}}\left(\frac{1}{\sqrt{1+((x-a)/b)^2}}\right)\exp\left[-\frac{1}{2}\left(d+c\log\left(\left(\frac{x-a}{b}\right)+\sqrt{\left(\frac{x-a}{b}\right)+1}\right)\right)^2\right]; x > a$

In all the models, $x > 0$ and the parameters are subject to $a \geq 0$, $b > 0$, $c > 0$, $-\infty < d < \infty$, except in the beta function, where $d > 0$. The expression $\Gamma(\alpha)$ is the gamma function of a constant calculated as $(\alpha - 1)!$

Table 2. **Values of RMSE, EI, and AIC obtained with the three best PDFs according to species and planting density 28 months after field establishment.**

Species	Density (trees ha^{-1})	Beta			Weibull			Johnson S_B		
		RMSE	AIC	EI	RMSE	AIC	EI	RMSE	AIC	EI
A. melanoxylon	5,000	0.0184	−228.0	0.0126	0.0183	−230.4	0.0146	0.0189	−226.5	0.0127
	7,500	0.0323	−154.3	0.0134	0.0305	−158.9	0.0143	0.0352	−150.3	0.0135
	10,000	0.0307	−156.6	0.0196	0.0288	−161.6	0.0206	0.0323	−154.3	0.0197
E. camaldulensis	5,000	0.0138	−270.4	0.0097	0.0142	−270.6	0.0089	0.0146	−266.8	0.0103
	7,500	0.0216	−226.4	0.0149	0.0210	−230.1	0.0153	0.0220	−225.3	0.0153
	10,000	0.0140	−243.5	0.0094	0.0146	−243.9	0.0109	0.0147	−241.1	0.0098
E. nitens	5,000	0.0187	−203.3	0.0147	0.0228	−195.0	0.0188	0.0188	−203.0	0.0148
	7,500	0.0270	−162.5	0.0174	0.0351	−152.5	0.0274	0.0272	−162.2	0.0172
	10,000	0.0302	−171.4	0.0171	0.0280	−177.1	0.0178	0.0312	−169.7	0.0177

RMSE, root mean squared error; AIC, Akaike's index; EI, values from Reynolds et al. (1988).

plantation according to the expression $\hat{a} = 0.5x_{\min}$. In all the functions, the other parameters were estimated by the maximum likelihood method using SAS software (SAS Institute, Inc. 2002).

Evaluation of Functions

The quality of the adjusted functions was evaluated based on the root mean square error (RMSE), the Akaike index (AIC), and the error index (EI) proposed by Reynolds et al. (1988). The RMSE is an indicator of the precision of each PDF. Similar to the RMSE, the EI is an indicator obtained from the sum of weighted absolute differences between observed and estimated distributions. The EI considers the magnitude of the differences and the size class in which they are produced, evaluating the importance of the size class for variables derived from the PDF (e.g., basal area). The AIC, however, was useful for comparison between functions with different numbers of parameters. The RMSE, EI, and AIC were calculated, respectively, using

$$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{n} (F(x_i) - \hat{F}(x_i))^2/n},$$

$$\mathrm{EI} = \sum_{i=1}^{n} w(x_i)\left|F(x_i) - \hat{F}(x_i)\right|,$$

$$\mathrm{AIC} = n \ln(SSE/n) + 2p.$$

In these criteria, $F(x_i)$ and $\hat{F}(x_i)$ are the observed and estimated relative frequencies, respectively, n is the number of diametric classes, p is the number of parameters in the PDF, and $w(x_i)$ is a weighting function that depends on some variable of interest. Following the methodology of Mehtätalo (2004), we also used the basal area of each diametric class type (g_i) as a weighting factor. However, unlike the Mehtätalo (2004) we expressed this as $w(x_i) = g_i/\Sigma_{i=1}^{n} g_i$ in the EI to be an indicator of precision that can be used to compare distributions with different numbers of diametric classes.

Results and Discussion

The maximum likelihood method used for the fit achieved good estimates in all the PDFs analyzed. Zarnoch and Dell (1985) reported similar results from a study based on adult plantations under traditional silviculture. Those authors highlight the capacity of the method to obtain good estimators. The estimation of the parameter of location (a) of the PDFs revealed good results, coinciding with several investigations that have used this methodology (Frazier 1981, Cao 2004, Lei 2008, Jiang and Brooks 2009).

In general, the indicators RMSE, EI, and AIC revealed that the best results in terms of precision and parsimony were obtained with the beta PDF, followed in decreasing order by the Weibull, Johnson S_B, Johnson S_U, gamma, and lognormal functions. Although the gamma, lognormal, and Johnson S_U functions exhibited good precision values (RMSE), they were less precise than the beta, Weibull, and Johnson S_B PDFs (Table 2). Therefore, the remaining analyses were limited to the latter three functions.

Precision varied between planting densities and between species. Twenty-eight months after the beginning of the trial, precision was consistently highest at the lowest planting density (5,000 trees) for the three species and the three best PDFs (Table 2). For *A. melanoxylon* and *E. camaldulensis*, the three PDFs showed the best results in terms of precision, in declining order, at 5,000, 10,000, and 7,500 trees. For *E. nitens,* however, the same result was only observed with the Weibull PDF, whereas the beta and Johnson S_B functions showed greater precision, in declining order, at 5,000, 7,500, and 10,000 trees. For *A. melanoxylon,* the highest precision evaluated by RMSE was achieved with the Weibull PDF at the three planting densities. For *E. camaldulensis,* the best results were obtained with the beta PDF for 5,000 and 10,000 trees, whereas at 7,500 trees, the best precision was obtained with the Weibull PDF. For *E. nitens,* the beta PDF showed the best results for 5,000 and 7,500 trees, and the Weibull PDF showed the highest precision for 10,000 trees.

The effects of planting density and species on the precision of the PDFs are difficult to analyze based on the EI. No clear tendency was observed in terms of the effect of these factors (Table 2). For *A. melanoxylon,* the best results were

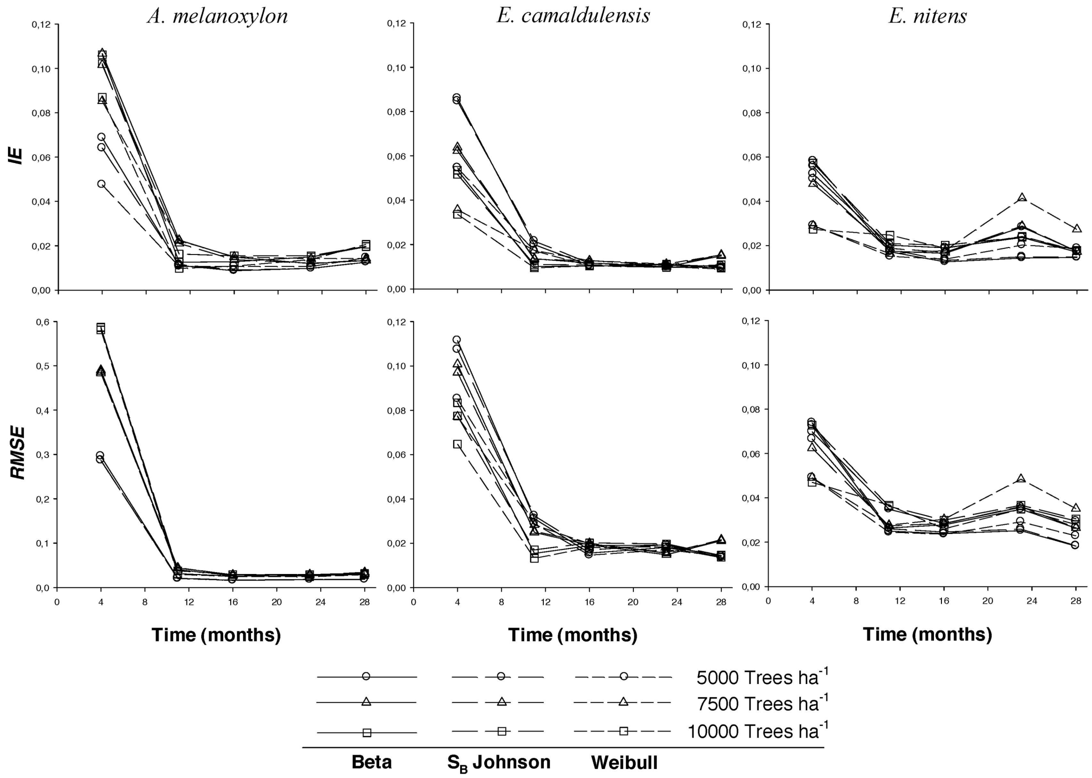

Figure 1. Comparative graphs showing EI and RMSE values obtained for the three best PDFs according to species and planting density over time.

obtained for 5,000 trees with the beta and Johnson S_B PDFs, whereas with the Weibull PDF the best result was obtained for 7,500 trees. For *E. camaldulensis,* the EI was lower for 10,000 trees for the beta and Johnson S_B PDFs, and the best result was obtained with the Weibull PDF for 5,000 trees. For *E. nitens,* there was also no clear effect of stocking on the EI. Here the beta and Johnson S_B PDFs showed the best results for 5,000 trees and the Weibull PDF for 10,000 trees.

In general, as the age of the plantation increases, the precision of the PDFs tended to increase (Figure 1). RMSE and EI were higher from the 4th to the 11th month of the plantation for all three species and the three best PDFs. This is because the distribution had a greater number of diameter classes at 11 months, allowing the fit to improve substantially in relation to the fourth month (Figures 2, 3, and 4).

In most cases, 28 months after trial establishment and with use of EI as the indicator, the beta PDF showed the best results (Table 2). For *A. melanoxylon,* the best results were obtained with the beta PDF at all three planting densities. For *E. camaldulensis,* we observed the best results for 7,500 and 10,000 trees with the beta PDF, whereas for 5,000 trees, the Weibull PDF generated the lowest value of the EI. For *E. nitens* at 5,000 and 10,000 trees, the best EI values were obtained with the beta PDF, but for 7,500 trees, the Johnson S_B PDF was more precise.

The response in terms of precision (RMSE) varied widely between the three best PDFs tested. Nonetheless, the AIC indicated that, in terms of parsimony, the Weibull PDF was advantageous in most cases (Table 2). For *A. melanoxylon* and *E. camaldulensis,* the Weibull PDF showed the lower AIC values at the three planting densities, whereas for *E. nitens,* the best values of the AIC were obtained with the Weibull PDF for 10,000 trees and with the beta PDF for 5,000 and 7,500 trees.

The shape of the distributions changed as the crop grew, independent of the species and planting density (Figures 2, 3, and 4). In the first stages of development, few diameter classes were observed, indicating that the trees were very similar in size and giving rise to a leptokurtic and asymmetric distribution skewed to the right. In the following stages, the distribution became mesokurtic and symmetric. Finally, because of heightened competition levels, tree development is more limited, resulting in platykurtic, asymmetric distributions skewed to the left. The change in the shape of the distribution has been studied previously in population ecology, as distributions often become more skewed to the left as a crop's age increases (White and Harper 1970). Bullock and Burkhart (2005) analyzed growth of *Pinus taeda* L. juvenile plantations, finding that as the plantation aged, the shape parameter (c) of the Weibull PDF tended to increase, causing the distribution to show a greater negative skew, as occurred in this study.

The Weibull PDF accurately modeled the change in the shape of the distribution as the crop grew. In fact, this PDF was able to represent all acquired shapes by the observed distributions over time. The beta and Johnson S_B PDFs presented problems for modeling distributions that were too platykurtic, as seen for *A. melanoxylon* at 5,000 trees; these functions do not fit well at the upper end of the distribution (Figure 2). For *E. camaldulensis* and *E. nitens,* the estimate of the Weibull PDF differed slightly in shape from that of

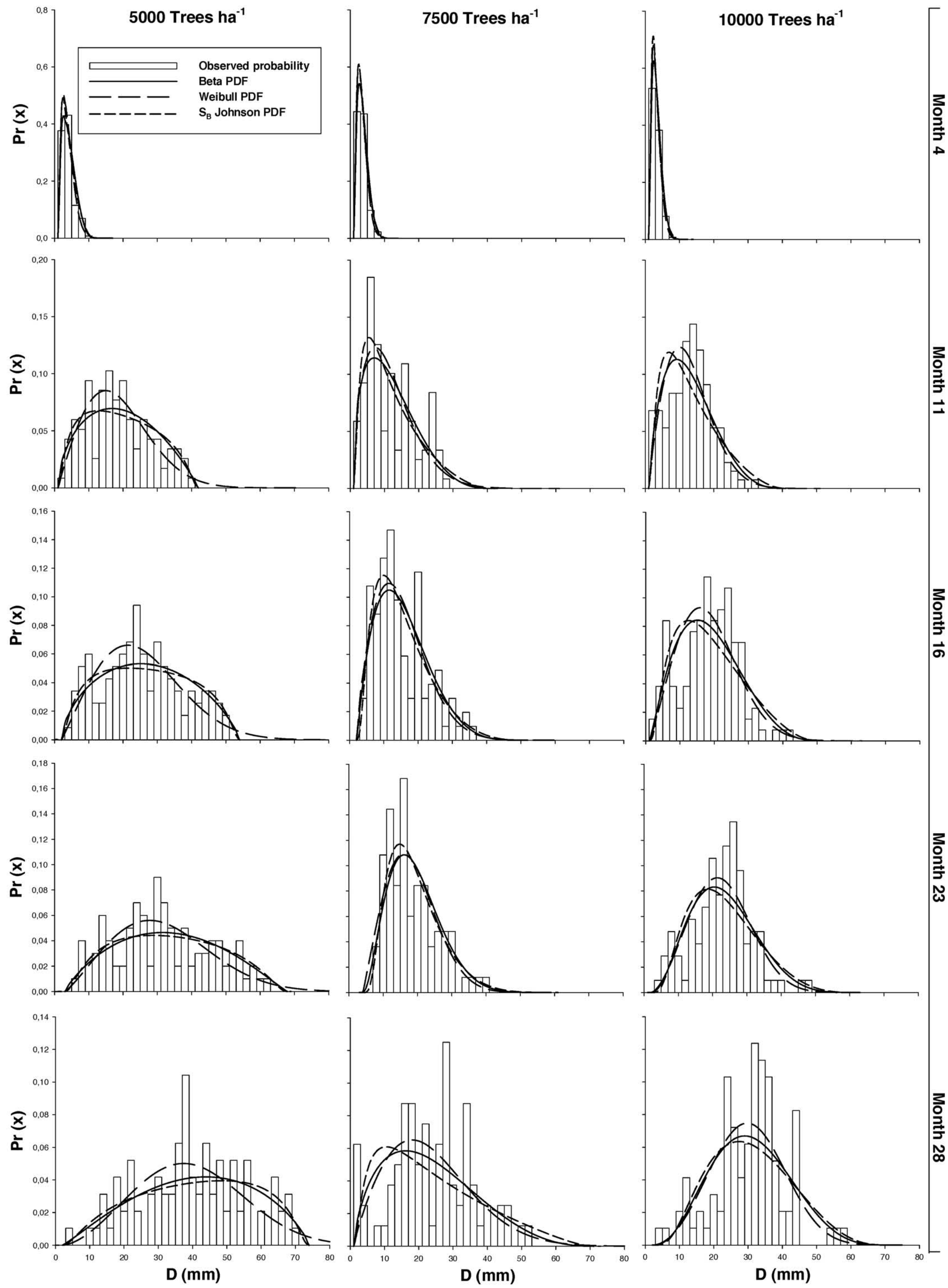

Figure 2. Distributions of observed and estimated probability for *A. melanoxylon* over time. The observed distribution represent the average of the plots measured.

the beta and Johnson S_B PDFs. The Weibull PDF represented the middle section of the distribution better than the beta and Johnson S_B PDFs. This observation exemplifies the flexibility of the Weibull PDF, which requires the estimation of only three parameters, compared with the four parameters that must be estimated for the beta and Johnson S_B PDFs, based on the parsimony evidenced by the AIC. This characteristic has been underscored by several authors, who also mentioned the capacity of the Weibull distribution to describe a wide range of unimodal distributions (e.g., j-inverted, exponential, and normal) and highlighted the capacity of this PDF to be expressed in its closed shape as an cumulative distribution function (Bailey and Dell 1973, Schreuder and Swank 1974, Schreuder et al. 1979, Little 1983, Rennolls et al. 1985, Mabvurira et al. 2002, Lei 2008).

Stocking also affected the shape and amplitude of the diametric distribution. *A. melanoxylon,* for example, showed important changes in the distribution shape as stocking

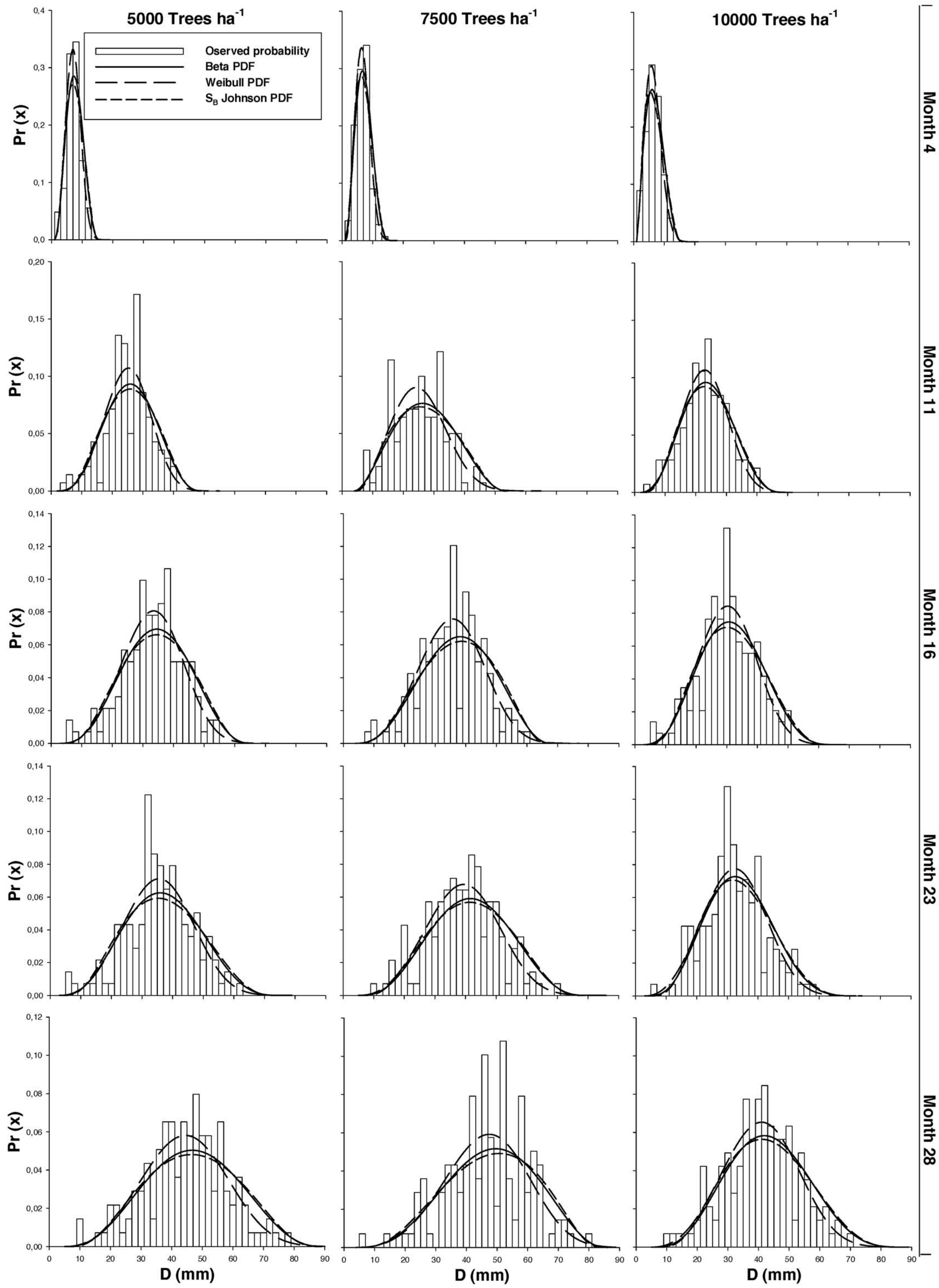

Figure 3. Distributions of observed and estimated probability for *E. camaldulensis* over time. The observed distribution represent the average of the plots measured.

increased (Figure 2). The distribution for 5,000 trees was more platykurtic than the distribution observed for 10,000 trees. Furthermore, for 5,000 trees, the number of diameter classes was greater because these experimental units had lower levels of competition, as manifested by the presence of larger trees. The three species studied here exhibited this effect, which was observed in the first stages of growth, although to a lesser degree. This finding agrees with the findings of White and Harper (1970), who reported distributions further skewed to the right in forest populations with high levels of competition. Bullock and Burkhart (2005) reported the same tendency in their analysis of juvenile plantations of *Pinus taeda* at several planting densities (747–6,727 trees), finding that as stocking increases, the value of the parameter that defined the shape of the Weibull PDF diminished (i.e., parameter *c*), thereby causing a positive skew of the distribution, which increases with the age of the crop. Although no studies have been published

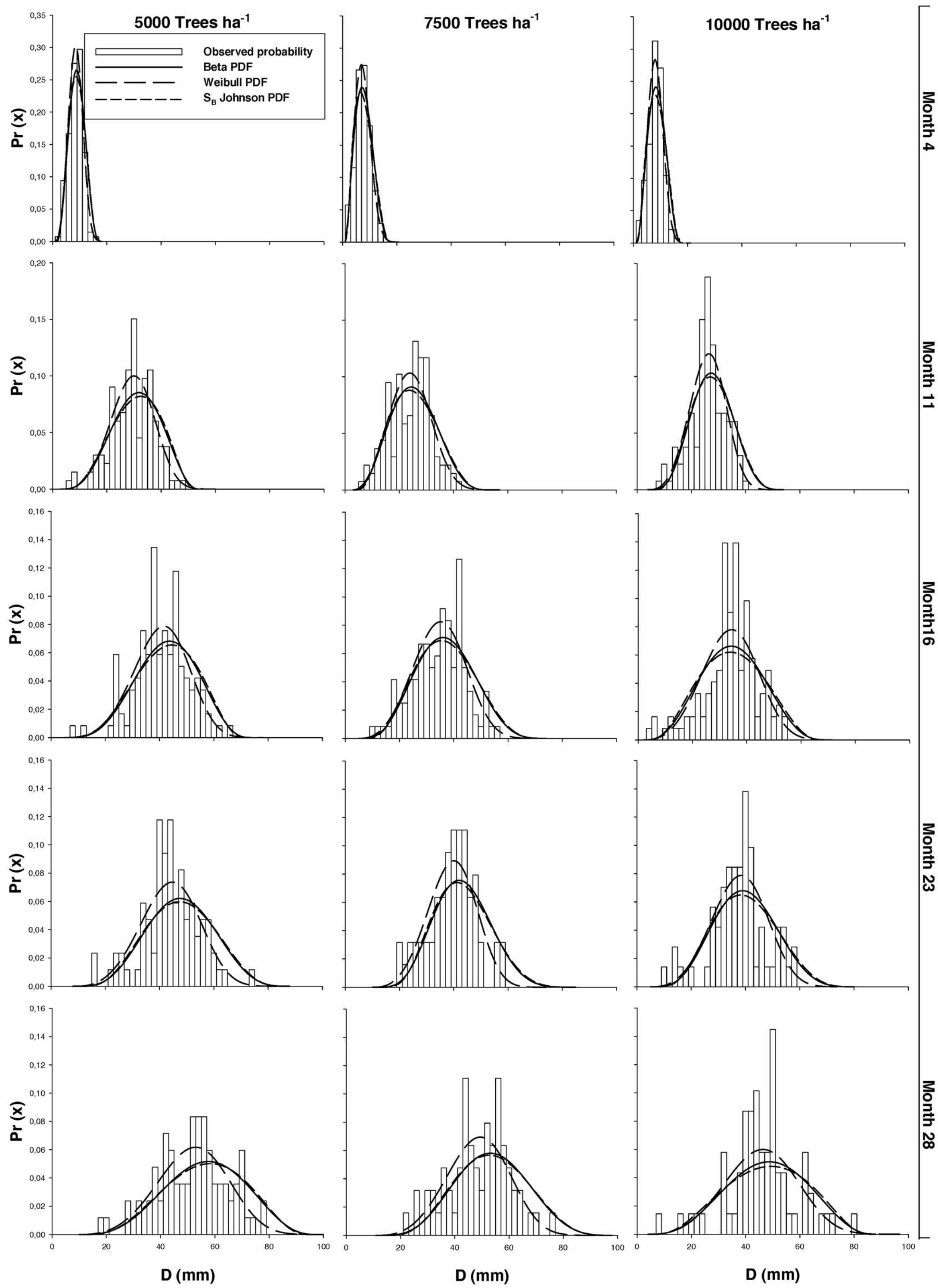

Figure 4. Distributions of observed and estimated probability for *E. nitens* over time. The observed distribution represent the average of the plots measured.

for diameter distributions of dendroenergetic crops, several authors results agree with the results obtained, suggesting that diameter size decreases with greater planting densities (Bernardo et al. 1998, Pinkard and Neilsen 2003, Harmand et al. 2004, Barton and Montagu 2006, Wilkinson et al. 2007) and offering clear evidence of the effect on diameter distribution caused by high levels of competition at early stages of development.

Some stand variables were closely related to the parameters estimated for the Weibull PDF for the three species analyzed (Figure 5). The parameters of location (a) and scale (b) maintained a clear direct relationship with mean height and quadratic mean diameter but showed little relationship to stocking and showed a tendency to decrease as stocking increased. The parameter of shape (c) showed little relationship with these stand variables. The parameters increased slightly along with mean height and quadratic mean diameter, whereas the real number of trees did not

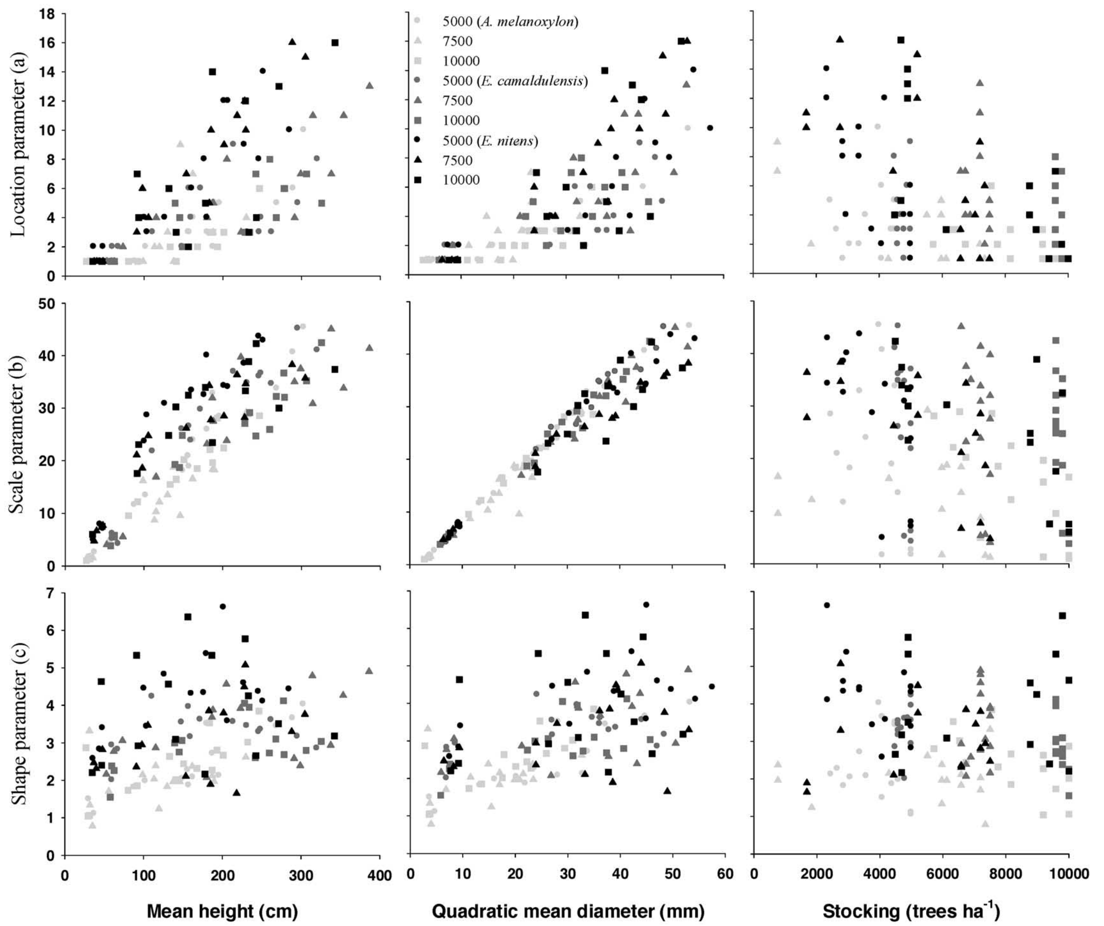

Figure 5. Relationship between parameters of the Weibull PDF and stand variables for the three species at the three stockings.

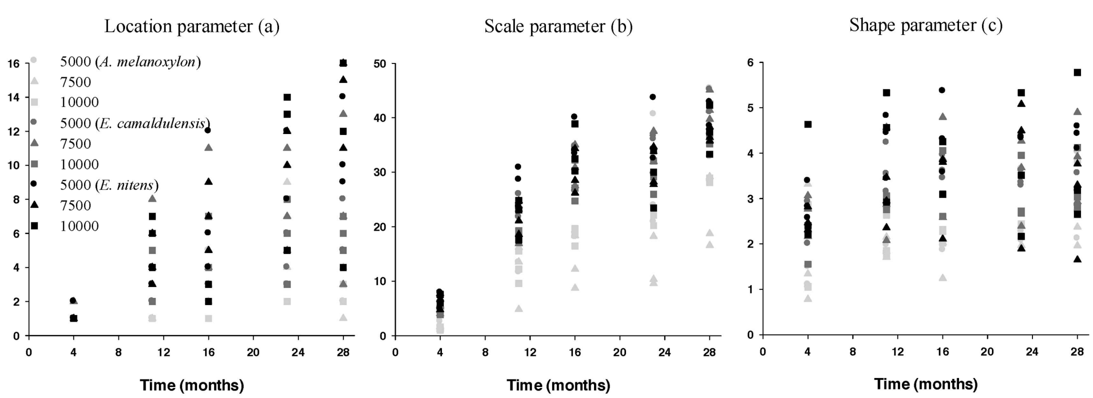

Figure 6. Evolution of the parameters estimated with the Weibull PDF over time for the three species at the three stockings.

seem to explain the parameter of shape. In this study we observed a direct relationship between the estimated Weibull PDF parameters and the age of the crop for all the three species (Figure 6). Bullock and Burkhart (2005) found the same relationships studying diameter distributions in juvenile plantations of *Pinus taeda* L. These authors showed that both the scale (*b*) and the shape (*c*) parameters incremented as the age of the crop increased. However, the parameter of shape showed a lower slope, similar to our results.

Conclusion

The precision of the Weibull, beta, and Johnson S_B PDFs was similar when evaluated using the RMSE. In most cases, the beta PDF showed the highest precision, evaluated using the EI. According to the indicator of precision in terms of parsimony (AIC), the Weibull PDF produced the best results in relation to the other functions. Therefore, despite the lower weighted estimation errors of the beta PDF, the Weibull function presented the best fit throughout the diametric range, offering advantages in terms of the estimation of the number of trees per unit area. Moreover, the Weibull PDF was more flexible than the other functions tested, fitting all distribution shapes. Therefore, the Weibull PDF should be recommended for modeling diametric distributions in dendroenergetic crops similar to those tested in this study. Our results suggest that this function is more adequate for modeling probability distributions in terms of precision and parsimony to implement growth and yield dendroenergetic biomass crops simulation models.

The shape of the diameter distribution changed with the age of the stand independent of the species. The distribution shifted from leptokurtic, asymmetric, and skewed to the right in the first stages of development to more platykurtic, asymmetric, and skewed to the left. In turn, the amplitude of the diametric distribution was directly related to the age of the plantation with older crops showing greater amplitude.

Although stocking affected the shape and amplitude of the diametric distributions for the three species analyzed, the effect on the shape of the distribution is not yet evident. The distribution shifted from platykurtic to leptokurtic only for *A. melanoxylon*, as planting density incremented. Broader diameter amplitude was observed for lower stockings, an effect that incremented with the age of the crop.

The Weibull PDF parameters evolved in direct relationship to the crop age and could be estimated from stand variables. Mean height and quadratic mean diameter were useful variables for predicting the Weibull PDF parameters. Conversely, the number of trees per hectare did not seem to be a useful variable for predicting these parameters.

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